CLASSMATE

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VISCOUS FLOW OF INCOMPRESIBLE FLOW

(LAMINAR FLOW)

- **Reynold's Number (Re) :-**

It is defined as the ~~ratio~~ ratio of Inertia force to viscous force.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{F_i}{F_v} = \frac{\rho V L}{\mu} = \frac{VL}{\mu/\rho}$$

$$\boxed{Re = \frac{VL}{\nu}}$$

For pipe flow,

$Re < 2000 \longrightarrow$ Laminar (or viscous) flow.

$Re > 4000 \longrightarrow$ Turbulent flow.

$2000 < Re < 4000 \rightarrow$ Transition flow.

NOTE- For pipe flow the characteristic length L is taken as dia. of the pipe.

$$\boxed{Re = \frac{VD}{\nu}}$$

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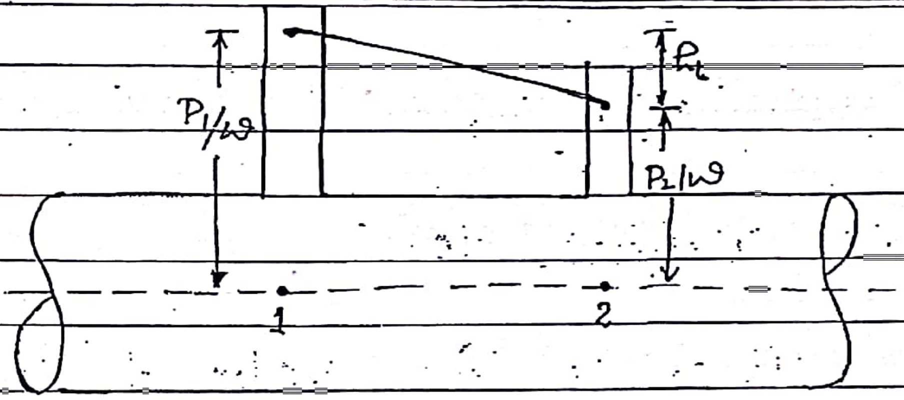


Apply continuity eqn. b/w 1 & 2.

$$Q = A_1 V_1 = A_2 V_2$$

$$\boxed{V_1 = V_2} \qquad [\because A_1 = A_2]$$

Apply bernoulli eqn. b/w 1 & 2.

$$\frac{P_1}{w} + \cancel{\frac{V_1^2}{2g}} = \frac{P_2}{w} + \cancel{\frac{V_2^2}{2g}} + h_L$$

$$\frac{P_1}{w} = \frac{P_2}{w} + h_L$$

$$\boxed{h_L = \frac{P_1 - P_2}{w}}$$

In the direction of flow press. decreases in overcome to losses because press. gradient in the direction of flow is negative.

Darcy - Weisbach Equation :-

This equation is used for finding out head loss due to friction in steady laminar & turbulent flow.

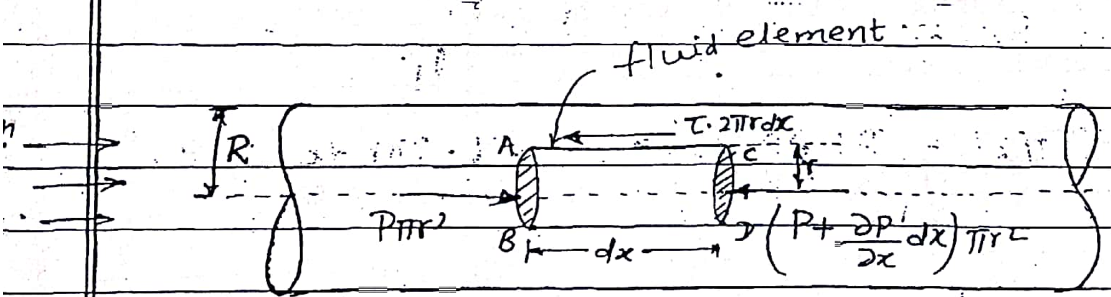
$$h_L = \frac{4f'LV^2}{2gD}$$ ← oldest formula.

where f' = friction co-efficient

$$\boxed{h_L = \frac{fLV^2}{2gD}}$$ ← New formula. (use this)

where f = friction factor $= 4f'$

:- Laminar flow through circular pipe (Hagen-poiseuille flow) :-



Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown in fig.

Consider a fluid element of radius r, sliding in

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a cylindrical fluid element of radius $(r+dr)$: let the length of the fluid element be dx.

If 'P' is the intensity of pressure on the face AB, then intensity of pressure on face CD will be $\left(P + \frac{\partial P}{\partial x} dx\right)$. [from Euler's Expansion series]

Then the force acting on fluid element -

1. The press. force, $P \times \pi r^2$ on face AB.
2. The press. force, $\left(P + \frac{\partial P}{\partial x} dx\right) \pi r^2$ on face CD.
3. The shear force, $\tau \times 2\pi r\, dx$ on the surface of fluid element.

As there is no acceleration, hence the summation of all forces in the direction of flow must be zero i.e.

$$P\pi r^2 - \left(P + \frac{\partial P}{\partial x} dx\right)\pi r^2 - \tau \cdot 2\pi r\, dx = 0.$$

$$Pr - \left(P + \frac{\partial P}{\partial x} dx\right) r = 2\tau \cdot dx$$

$$\cancel{Pr} - \cancel{Pr} - \frac{\partial P}{\partial x} dx \cdot r = 2\tau \cdot dx$$

$$-\frac{\partial P}{\partial x}\,\cancel{dx}\cdot r = 2\tau\,\cancel{dx}$$

$$\boxed{\tau = \left(-\frac{\partial P}{\partial x}\right)\cdot\frac{r}{2}}$$ *****

NOTE- The shear stress τ across a section varie's with 'r' as $\frac{\partial P}{\partial x}$ across the section is constant. Hence shear stress distribution across a section is <u>linear</u>

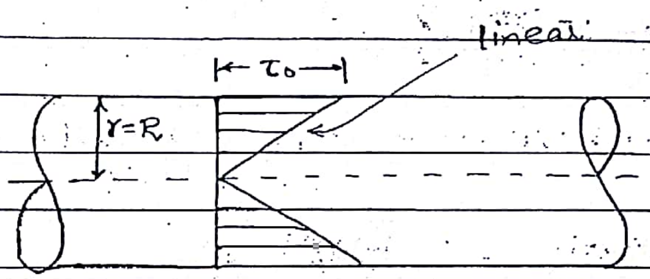
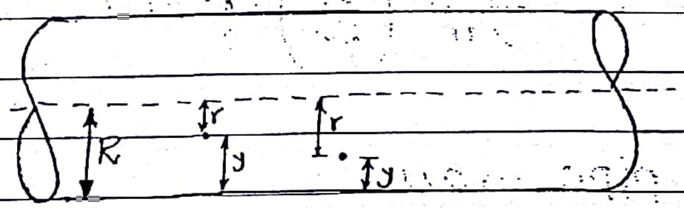


fig. shear stress distribution across a section.

maximum shear stress τ_o at the pipe wall.

• <u>velocity distribution</u> :-



$\because \quad R = r + y$ [$\because$ R is const. differentiate the eqn. we get]

$0 = dr + dy$

$\Rightarrow \quad dy = -dr$

but we know that;

$$\tau = \mu \cdot \frac{du}{dy}$$

$$\tau = \mu \frac{du}{(-dr)}$$

$$\tau = -\mu \frac{du}{dr}$$

& also $\tau = \left(-\frac{\partial P}{\partial x}\right) \cdot \frac{r}{2}$

equating above two eqn, we get:

$$-\mu \frac{du}{dr} = \left(-\frac{\partial P}{\partial x}\right) \frac{r}{2}$$

$$du = \frac{1}{2\mu} \left(\frac{\partial P}{\partial x}\right) \cdot r \cdot dr$$

Integrate the above eqn. we find:

$$u = \frac{1}{2\mu} \left(\frac{\partial P}{\partial x}\right) \frac{r^2}{2} + C$$

At the pipe wall;

$$r = R \quad \& \quad u = 0$$

then, $$0 = \frac{1}{2\mu} \left(\frac{\partial P}{\partial x}\right) \frac{R^2}{2} + C$$

$$C = \frac{-1}{4\mu}\left(\frac{\partial P}{\partial x}\right)R^2$$

then,
$$u = \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)\frac{r^2}{2} - \frac{1}{4\mu}\left(\frac{\partial P}{\partial x}\right)R^2$$

$$u = \frac{-1}{4\mu}\left(\frac{\partial P}{\partial x}\right)R^2\left[1 - \frac{r^2}{R^2}\right]$$

At the centre,

$$r = 0 \quad \& \quad u = u_{max}.$$

$$\boxed{u_{max} = \frac{-1}{4\mu}\left(\frac{\partial P}{\partial x}\right)R^2} \quad *****$$

&
$$\boxed{u = U_{max}\left(1 - \frac{r^2}{R^2}\right)} \quad *****$$

NOTE- value of μ, $\frac{\partial P}{\partial x}$ & R are constant, which mean's the velocity, u varie's with the square of r. Thus the eqn. is parabola eqn. This show's the velocity distribution across the section of a pipe is parabolic.

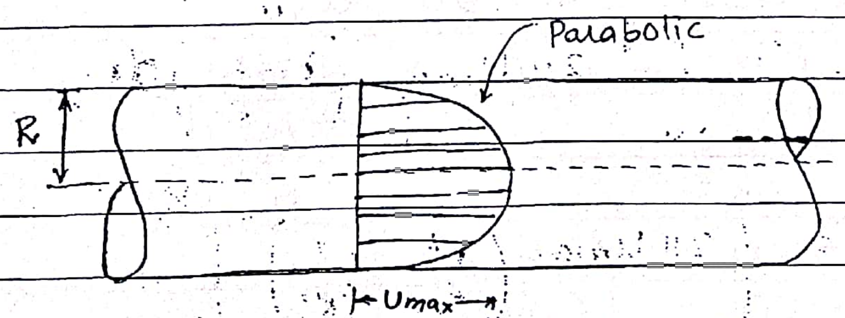


fig- velocity distribution across a section

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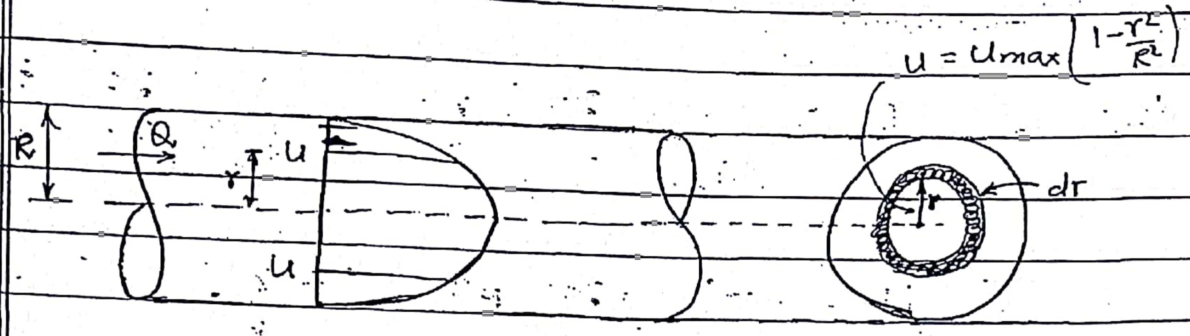
Discharge (Q) :-



$$u = u_{max}\left(1-\frac{r^2}{R^2}\right) \quad u \text{ vary with } r$$

local velocity

discharge through circular ring element of radius r & thickness dr is,

dQ = velocity at radius r × area of ring element

$$dQ = u \times 2\pi r\, dr$$

$$= u_{max}\left(1-\frac{r^2}{R^2}\right) \times 2\pi r\, dr$$

Integrate the above eqn. we get,

then, $$Q = \int_0^R 2\pi u_{max}\left(r - \frac{r^3}{R^2}\right) dr.$$

$$Q = \left[2\pi u_{max}\left(\frac{r^2}{2} - \frac{r^4}{4R^2}\right)\right]_0^R$$

$$Q = 2\pi U_{max}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$

$$Q = 2\pi U_{max}\left[\frac{R^2}{4}\right]$$

$$Q = \frac{\pi U_{max} R^2}{2}$$

$$Q = \frac{\pi R^2}{2} \times \frac{1}{4\mu}\left(\frac{-\partial P}{\partial x}\right) R^2$$

$$\boxed{Q = \frac{\pi}{8\mu}\left(\frac{-\partial P}{\partial x}\right) R^4}$$ *****

- Average velocity :-

$$Q = AV$$

$$V = \frac{Q}{A}$$

$$V = \frac{\frac{\pi U_{max} R^2}{2}}{\pi R^2}$$

$$\boxed{V = \frac{U_{max}}{2}}$$ *****

It is defined as the ratio of total discharge to the total area of flow.

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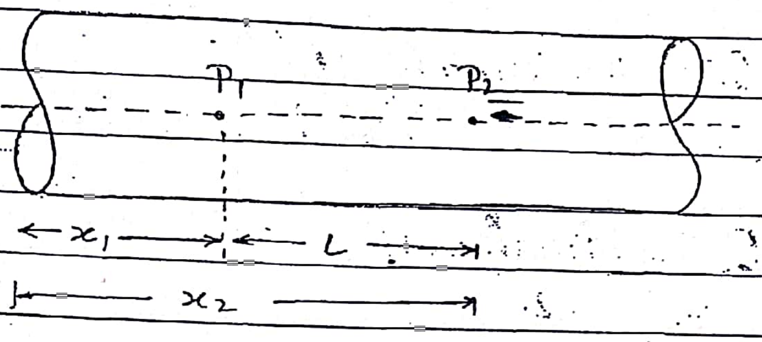
- ## Pressure drop in a given length (l) :-



$$V = \frac{U_{max}}{2}$$

$$V = \frac{1}{2}\left[\frac{1}{4\mu}\left(\frac{-\partial P}{\partial x}\right)R^2\right]$$

$$\frac{8\mu V}{R^2}\,\partial x = -\partial P$$

Integrate the above eqn.

$$\int_{x_1}^{x_2} \frac{8\mu V}{R^2}\,\partial x = \int_{P_1}^{P_2} -\partial P$$

$$\frac{8\mu V}{R^2}(x_2 - x_1) = -(P_2 - P_1)$$

$$\frac{8\mu V L}{R^2} = P_1 - P_2$$

$$P_1 - P_2 = \frac{8\mu V L}{R^2}$$

$$P_1 - P_2 = \frac{8 \mu V L}{(D/2)^2}$$

avg. velocity ****

$$\boxed{P_1 - P_2 = \frac{32 \mu V L}{D^2}}$$

Heat of the laminar flow eqn.

We know that,

$$\frac{P_1 - P_2}{w} = h_L$$

$$P_1 - P_2 = w \times h_L$$

$$P_1 - P_2 = \rho g \times h_L$$

$$\frac{32 \mu V L}{D^2} = \rho g \times \frac{f L V^2}{2 g D}$$

$$\frac{32 \mu}{D} = \frac{\rho f V}{2}$$

$$\frac{64 \mu}{\rho V D} = f$$

$$\frac{64}{\left(\frac{\rho V D}{\mu}\right)} = f$$

*****

$$\boxed{f = \frac{64}{Re}}$$

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Imp.

NOTE - Friction factor in laminar flow through circular pipe depend's upon Reynold number only.

Shear velocity (V^*) :-

We know that,

$$\tau = \left(\frac{-\partial P}{\partial x}\right)\frac{r}{2}$$

At the wall of the pipe,

$$\tau = \tau_0 \quad \& \quad r = R$$

then,
$$\tau_0 = \left(\frac{-\partial P}{\partial x}\right)\frac{R}{2}$$

$$\tau_0 = -\frac{(P_2 - P_1)}{x_2 - x_1} \cdot \frac{R}{2}$$

$$\tau_0 = \frac{P_1 - P_2}{x_2 - x_1} \cdot \frac{D}{2} \cdot \frac{1}{2}$$

$$\tau_0 = \frac{P_1 - P_2}{L} \cdot \frac{D}{4} \quad [\because x_2 - x_1 = l]$$

$$\tau_0 = (P_1 - P_2) \cdot \frac{D}{4l}$$

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But we know that,

$$\frac{P_1 - P_2}{w} = h_L$$

$$\Rightarrow \quad P_1 - P_2 = w h_L$$

then, $\tau_0 = w h_L \cdot \frac{D}{4L}$

$$\tau_0 = \rho g \times \frac{f L V^2}{2gD} \cdot \frac{D}{4L}$$

$$\boxed{\tau_0 = \frac{\rho f V^2}{8}}$$ *****

$$\Rightarrow \quad \frac{\tau_0}{\rho} = \frac{f}{8} V^2$$

$$\sqrt{\frac{\tau_0}{\rho}} = \sqrt{\frac{f}{8}}\, V$$

$$\boxed{V^* = \sqrt{\frac{f}{8}}\, V}$$ *****

Where $\sqrt{\frac{\tau_0}{\rho}} = V^*$ = shear velocity.