Substituting the values in (*i*), we get

$$q = \frac{(1300 - 40)}{0.4843} = \mathbf{2601.7\ W/m^2} \qquad \textbf{(Ans.)}$$

(*ii*) Maximum temperature to which common brick is subjected, t_2 :

$$q = h_i (t_{hf} - t_1) = \frac{(t_1 - t_2)}{L_A / k_A}$$

or, $$2601.7 = 33.9\,(1300 - t_1)$$

or, $$t_1 = 1300 - \frac{2601}{33.9} = 1223.27°\text{C}$$

and, $$2601.7 = \frac{(1223.27 - t_2)}{(0.22/3.5)}$$

or, $$t_2 = 1223.27 - 2601.7 \times \frac{0.22}{3.5}$$

$$= \mathbf{1059.73°C}\ \ \textbf{(Ans.)}$$

2.6. HEAT CONDUCTION THROUGH HOLLOW AND COMPOSITE CYLINDERS

2.6.1. HEAT CONDUCTION THROUGH A HOLLOW CYLINDER

Case I. Uniform conductivity :

Refer to Fig. 2.45. Consider a hollow cylinder made of material having constant thermal conductivity and insulated at both ends.

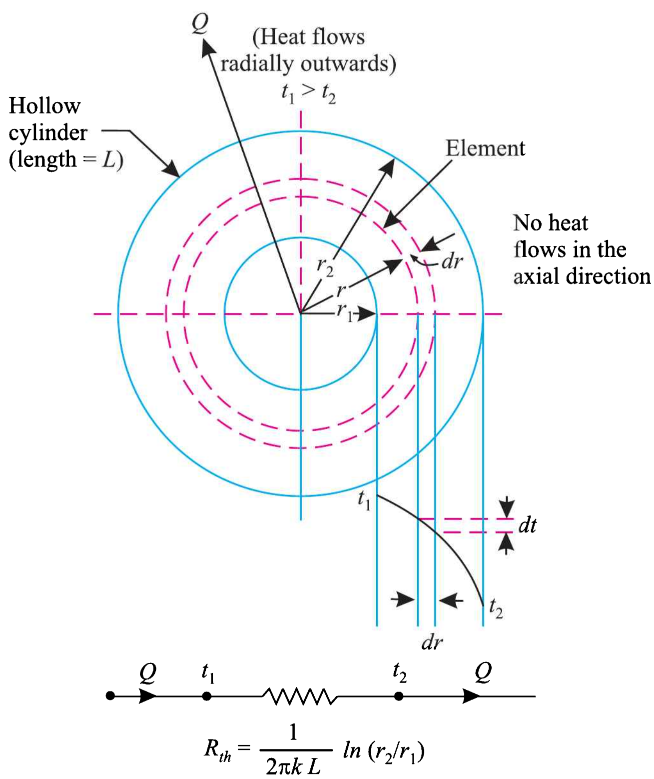

Fig. 2.45.

Let, r_1, r_2 = Inner and outer radii;

t_1, t_2 = Temperatures of inner and outer surfaces, and

k = Constant thermal conductivity within the given temperature range.

The general heat conduction equation in cylindrical coordinates is given by,

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}\right] + \frac{q_g}{k} = \frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau} \qquad \text{...(Eqn. 2.22)}$$

For steady state $\left(\frac{\partial t}{\partial \tau} = 0\right)$, unidirectional $[t \neq f(\phi, x)]$ heat flow in radial direction and with no internal heat generation ($q_g = 0$), the above equation reduces to

$$\frac{d^2 t}{dr^2} + \frac{1}{r}\cdot\frac{dt}{dr} = 0$$

or,
$$\frac{1}{r}\cdot\frac{d}{dr}\left[r\cdot\frac{dt}{dr}\right] = 0$$

Since, $\frac{1}{r} \neq 0$, therefore, $\frac{d}{dr}\left(r\cdot\frac{dt}{dr}\right) = 0$

or,
$$r\cdot\frac{dt}{dr} = C \text{ (a constant)} \qquad \text{...(2.54)}$$

Integrating the above equation, we get

$$t = C\ln(r) + C_1 \qquad \text{...(2.55)}$$

(where C_1 = Constant of integration)

Using the following boundary conditions, we have :

At $r = r_1$, $t = t_1$; At $r = r_2$, $t = t_2$

$\therefore$
$$t_1 = C\ln(r_1) + C_1 \qquad \text{...}(i)$$
$$t_2 = C\ln(r_2) + C_1 \qquad \text{...}(ii)$$

From (*i*) and (*ii*), we have

$$C = -\frac{(t_1 - t_2)}{\ln(r_2/r_1)} \text{ and } C_1 = t_1 + \frac{t_1 - t_2}{\ln(r_2/r_1)}\ln(r_1) \qquad \text{...(2.56)}$$

Substituting the values of these constants in eqn. (2.55), we have

$$t = t_1 + \frac{t_1 - t_2}{\ln(r_2/r_1)}\ln(r_1) - \frac{(t_1 - t_2)}{\ln(r_2/r_1)}\cdot\ln(r) \qquad \text{...(2.57)}$$

[Equation 2.57 is the expression for *temperature distribution* in a hollow cylinder].

or,
$$(t - t_1)\ln(r_2/r_1) = (t_1 - t_2)\ln(r_1) - (t_1 - t_2)\ln(r)$$
$$= (t_2 - t_1)\ln(r) - (t_2 - t_1)\ln(r_1) = (t_2 - t_1)\ln(r/r_1)$$

or,
$$\frac{t - t_1}{t_2 - t_1} = \frac{\ln(r/r_1)}{\ln(r_2/r_1)} \text{ (Dimensionless form)} \qquad \text{...(2.58)}$$

From the above equation, the following points are worth noting :

(*i*) The *temperature distribution is logarithmic* (not linear as in the case of plane wall).

(*ii*) *Temperature at any point* in the cylinder can be expressed as a *function of radius only.* Isotherms (or lines of constant temperatures) are then concentric circles lying between the inner and outer boundaries of the hollow cylinder.

(*iii*) The temperature profile [Eqn. (2.57)] is *nearly linear for values of* (r_2/r_1) *of the order of unity, but decidely non-linear for large values of* (r_2/r_1)

Determination of conduction heat transfer rate (Q) :

The conduction heat transfer rate is determined by utilizing the temperature distribution [Eqn. (2.57)] in conjunction with Fourier's equation as follows :

$$Q = -kA\frac{dt}{dr}$$

$$= -kA\frac{d}{dr}\left[t_1 + \frac{t_1 - t_2}{\ln(r_2/r_1)}\ln(r_1) - \frac{(t_1 - t_2)}{\ln(r_2/r_1)}\ln(r)\right]$$

[Substituting the value of t from Eqn. (2.57)]

$$= -k(2\pi r.L)\left[\frac{-(t_1 - t_2)}{r\ln(r_2/r_1)}\right]$$

$$= 2\pi k L\frac{(t_1 - t_2)}{\ln(r_2/r_1)} = \frac{(t_1 - t_2)}{\dfrac{\ln(r_2/r_1)}{2\pi k L}}\left[=\frac{\Delta t}{R_{th}}\right] \quad \left(\text{where, } R_{th} = \frac{\ln(r_2/r_1)}{2\pi k L}\right)$$

Hence,

$$Q = \frac{(t_1 - t_2)}{\dfrac{\ln(r_2/r_1)}{2\pi k L}} \quad ...(2.59)$$

Alternative method :

Refer to Fig. 2.45 Consider an element at radius ‘r’ and thickness ‘*dr*’ for a length of the hollow cylinder through which heat is transmitted. Let *dt* be the temperature drop over the element.

Area through which heat is transmitted, $A = 2\pi\ r.\ L$.

Path length = *dr* (over which the temperature falls is *dt*)

$$\therefore \quad Q = -kA\cdot\left(\frac{dt}{dr}\right)$$

$$= -k.2\pi r.L\frac{dt}{dr} \text{ per unit time}$$

Induction eddy current heating.

or, $$Q.\frac{dr}{r} = -k.2\pi L.dt$$

Integrating both sides, we get

$$Q\int_{r_1}^{r_2}\frac{dr}{r} = -k.2\pi L\int_{t_1}^{t_2}dt$$

or, $$Q[\text{In}(r)]_{r_1}^{r_2} = -k.2\pi L[t]_{t_1}^{t_2}$$

or, $$Q.\ln(r_2/r_1) = -k.2\pi L(t_2 - t_1) = k.2\pi L(t_1 - t_2)$$

$\therefore$ $$Q = \frac{k.2\pi L(t_1 - t_2)}{\ln(r_2/r_1)} = \frac{(t_1 - t_2)}{\left[\dfrac{\ln(r_2/r_1)}{2\pi k L}\right]} \qquad ...(2.60)$$

Case II. Variable thermal conductivity :

A. *Temperature variation in terms of interface temperatures* (t_1, t_2) :

The heat flux equation is given by

$$Q = -kA\frac{dt}{dr} \quad [\text{where } k = k_0(1 + \beta t)]$$

$$= -k_0(1 + \beta t)\, 2\pi.r.L.\frac{dt}{dr}$$

or, $$Q.\frac{dr}{r} = -k_0.2\pi L(1 + \beta t)\,dt \qquad ...(2.61)$$

Integrating both sides, we have

$$Q\int_{r_1}^{r_2}\frac{dr}{r} = -k_0.2\pi L\int_{t_1}^{t_2}(1 + \beta t)\,dt$$

$$Q[\ln(r)]_{r_1}^{r_2} = -k_0.2\pi L\left[t + \beta.\frac{t^2}{2}\right]_{t_1}^{t_2}$$

$$Q.\ln(r_2/r_1) = -k_0.2\pi L\left[(t_2 - t_1) + \frac{\beta}{2}(t_2^2 - t_1^2)\right]$$

$$= -k_0.2\pi L\left[(t_2 - t_1) + \frac{\beta}{2}(t_2 + t_1)(t_2 - t_1)\right]$$

$$= k_0.2\pi L\left[1 + \frac{\beta}{2}(t_1 + t_2)\right][t_1 - t_2]$$

$\therefore$ $$Q = \frac{k_0.2\pi L\left[1 + \frac{\beta}{2}(t_1 + t_2)\right][t_1 - t_2]}{\ln(r_2/r_1)} \qquad ...(2.62)$$

Integrating between r_1 and r, we obtain

$$Q = \frac{k_0.2\pi L\left[1 + \frac{\beta}{2}(t_1 + t)\right][t_1 - t]}{\ln(r/r_1)} \qquad ...(2.63)$$

Equating eqns. (2.62) and (2.63), we get

$$\frac{k_0.2\pi L\left[1 + \frac{\beta}{2}(t_1 + t_2)\right][t_1 - t_2]}{\ln(r_2/r_1)} = \frac{k_0.2\pi L\left[1 + \frac{\beta}{2}(t_1 + t)\right][t_1 - t]}{\ln(r/r_1)}$$