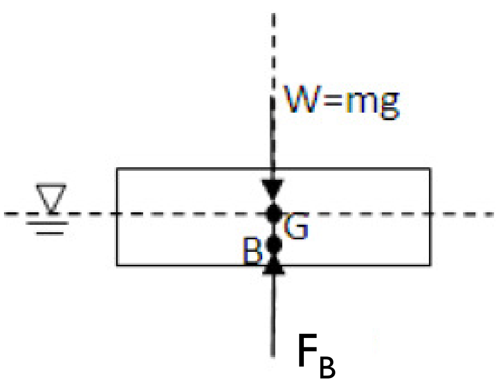
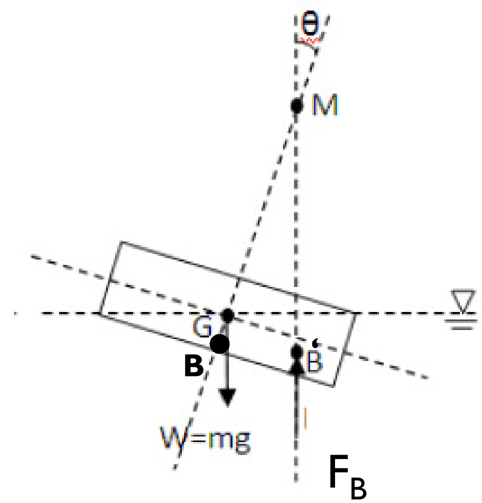
Metacenter and Metacentric Height

- **Center of Buoyancy** (B) The point of application of the force of buoyancy on the body is known as the center of buoyancy.
- **Metacenter (M):** The point about which a body in stable equilibrium start to oscillate when given a small angular displacement is called metacenter.

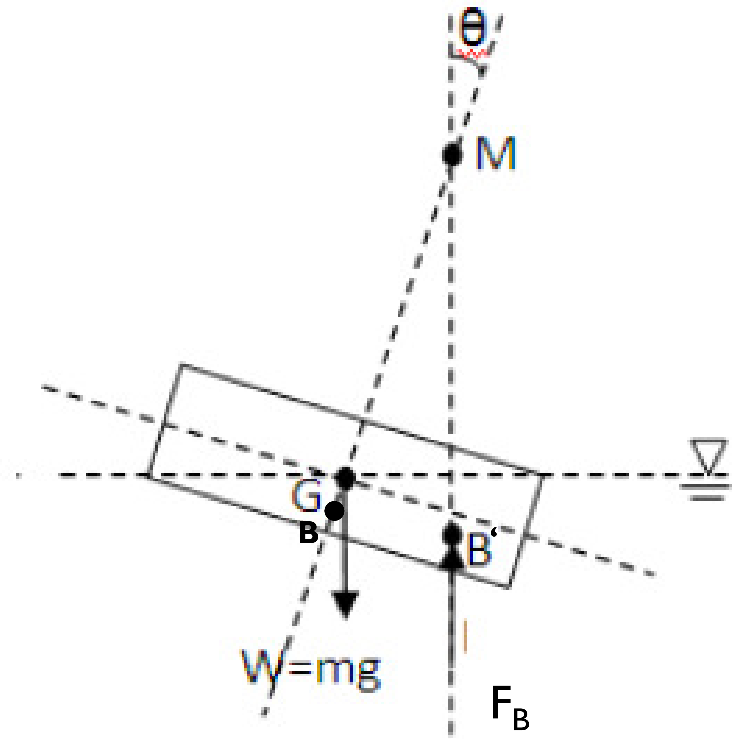
It may also be defined as point of intersection of the axis of body passing through center of gravity (CG or G) and original center of buoyancy (B) and a vertical line passing through the center of buoyancy (B') of tilted position of body.





Metacenter and Metacentric Height

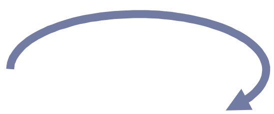
- **Metacentric height (GM):** The distance between the center of gravity (G) of floating body and the metacenter (M) is called metacentric height. (i.e., distance GM shown in fig)



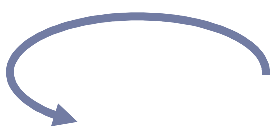
GM=BM-BG

Condition of Stability

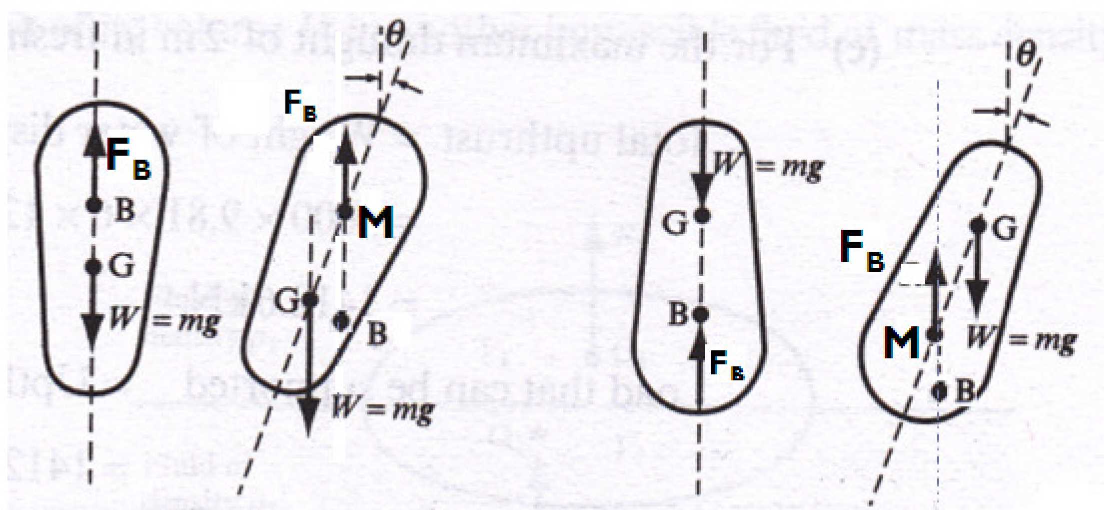
- **For Stable Equilibrium**
- Position of metacenter (M) is above than center of gravity (G)
- **For Unstable Equilibrium**
- Position of metacenter (M) is below than center of gravity (G)
- **For Neutral Equilibrium**
- Position of metacenter (M) coincides center of gravity (G)



Overturning moment



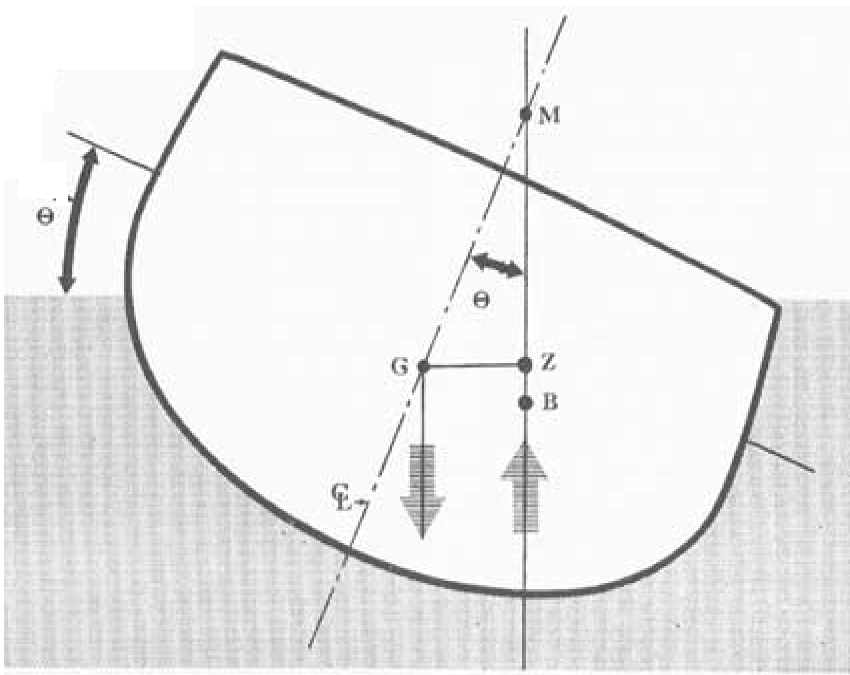
Restoring moment



(a) Stable (b) Unstable

Determination of Metacentric height

- The metacentric height may be determined by the following two methods
- **<u>1. Analytical method</u>**
- 2. Experimental method

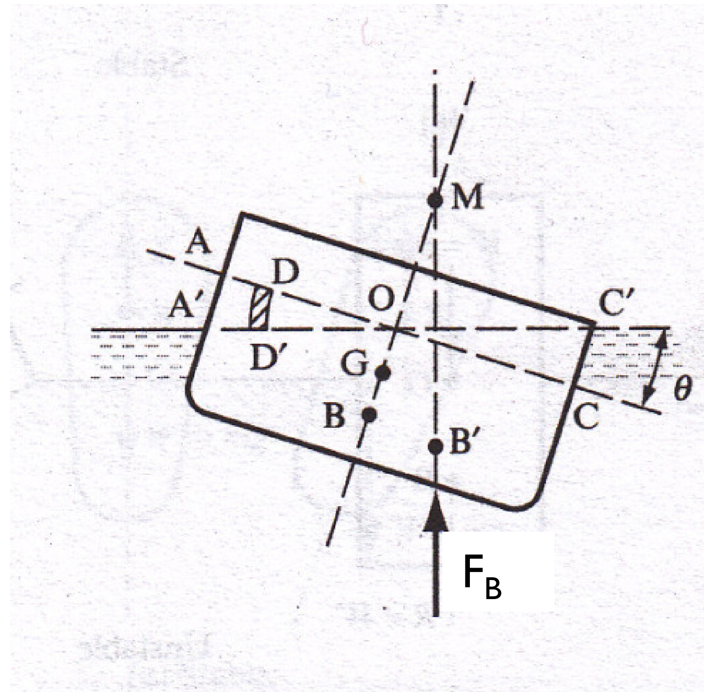


Determination of Metacentric height

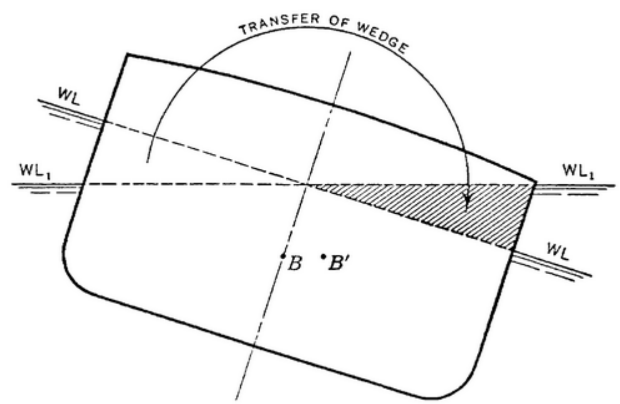
- In Figure shown AC is the original waterline plane and B the center of buoyancy in the equilibrium position.

- When the vessel is tilted through small angle θ, the center of buoyancy will move to B' as a result of the alteration in the shape of displaced fluid.

- A'C' is the waterline plane in the displaced position.



Cross-section



Determination of Metacentric height

- To find the **metacentric height GM**, consider a small area dA at a distance x from O. The height of elementary area is given by xθ.
- Therefore, volume of the elementary area becomes

$$dV = (x\theta)dA$$

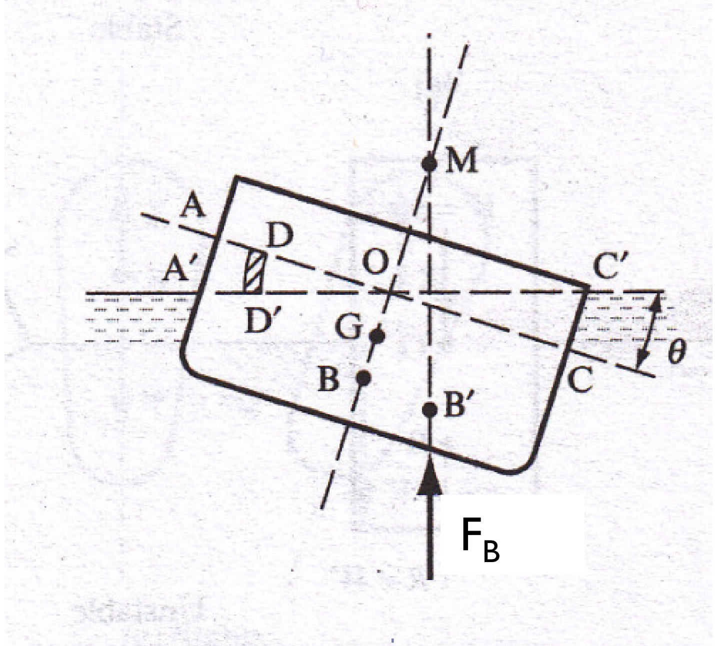
- The upward force of buoyancy on this elementary area is then

$$dF_B = \gamma(x\theta)dA$$

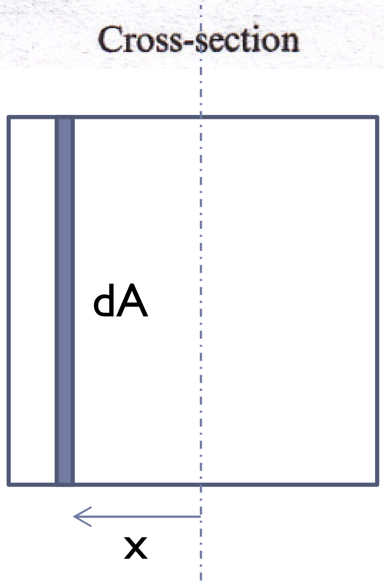
- Moment of dF_B (moment due to movement of wedge) about O is given by;

$$\int x.dF_B = \int x\gamma(x\theta)dA = (\gamma\theta)\int x^2 dA$$

$$\int x.dF_B = \gamma\theta I$$



Cross-section



Determination of Metacentric height

- The change in the moment of the buoyancy Force, FB is

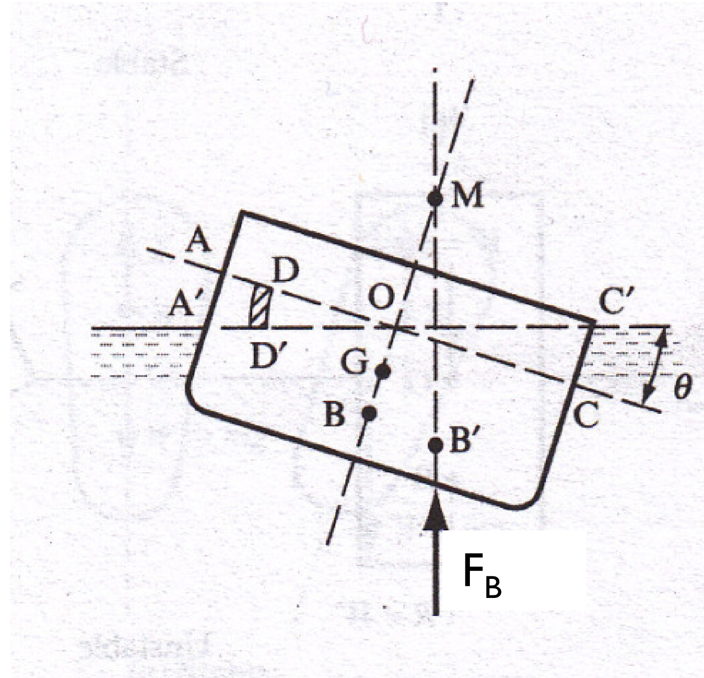
$$F_B = F_B BB' = \gamma V\left(BM\theta\right)$$

- For equilibrium, the moment due to movement of wedge=change in moment of buoyancy force

$$\gamma\theta I = \gamma V\left(BM\theta\right)$$

$$BM = \frac{I}{V}$$

$$GM = BM - BG$$

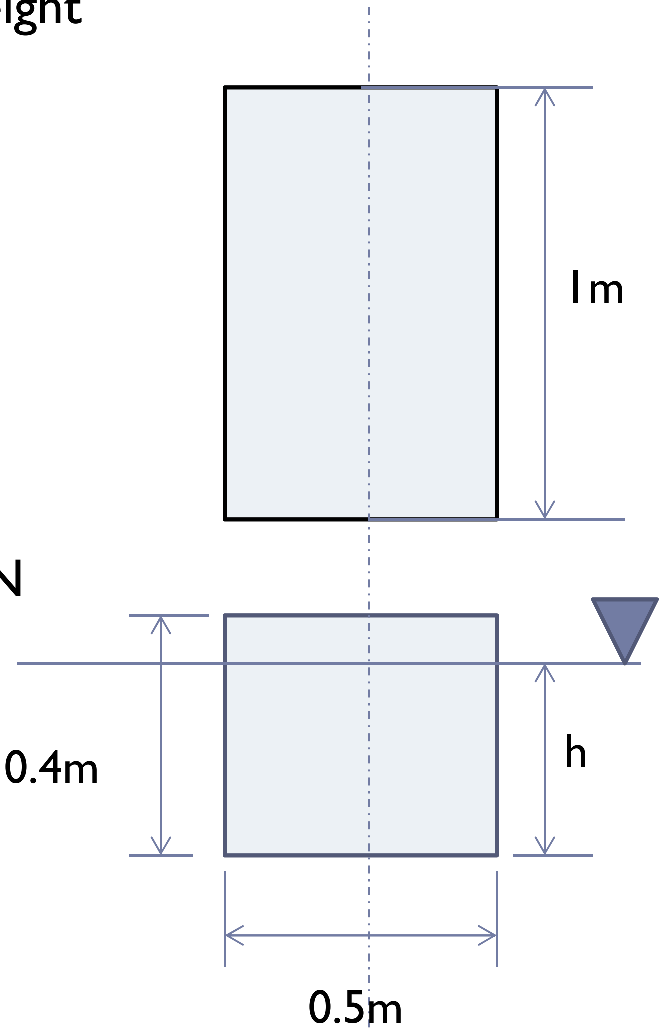


Cross-section

Part II

NUMERICALS

Q. 1 A wooden block of specific gravity 0.75 floats in water. If the size of block is 1mx0.5mx0.4m, find its meta centric height



Solution: Given Data:

Size of wooden block= 1mx0.5mx0.4m,

Specific gravity of wood=0.75

Specific weight of wood=0.75(9.81)=7.36kN/m2

Weight of wooden block=(specific weight)x(volume)

Weight of wooden block=7.36(1x0.5x0.4)=1.47kN

Let h is depth of immersion=?

For equilibrium

Weight of water displaced = weight of wooden block

9.81(1x0.5xh)=1.47 >> h=0.3m

NUMERICALS

Distance of center of buoyancy=OB=0.3/2=0.15m

Distance of center of gravity=OG=0.4/2=0.2m

Now; BG=OG-OM=0.2-0.15=0.05m

Also; BM=I/V

I=moment of inertia of rectangular section

I=(1)x(0.5)^3/12=0.0104m

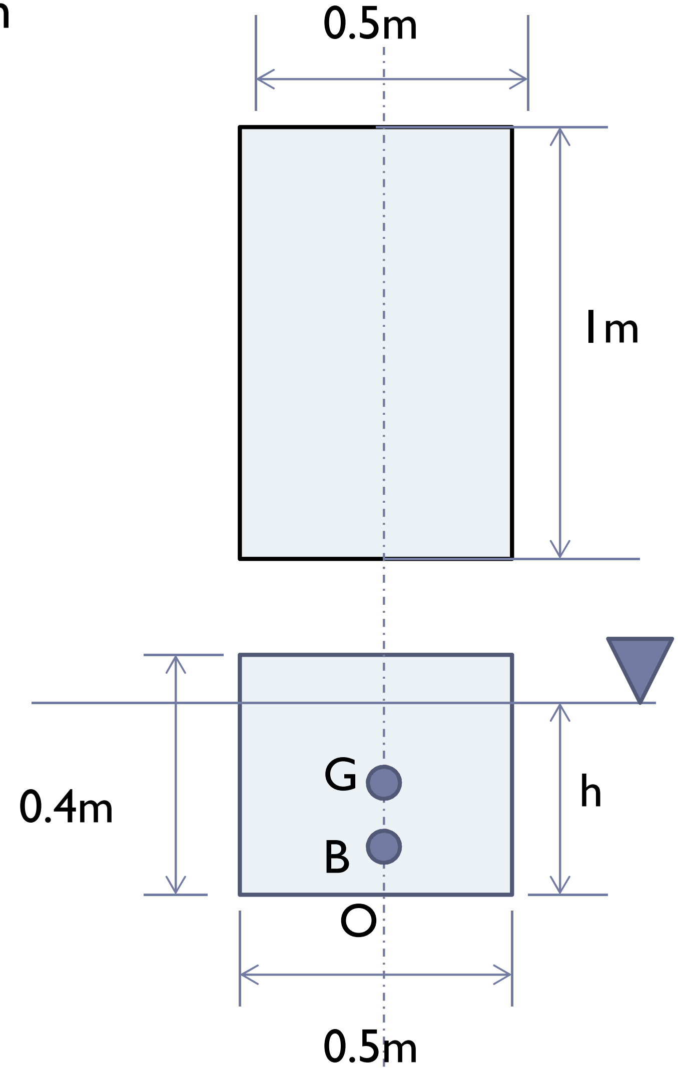
V=volume of water displaced by wooden block

V=(1)x(0.5)x(0.3)=0.15m3

BM=I/V=0.0104/0.15=0.069m

Therefore, meta centric height=GM=BM-BG

GM=0.069-0.05=0.019m



NUMERICALS

- Q 2. A solid cylinder 2m in diameter and 2m high is floating in water with its axis vertical. If the specific gravity of the material of cylinder is 0.65, find its meta-centric height. State also whether the equilibrium is stable of unstable.

<u>Solution: Given Data:</u>

Size of solid cylinder= 2m dia, & 2m height

Specific gravity solid cylinder=0.65

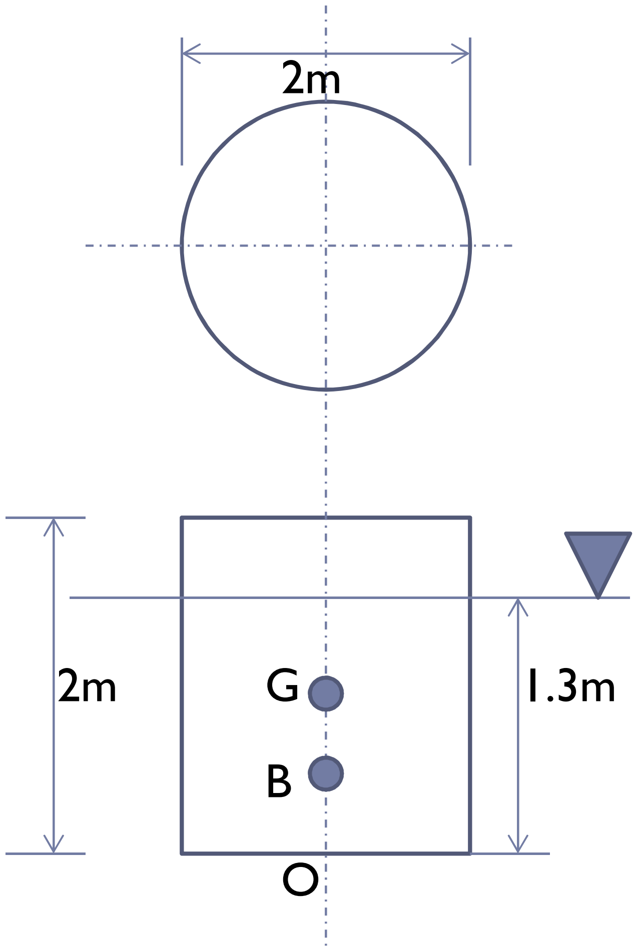
Let h is depth of immersion=?

For equilibrium

Weight of water displaced = weight of wooden block

$9.81(\pi/4(2)^2(h))=9.81(0.65).(\pi/4(2)^2(2))$

$h=0.65(2)=1.3m$



NUMERICALS

Center of buoyancy from O=OB=1.3/2=0.65m

Center of gravity from O=OG=2/2=1m

BG=1-0.65=0.35m

Also; BM=I/V

Moment of inertia=I=$(\pi/64)(2)^4=0.785m^4$

Volume displaced=V=$(\pi/4)(2)^4(1.3)=4.084m^3$

BM=I/V=0.192m

GM=BM-BG=0.192-0.35=-0.158m

-ve sign indicate that the metacenter (M) is below the center of gravity (G), therefore, the cylinder is in **unstable equilibrium**