INPUT

3.0 PRESSURE MEASUREMENT

3.1 PIEZOMETER (Pressure Tube)

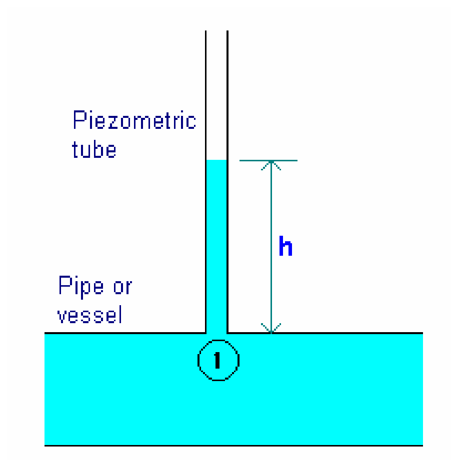


Figure 3.1: Piezometer inside a pipe

A Piezometer is used for measuring pressure inside a vessel or pipe in which liquid is there. A tube may be attached to the walls of the container (or pipe) in which the liquid resides so that liquid can rise in the tube. By determining the height to which liquid rises and using the relation $p_1 = \rho gh$, gauge pressure of the liquid can be determined. It is important that the opening of the device is to be tangential to any fluid motion, otherwise an erroneous reading will result.

Although the Piezometer tube is a very simple and accurate pressure measuring device, it has several disadvantages. It is only suitable if the pressure in the container (pipe or vessel) is greater than the atmospheric pressure (otherwise air would be sucked into system), and the pressure to be measured must be relatively small so that the required height of column is reasonable. Also, the fluid in the container in which the pressure is to be measured must be a liquid rather than a gas.

Example 3.1

A pressure tube is used to measure the pressure of oil (mass density, $640\ kg/m^3$) in a pipeline. If the oil rises to a height of 1.2 above the centre of the pipe, what is the gauge pressure in N/m^2 at that point? (gravity = 9.81 m/s^2)

Solution to Example 3.1

Putting $\rho = 640 kg/m^3$
and $h = 1.2m$

We know that, $p = \rho g h$

So, $p = 640 \times 9.81 \times 1.2$

$p = \underline{\underline{7.55\ kN/m^2}}$

3.2 BAROMETERS

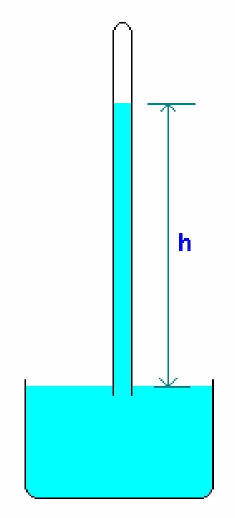


Figure 3.2: Mercury Barometer

A Barometer is a device used for measuring atmospheric pressure. A simple Barometer consists of a tube of more than 30 *inch* (760 *mm*) long inserted into an open container of mercury with a closed and evacuated end at the top and open tube end at the bottom and with mercury extending from the container up into the tube. Strictly, the space above the liquid cannot be a true vacuum. It contains mercury vapour at its saturated vapour pressure, but this is extremely small at room temperatures (e.g. 0.173 *Pa* at 20^oC). The atmospheric pressure is calculated from the relation $p_{atm} = \rho gh$ where ρ is the density of fluid in the barometer. There are two types of Barometer; Mercury Barometer and Aneroid Barometer.

Example 3.2

Describe with a sketch, one method of measuring atmospheric pressure.

Solution to Example 3.2

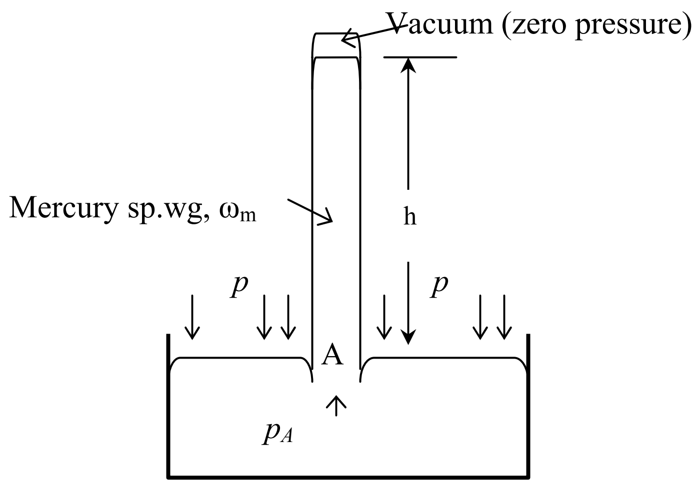


Figure 3.3

A Mercury Barometer in its simplest form consists of a glass tube, about 1 *m* long and closed at one end, which is completely filled with mercury and inverted in a bowl of mercury (Figure 3.3). A vacuum forms at the top of the tube and the atmospheric pressure acting on the surface of the mercury in the bowl supports a column of mercury in the tube height, *h*.

Example 3.3

What is the atmospheric pressure in N/m^2 if the level of mercury in a Barometer (Figure 3.3) tube is 760 *mm* above the level of the mercury in the bowl? Given the specific gravity of mercury is 13.6 and specific weight of water is $9.81 \times 10^3 \, N/m^3$.

Solution to Example 3.3

If A is a point in the tube at the same level as the free surface outside, the pressure p_A at A is equal to the atmospheric pressure p at the surface because, if the fluid is at rest, pressure is the same at all points at the same level.

The column of mercury in the tube is in equilibrium under the action of the force due to p_A acting upwards and its weight acting downwards; there is no pressure on the top of the column as there is a vacuum at the top of the tube.

So,

$$p_A \times area\ of\ column\ A\ =\ specific\ weight\ of\ mercury \times specific\ weight\ of\ water$$

$$p_A \times\ A = \omega_m \times ah$$

or

$$p_A = \omega_m \times h$$

Putting

$$h = 760\ mm = 0.76\ mm$$

While

$$\omega_m = specific\ gravity\ of\ mercury \times specific\ weight\ of\ water$$

$$\omega_m = 13.6 \times 9.81 \times 10^3\ N/m^2$$

From

$$p_A = \omega_m \times h$$

So

$$p_A = 13.6 \times 9.81 \times 10^3 \times 0.76\ N/m^2$$

$$= \underline{\underline{101.3\ kN/m^2}}$$

INPUT

3.4 MANOMETERS

The relationship between pressure and head is utilized for pressure measurement in the manometer or liquid gauge. We can measure comparatively high pressures and negative pressures with the manometer. The following are a few types of manometers:

a) Simple manometer,
b) Differential manometer and
c) Inverted differential manometer.

3.4.1 SIMPLE MANOMETER

A simple manometer is a tube bent in U-shape. One end of which is attached to the gauge point and the other is open to the atmosphere as shown in (Figure 3.5)

The liquid used in the bent tube or simple manometer is generally mercury which is 13.6 times heavier than water. Hence, it is also suitable for measuring high pressure.

Now consider a simple manometer connected to a pipe containing a light liquid under high pressure. The high pressure in the pipe will force the heavy liquid, in the left-hand limb of the U-tube, to move downward. This downward movement of the heavy liquid in the left-hand limb will cause a corresponding rise of the heavy liquid in the right-hand limb. The horizontal surface, at which the heavy and light liquid meet in the left-hand limb is known as a common surface or datum line. Let B-C be the datum line, as shown in Figure 3.5.

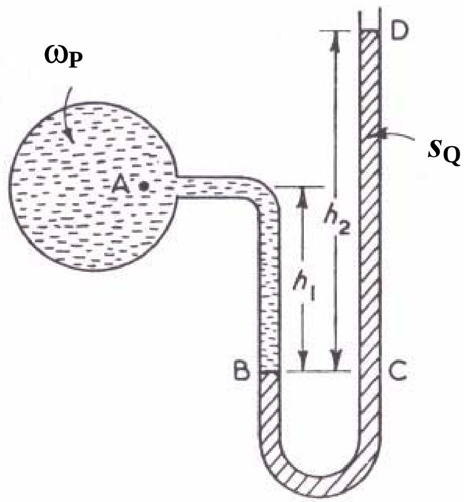


Figure 3.5

Let h_1 = Height of the light liquid in the left-hand limb above the common surface in cm.

h_2 = Height of the heavy liquid in the right-hand limb above the common surface in cm.

p_A = Pressure in the pipe, expressed in terms of head of water in cm.

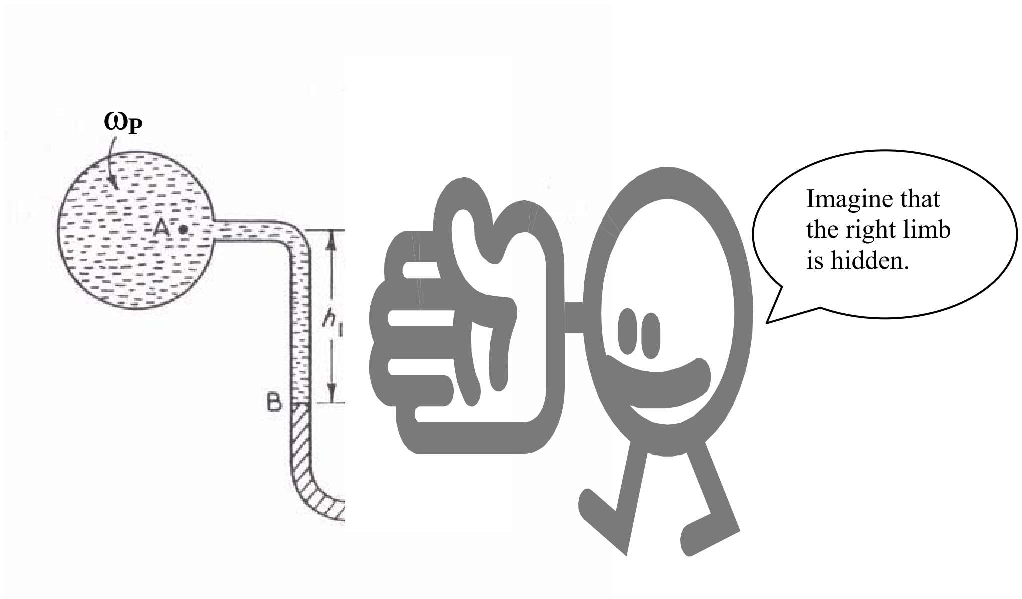
ω_P = Specific weight of the light liquid

s_Q = Specific gravity of the heavy liquid.

The pressure in the left-hand limb and the right-hand limb above the datum line is equal.

Pressure p_B at B = Pressure p_C at C

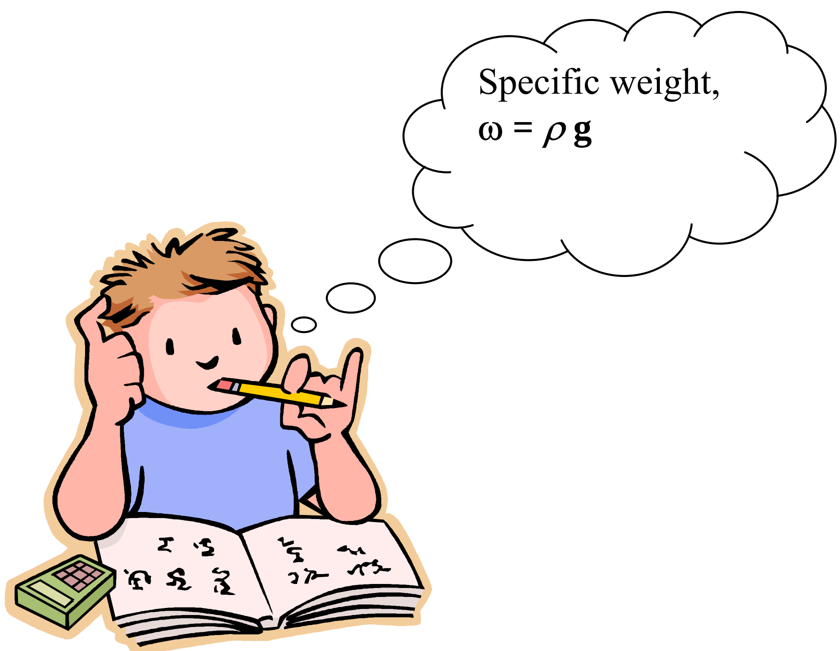
Pressure in the **left-hand limb** above the datum line



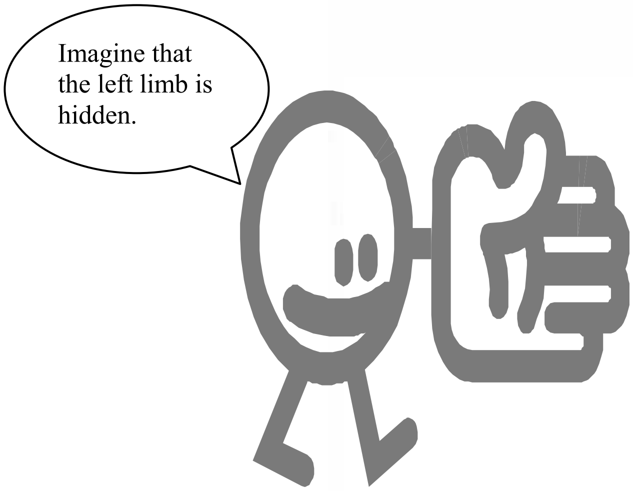
p_B = Pressure, p_A at A + Pressure due to depth, h_1 of fluid P

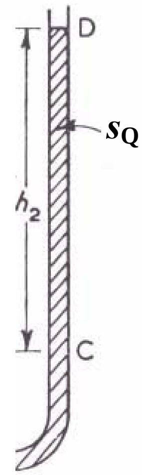
$= p_A + \omega_P h_1$

$= p_A + \rho_P g h_1$



Thus pressure in the **right-hand limb** above the datum line;





p_C = Pressure p_D at D + Pressure due to depth h_2 of liquid Q

But p_D = Atmospheric pressure = Zero gauge pressure

And so, $p_C = 0 + \omega_Q h_2$

$= 0 + \rho_Q g h_2$

Since $p_B = p_C$,

$$p_A + \rho_P g h_1 = \rho_Q g h_2$$

so,

$$p_A = \rho_Q g h_2 - \rho_P g h_1$$

Example 3.4

A U-tube manometer similar to that shown in Figure 3.6 is used to measure the gauge pressure of water (mass density ρ = 1000 kg/m^3). If the density of mercury is 13.6 × 10^3 kg/m^3, what will be the gauge pressure at A if h_1 = 0.45 m and D is 0.7 m above BC.

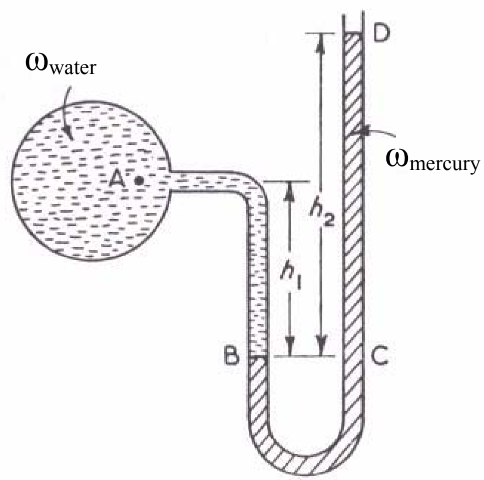


Figure 3.6

Solution to Example 3.4

Considering

$\rho_Q = 13.6 \times 10^3 \ kg/m^3$

$\rho_P = 1.0 \times 10^3 \ kg/m^3$

$h_1 = 0.45 \ m$

$h_2 = 0.7 \ m$

the pressure at left-hand limb;

p_B = Pressure, p_A at A + Pressure due to depth, h_1 of fluid P

= $p_A + \omega_P h_1$

= $p_A + \rho_P g h_1$

the pressure at right-hand limb;

p_C = Pressure p_D at D + Pressure due to depth h_2 of liquid Q

$p_C = 0 + \omega_Q h_2$

$= 0 + \rho_Q g h_2$

Since $p_B = p_C$

$p_A + \rho_P g h_1 = \rho_Q g h_2$

$$p_A = \rho_Q g h_2 - \rho_P g h_1$$
$$= 13.6 \times 10^3 \times 9.81 \times 0.7 - 1.0 \times 10^3 \times 9.81 \times 0.45$$
$$= 88976.7\ N/m^2$$
$$= \underline{\underline{88.97 \times 10^3\ N/m^2}}$$

If negative pressure is to be measured by a simple manometer, this can be measured easily as discussed below:

In this case, the negative pressure in the pipe will suck the light liquid which will pull up the heavy liquid in the left-hand limb of the U-tube. This upward movement of the heavy liquid, in the left-hand limb will cause a corresponding fall of the liquid in the right-hand limb as shown in Figure 3.7.

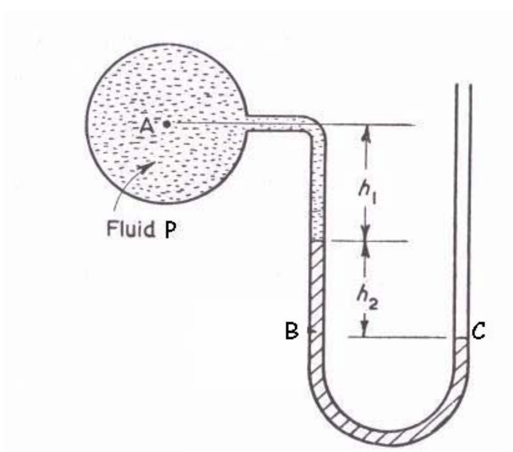


Figure 3.7

In this case, the datum line B-C may be considered to correspond with the top level of the heavy liquid in the right column as shown in the Figure 3.7.

Now to calculate the pressure in the left- hand limb above the datum line.

Let h_1 = Height of the light liquid in the left-hand limb above the common surface in cm.

h_2 = Height of the heavy liquid in the left-hand limb above the common surface in cm

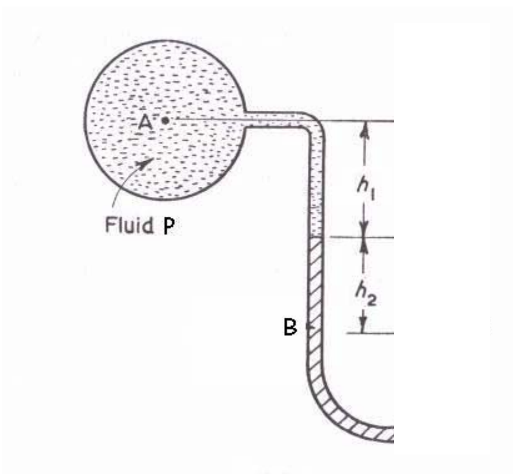
p_A = Pressure in the pipe, expressed in terms of head of water in cm.

s_P = Specific gravity of the light liquid

s_Q = Specific gravity of the heavy liquid.

Pressure p_B at B = Pressure p_C at C

Pressure in the left-hand limb above the datum line;

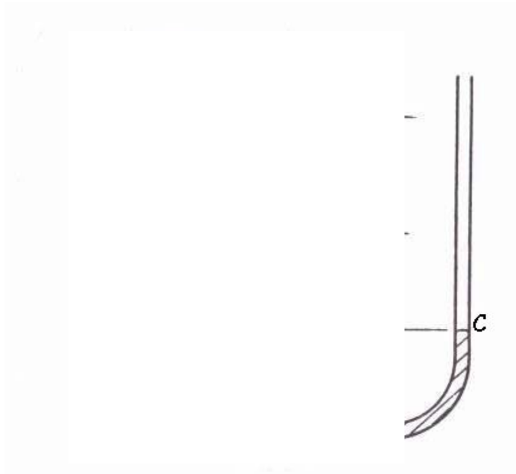


p_B = Pressure p_A at A + Pressure due to depth h_1 of fluid P + Pressure due to depth h_2 of liquid Q

$= p_A + \omega_P h_1 + \omega_Q h_2$

$= p_A + \rho_P g h_1 + \rho_Q g h_2$

Pressure in the right-hand limb above the datum line;



p_C = Pressure p_D at D

But p_D = Atmospheric pressure

And so, $p_C = p_{atm}$

Since $p_B = p_C$

$$p_A + \rho_P g h_1 + \rho_Q g h_2 = p_D$$

$$p_A = p_B - \left(\rho_P g h_1 + \rho_Q g h_2\right)$$

Example 3.5

A U-tube manometer similar to that shown in Figure 3.8 is used to measure the gauge pressure of a fluid P of density $\rho = 1000\ kg/m^3$. If the density of the liquid Q is $13.6 \times 10^3\ kg/m^3$, what will be the gauge pressure at A if $h_1 = 0.15$ m and $h_2 = 0.25\ m$ above BC. Take into consideration $p_{atm} = 101.3\ kN/m^2$.

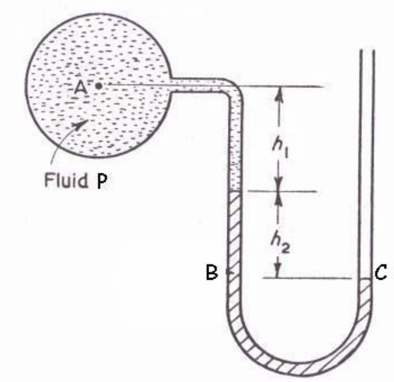


Figure 3.8

Solution to Example 3.5

Putting ,

$\rho_Q = 13.6 \times 10^3$
$\rho_P = 1000\ kg/m^3$
$h_1 = 0.15\ m$
$h_2 = 0.25\ m$

pressure at left-hand limb;

p_B = Pressure p_A at A + Pressure due to depth h_1 of fluid P + Pressure due to depth h_2 of liquid Q

$$= p_A + \omega_P h_1 + \omega_Q h_2$$

$$= p_A + \rho_P g h_1 + \rho_Q g h_2$$

pressure at right-hand limb;

p_C = Pressure p_D at D

p_D = Atmospheric pressure

$p_C = p_{atm}$

Since $p_B = p_C$,

$p_A + \rho_P g h_1 + \rho_Q g h_2 = p_D$

$$p_A = p_B - \left(\rho_P g h_1 + \rho_Q g h_2\right)$$
$$= 101.3 - \left(13.6 \times 103 \times 9.81 \times 0.15 + 1000 \times 9.81 \times 0.25\right)$$
$$= 70835.1\ N/m^2$$
$$= \underline{\underline{70.84\ kN/m^2}}$$

INPUT

3.4.2 DIFFERENTIAL MANOMETER

It is a device used for measuring the difference of pressures, between two points in a pipe, or in two different pipes.

A differential manometer consists of a U-tube, containing a heavy liquid with two ends connected to two different points. We are required to find the difference of pressure at these two points, as shown in Figure 3.9.

A differential manometer is connected to two different points *A* and *B*. A little consideration will show that the greater pressure at *A* will force the heavy liquid in the U-tube to move downwards. This downward movement of the heavy liquid, in the left-hand limb, will cause a corresponding rise of the heavy liquid in the right-hand limb as shown in Figure 3.9.

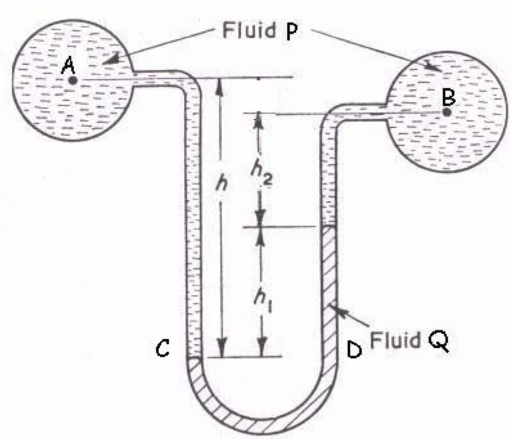


Figure 3.9

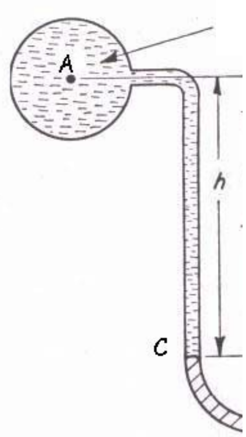
The horizontal surface *C-D*, at which the heavy liquid meet in the left-hand limb, is the datum line.

Let h = Height of the light liquid in the left-hand limb above the datum line.
h_1 = Height of the heavy liquid in the right-hand limb above the datum line
h_2 = Height of the light liquid in the right-hand limb above the datum line
p_A = Pressure in the pipe A, expressed in term of head of the liquid in cm
p_B = Pressure in the pipe B, expressed in term of head of the liquid in cm
ω_P = Specific weight of the light liquid
ω_Q = Specific weight of the heavy liquid

We know that the pressures in the left-hand limb and right-hand limb , above the datum line are equal.

Pressure p_C at C = Pressure p_D at D

Pressure in the **left-hand limb** above the datum line

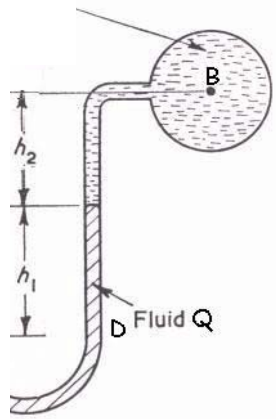


p_C = Pressure p_A at A + Pressure due to depth h of fluid P

$$p_C = p_A + \omega_P h$$

$$p_C = p_A + \rho_P gh$$

Pressure in the **right-hand limb** above the datum line



$p_D =$ Pressure p_A at A + Pressure due to depth h_1 of fluid P + Pressure due to depth h_2 of liquid Q

$$p_D = p_B + \omega_Q h_1 + \omega_P h_2$$
$$= p_B + \rho_Q g h_1 + \rho_P g h_2$$

Since, $p_C = p_D$

$$p_A + \rho_P g h = p_B + \rho_Q g h_1 + \rho_P g h_2$$

$$p_A - p_B = \rho_Q g h_1 + \rho_P g h_2 - \rho_P g h$$

Example 3.6

A U tube manometer measures the pressure difference between two points A and B in a liquid. The U tube contains mercury. Calculate the difference in pressure if h =1.5 m, h_2 = 0.75 m and h_1 = 0.5 m. The liquid at A and B is water ($\omega = 9.81 \times 10^3$ N/m^2) and the specific gravity of mercury is 13.6.

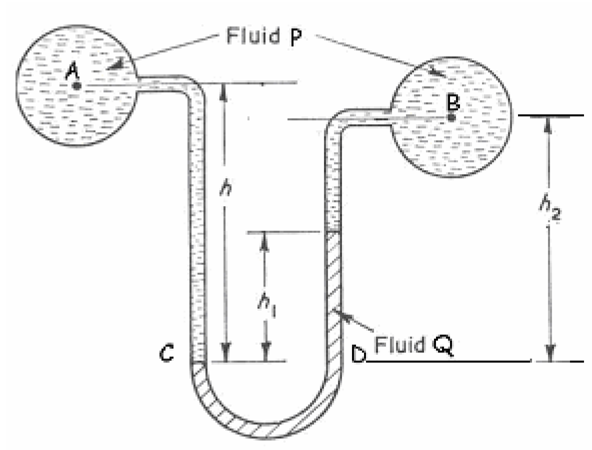


Figure 3.10

Solution to Example 3.6

Since C and D are at the same level in the same liquid at rest

Pressure p_P at C = Pressure p_Q at D

For the left hand limb

$$p_C = p_A + \omega h$$

For the right hand limb

$$p_D = p_B + \omega(h_2 - h_1) + s\omega h_1$$

$$= p_B + \omega h_2 - \omega h_1 + s\omega h_1$$

since $p_C = p_D$

$$p_A + \omega h = p_B + \omega h_2 - \omega h_1 + s\omega h_1$$

Pressure difference $p_A - p_B$

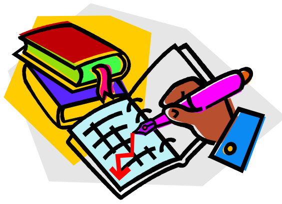
$$= \omega h_2 - \omega h_1 + s\omega h_1 - \omega h$$

$$= \omega h_2 - \omega h + s\omega h_1 - \omega h_1$$

$$= \omega(h_2 - h) + \omega h_1(s - 1)$$

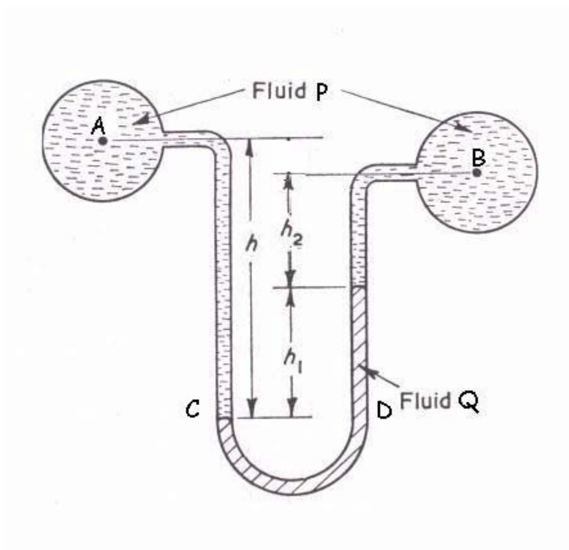
$$= 9.81 \times 10^3 (0.75 - 1.5) + 9.81 \times 10^3 (0.5)(13.6 - 1)$$

$$= 54445.5\, N/m^2 \qquad = \underline{\underline{54.44\, kN/m^2}}$$



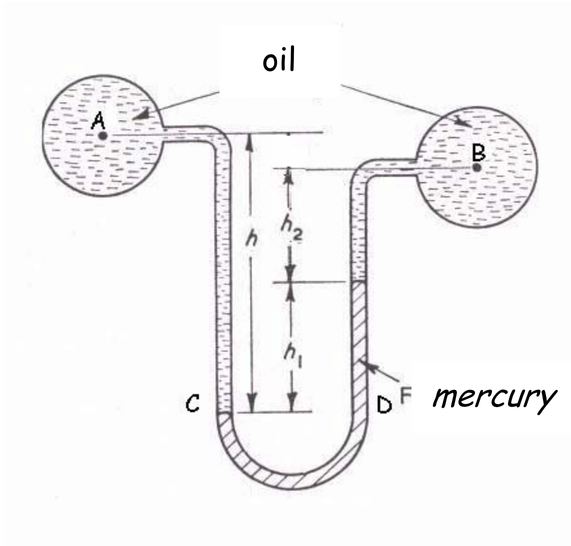
ACTIVITY 3C

3.4 A U tube manometer measures the pressure difference between two points A and B in a liquid. The U tube contains mercury. Calculate the difference in pressure if $h = 2.0\ m$, $h_2 = 0.35\ m$ and $h_1 = 0.5\ m$. The liquid at A and B is oil ($s = 0.85$) and the specific gravity of mercury is 13.6.



FEEDBACK ON ACTIVITY 3C

3.4



Since C and D are at the same level in the same liquid at rest
Pressure p_C at C = Pressure p_D at D

For the left hand limb

$$p_C = p_A + \omega h$$

For the right hand limb

$$p_D = p_B + \omega h_2 + s\omega h_1$$

since $p_P = p_Q$

$$p_A + \omega h = p_B + \omega h_2 + s\omega h_1$$

Pressure difference $p_A - p_B$

$$= \omega_{oil} h_2 + s\omega h_1 - \omega_{oil} h$$

$$= 0.85(9810)(0.35) + 13.6(9810)(0.5) + 0.85(9810)(2.0)$$

$$= 69626.475\ N/m^2$$

$$= \underline{\underline{69.626\ kN/m^2}}$$

INPUT

3.4.3 INVERTED DIFFERENTIAL MANOMETER

It is a particular type of differential manometer, in which an inverted U-tube is used. An inverted differential manometer is used for measuring the difference of low pressure, where accuracy is the prime consideration. It consists of an inverted U-tube, containing a *light liquid.* The two ends of the U-tube are connected to the points where the difference of pressure is to be found out as shown in Figure 3.10.

Now consider an inverted differential manometer whose two ends are connected to two different points *A* and *B*. Let us assume that the pressure at point *A* is more than that at point *B*, a greater pressure at *A* will force the light liquid in the inverted U-tube to move upwards. This upward movement of liquid in the left limb will cause a corresponding fall of the light liquid in the right limb as shown in Figure 3.10. Let us take *C-D* as the datum line in this case.

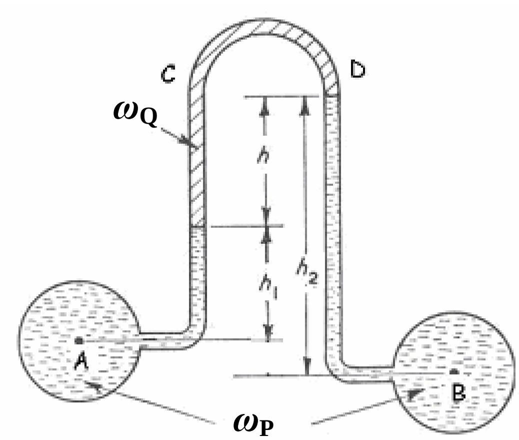


Figure 3.11

Let h = Height of the heavy liquid in the left-hand limb below the datum line,
h_1= Height of the light liquid in the left-hand limb below the datum line ,
h_2= Height of the light liquid in the right-hand limb below the datum line,
ω_P= Specific weight of the light liquid
ω_Q= Specific weight of the heavy liquid

We know that pressures in the left limb and right limb below the datum line are equal.

Pressure p_C at C = Pressure p_D at D

Example 3.7

The top of an inverted U tube manometer is filled with oil of specific gravity, s_{oil}=0.98 and the remainder of the tube with water whose specific weight of water, ω= 9.81×10^3 N/m^2. Find the pressure difference in N/m^2 between two points A and B at the same level at the base of the legs where the difference in water level h is 75 mm.

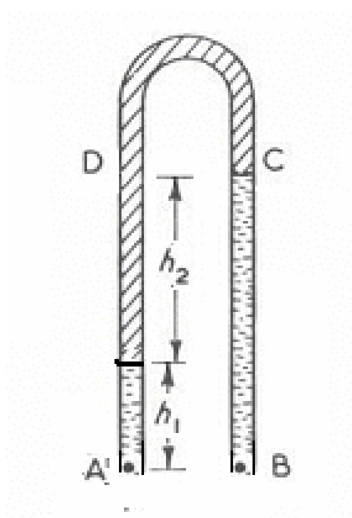


Figure 3.12

Solution to Example 3.7

For the left hand limb

$$p_D = p_A - \omega h_2 - s_o \omega h_1$$

for the right hand limb

$$p_C = p_B - \omega(h_1 - h_2)$$

$$= p_B - \omega h_1 - \omega h_2$$

since, $p_C = p_D$

$$p_B - \omega h_1 - \omega h_2 = p_A - \omega h_2 - s\omega h_1$$

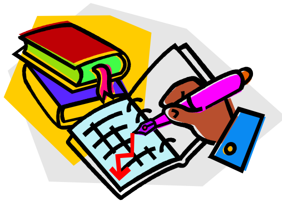
$$p_B - p_A = -\omega h_2 - s\omega h_1 + \omega h_1 + \omega h_2$$

$$= \omega h_1 - s\omega h_1$$

$$= \omega h_1(1 - s)$$

$$= 9.81 \times 10^3 (0.075)[1 - 0.98]$$

$$= \underline{\underline{14.715\, N/m^2}}$$

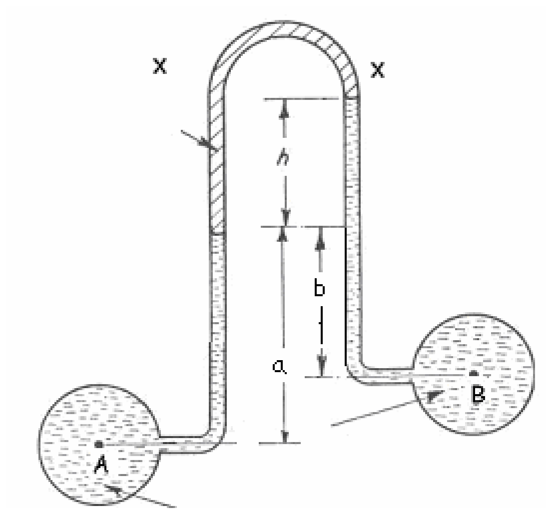


ACTIVITY 3D

3.5 An inverted U tube as shown in the figure below is used to measure the pressure difference between two points A and B which has water flowing. The difference in level $h = 0.3\ m$, $a = 0.25\ m$ and $b = 0.15\ m$. Calculate the pressure difference $p_B - p_A$ if the top of the manometer is filled with:

(a) air

(b) oil of relative density 0.8.



FEEDBACK ON ACTIVITY 3D

3.5

In either case, the pressure at X-X will be the same in both limbs, so that

$$p_{XX} = p_A - \rho g a - \rho_{mano} g h = p_B - \rho g(b+h)$$
$$p_B - p_A = \rho g(b-a) + gh(\rho - \rho_{mano})$$

(a) if the top is filled with air ρ_{mano} is negligible compared with ρ. Therefore,

$$p_B - p_A = \rho g(b-a) + \rho g h$$
$$= \rho g(b - a + h)$$

putting $\rho = \rho_{H_2O} = 10^3\ kg/m^3$, $b = 0.15\,m$, $a = 0.25\,m$, $h = 0.3\,m$:

$$p_B - p_A = 10^3 \times 9.81(0.15 - 0.25 + 0.3)$$
$$= \underline{\underline{1.962 \times 10^3\ N/m^2}}$$

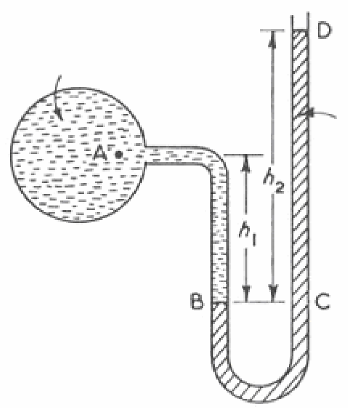
(b) if the top is filled with oil of relative density 0.8, $\rho_{mano} = 0.8\ \rho_{H_2O}$,

$$p_B - p_A = \rho g(b-a) + gh(\rho - \rho_{mano})$$
$$= 10^3 \times 9.81(0.15 - 0.25) + 9.81 \times 0.3 \times 10^3 (1 - 0.8)$$
$$= 10^3 \times 9.81(-0.1 + 0.06)$$
$$= \underline{\underline{-392.4\ N/m^2}}$$

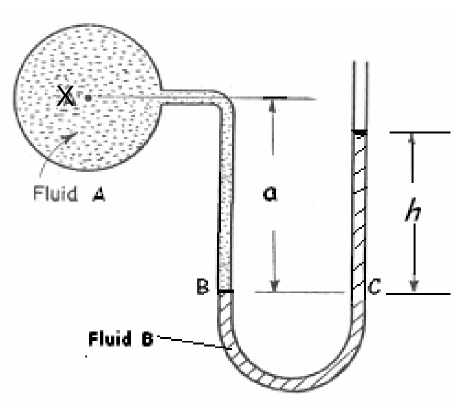
SELF-ASSESSMENT

You are approaching success. **Try all the questions** in this self-assessment section and check your answers with those given in the Feedback on Self-Assessment. If you face any problems, discuss it with your lecturer. Good luck.

3.1 What is the gauge pressure of the water at A if $h_1 = 0.6\ m$ and the mercury in the right hand limb, $h_2 = 0.9\ m$ as shown in the figure below?



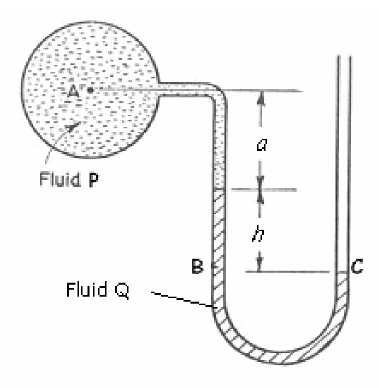
3.2 In the figure below, fluid at A is water and fluid B is mercury (s =13.6). What will be the difference in level h if the pressure at X is 140 kN/m^2 and a =1.5 m?



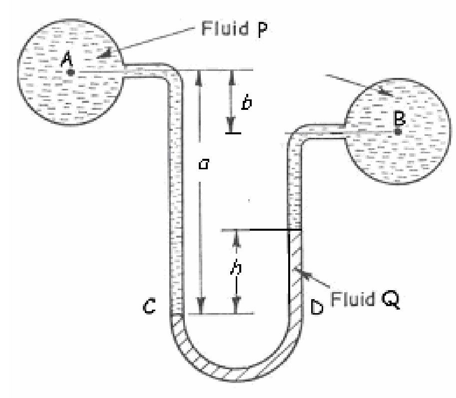
3.3 Assuming that the atmospheric pressure is 101.3 kN/m^2 find the absolute pressure at A in the figure below when

a) fluid P is water, fluid Q is mercury $\omega = 13.6$, $a = 1\ m$ and $h = 0.4\ m$.

b) fluid P is oil $\omega = 0.82$, fluid Q is brine $\omega = 1.10$, $a = 20\ cm$ and $h = 55\ cm$.

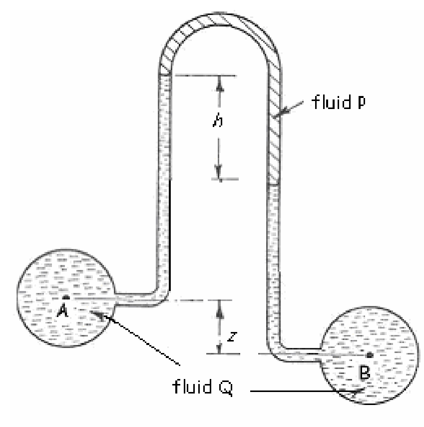


3.4 In the figure below, fluid P is water and fluid Q is mercury (specific gravity=13.6). If the pressure difference between A and B is 35 kN/m^2, $a = 1\ m$ and $b = 30\ cm$, what is the difference in level h?



3.5 According to the figure in question 3.4, fluid P is oil (specific gravity = 0.85) and fluid Q is water. If $a = 120\ cm$, $b = 60\ cm$ and $h = 45\ cm$, what is the difference in pressure in kN/m^2 between A and B?

3.6 In the figure below, fluid Q is water and fluid P is oil (specific gravity = 0.9). If h = 69 *cm* and z = 23 *cm*, what is the difference in pressure in kN/m^2 between A and B?



3.7 In question 6, fluid Q is water and fluid P is air. Assuming that the specific weight of air is negligible, what is the pressure difference in kN/m^2 between A and B?

FEEDBACK ON SELF-ASSESSMENT

Answers:

1. 114.188 kN/m^2

2. 1.164 m

3. a) 38.2 kN/m^2

   b) 93.8 kN/m^2

4. 30.7 cm

5. -5.23 kN/m^2

6. -1.57 kN/m^2

7. 4.51 kN/m^2