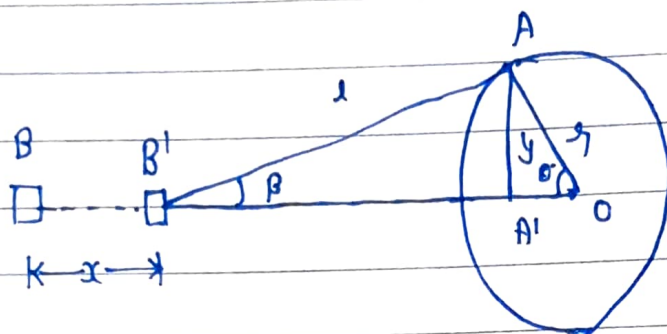


UNIT-1

Analytical method for finding out distance displacement



$$BB' = OB - OB'$$

$$x = (l+r) - (OA' + A'B')$$

$$x = (l+r) - (r \cos \theta + l \cos \beta)$$

$$x = (nr+r) - (r \cos \theta + nr \cos \beta)$$

$$x = r \left[(n+1) - (\cos \theta + n \cos \beta) \right]$$

①

$$\cos \beta = \sqrt{1 - \sin^2 \beta}$$

$$= \sqrt{1 - \frac{y^2}{r^2}}$$

$$= \sqrt{1 - \frac{r^2 \sin^2 \theta}{r^2}}$$

$$\sin \theta = \frac{AA'}{OA}$$

$$\sin \theta = \frac{y}{r}$$

$$y = r \sin \theta$$

$$\cos \beta = \sqrt{1 - \frac{\sin^2 \theta}{n^2}}$$

$$\cos \beta = \sqrt{\frac{n^2 - \sin^2 \theta}{n^2}}$$

$$\cos \theta = \frac{OA'}{r}$$

$$OA' = r \cos \theta$$

$$\cos \beta = \frac{A'B'}{l}$$

$$A'B' = \frac{l \cos \beta}{n}$$

$$n = \frac{l}{r}$$

$$l = nr$$

in equation (1)

$$x = r \left[(n+1) - \left(\cos \theta + n \sqrt{\frac{n^2 - \sin^2 \theta}{n^2}} \right) \right]$$

$$x = r \left[(n+1) - \left(\cos \theta + \frac{n}{n} \sqrt{n^2 - \sin^2 \theta} \right) \right]$$

$$x = r \left[n+1 - \cos \theta - \sqrt{n^2 - \sin^2 \theta} \right]$$

$$x = r \left[(1 - \cos \theta) + n - \sqrt{n^2 - \sin^2 \theta} \right]$$

if $\sin \theta$ is neglected ...

$$x = r \left[(1 - \cos \theta) + n - n \right]$$

$$x = r (1 - \cos \theta)$$

• Velocity \Rightarrow

$$v = \frac{dx}{dt}$$

$$v = \frac{dx}{d\theta} \cdot \frac{d\theta}{dt}$$

$$v = \frac{d}{d\theta} \left[r (1 - \cos \theta) + n - \sqrt{n^2 - \sin^2 \theta} \right] \frac{d\theta}{dt}$$

$$v = r \left[(0 + \sin \theta) + 0 - \frac{d}{d\theta} (n^2 - \sin^2 \theta)^{1/2} \right] \omega$$

$$V = \mu \left[\sin \theta \cdot \frac{1}{2} (n^2 - \sin^2 \theta)^{-1/2} \cdot \frac{d}{d\theta} (-\sin^2 \theta) \right] \omega$$

$$V = \mu \left[\sin \theta \cdot -\frac{1}{2} (n^2 - \sin^2 \theta)^{-1/2} \times (-2 \sin \theta \cdot \cos \theta) \right] \omega$$

$$V = \mu \left[\sin \theta \cdot -\frac{1}{2} \frac{1}{\sqrt{n^2 - \sin^2 \theta}} \times (-\sin 2\theta) \right] \omega$$

$$V = \mu \left[\sin \theta \cdot \frac{\sin 2\theta}{2 \sqrt{n^2 - \sin^2 \theta}} \right] \omega$$

$\sin^2 \theta = \frac{1}{n^2}$
neglect

$$V = \omega \mu \left[\sin \theta + \frac{\sin 2\theta}{2n} \right]$$