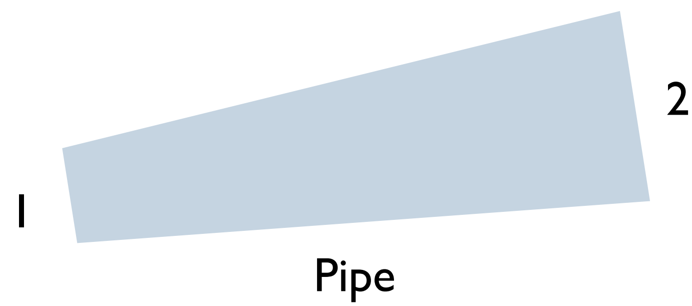
Bernoulli's Equation

- *It states that the sum of kinetic, potential and pressure heads of a fluid particle is constant along a streamline during steady flow when compressibility and frictional effects are negligible.*
- *i.e. , For an ideal fluid, Total head of fluid particle remains constant during a steady-incompressible flow.*
- *Or total head along a streamline is constant during steady flow when compressibility and frictional effects are negligible.*

$$\text{Total Head} = Z + \frac{P}{\gamma} + \frac{V^2}{2g} = constt$$

$$Z_1 + \frac{P_1}{\gamma} + \frac{V^2{}_1}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V^2{}_2}{2g}$$

$$H_1 = H_2$$



Derivation of Bernoulli's Equation

- Consider motion of flow fluid particle in steady flow field as shown in fig.
- Applying Newton's 2nd Law in s-direction on a particle moving along a streamline give

$$F_s = ma_s \qquad \text{Eq(1)}$$

- Where F is resultant force in s-direction, m is the mass and a_s is the acceleration along s-direction.

$$a_s = \frac{dV}{dt} = \frac{dsdV}{dsdt} = \frac{dsdV}{dtds} = V\frac{dV}{ds} \qquad \text{Eq(2)}$$

Assumption:

Fluid is ideal and incompressible
Flow is steady
Flow is along streamline
Velocity is uniform across the section and is equal to mean velocity
Only gravity and pressure forces are acting

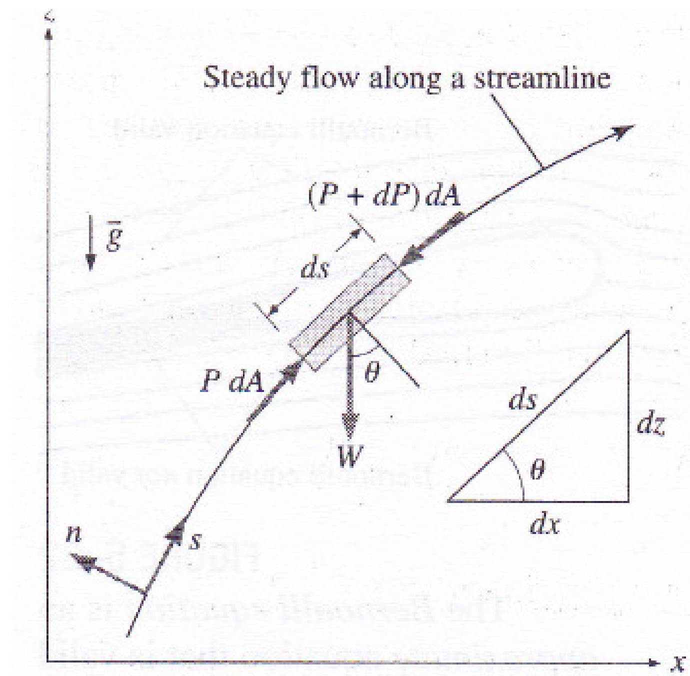


Fig. Forces acting on particle along streamline

Derivation of Bernoulli's Equation

$$F_s = PdA - (P + dp)dA - W \sin\theta \quad \text{Eq(3)}$$

Substituting values from Eq(2) and Eq(3) to Eq(1)

$$PdA - (P + dp)dA - W \sin\theta = mV \frac{dV}{ds}$$

$$-dpdA - \rho g dA ds \frac{dz}{ds} = \rho dA ds V \frac{dV}{ds}$$

Cancelling *dA* and simplifying

$$-dp - \rho g dz = \rho V dV \quad \text{Eq(4)}$$

Note that $VdV = \frac{1}{2} dV^2$

$$-dp - \rho g dz = \rho \frac{1}{2} dV^2 \quad \text{Eq(5)}$$

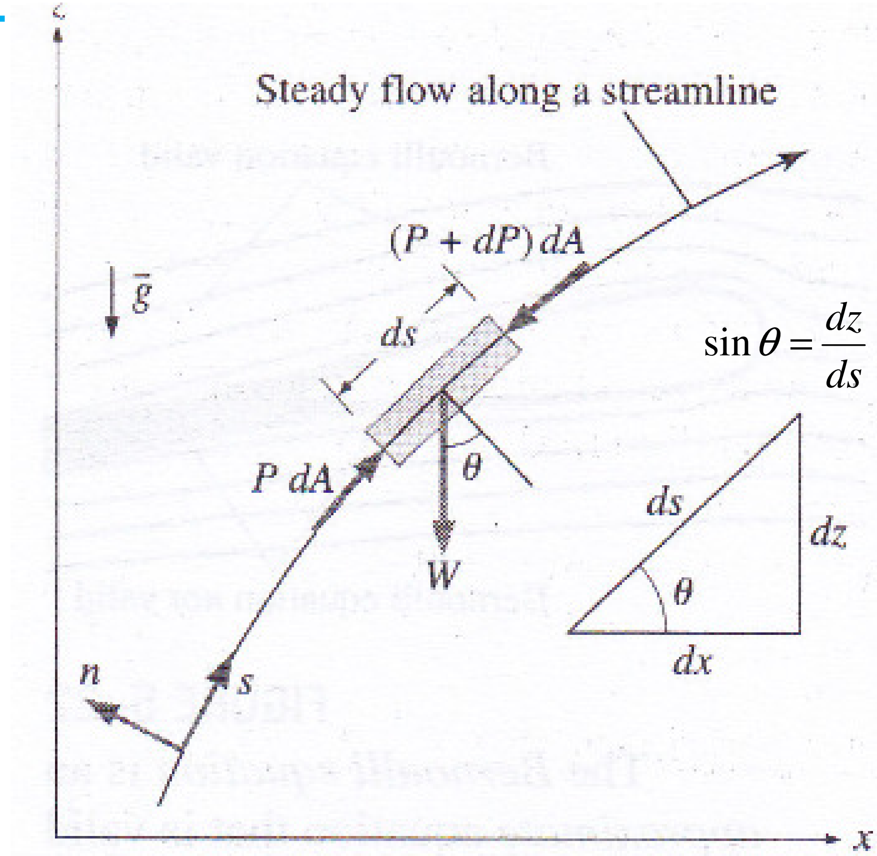


Fig. Forces acting on particle along streamline

W=weight of fluid $W = mg = (\rho dA ds)g$

Wsin(θ)= component acting along s-direction

dA= Area of flow

ds=length between sections along pipe

Derivation of Bernoulli's Equation

- Dividing eq (5) by ρ

$$\frac{dp}{\rho} + gdz + \frac{1}{2} dV^2 = 0 \qquad \text{Eq (6)}$$

- Integrating

$$\int \left(\frac{dp}{\rho} + gdz + \frac{1}{2} dV^2 \right) = contt \qquad \text{Eq (7)}$$

- Assuming incompressible and steady flow

$$\frac{P}{\rho} + gz + \frac{1}{2} V^2 = contt \qquad \text{Eq (8)}$$

- Dividing each equation by g

$$\frac{P}{\rho g} + z + \frac{V^2}{2g} = contt \qquad \text{Eq (9)}$$

- Hence Eq (9) for stead-incompressible fluid assuming no frictional losses can be written as

$$Z_1 + \frac{P_1}{\gamma} + \frac{V^2{}_1}{2g} = Z_2 + \frac{P_2}{\gamma} + \frac{V^2{}_2}{2g}$$
$$(\text{Total Head})_1 = (\text{Total Head})_2 \qquad \text{Eq (10)}$$

Above Eq(10) is general form of Bernoulli's Equation

Energy Line and Hydraulic Grade line

$$\frac{P}{\gamma} + z + \frac{V^2}{2g} = H$$

$$\text{Pressure head} + \text{Elevation head} + \text{Velocity head} = \text{Total Head}$$

Multiplying with unit weight,γ,

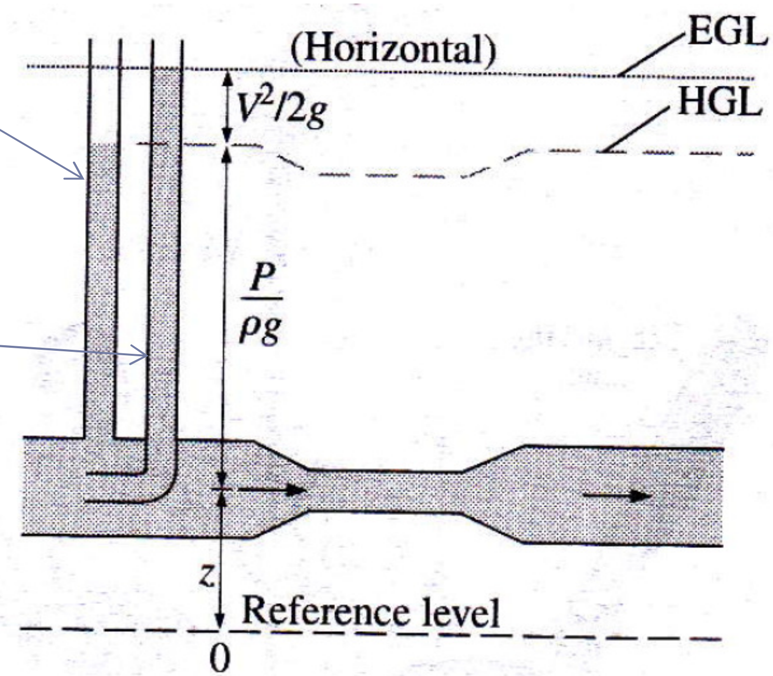
$$P + \rho g z + \rho \frac{V^2}{2} = contt$$

- Static Pressure : P
- Dynamic pressure : $\rho V^2 / 2$
- Hydrostatic Pressure: $\rho g Z$
- Stagnation Pressure: Static pressure + dynamic Pressure

$$P + \rho \frac{V^2}{2} = P_{stag}$$

Energy Line and Hydraulic Grade line

- **Measurement of Heads**
- Piezometer: It measures pressure head (P/γ).



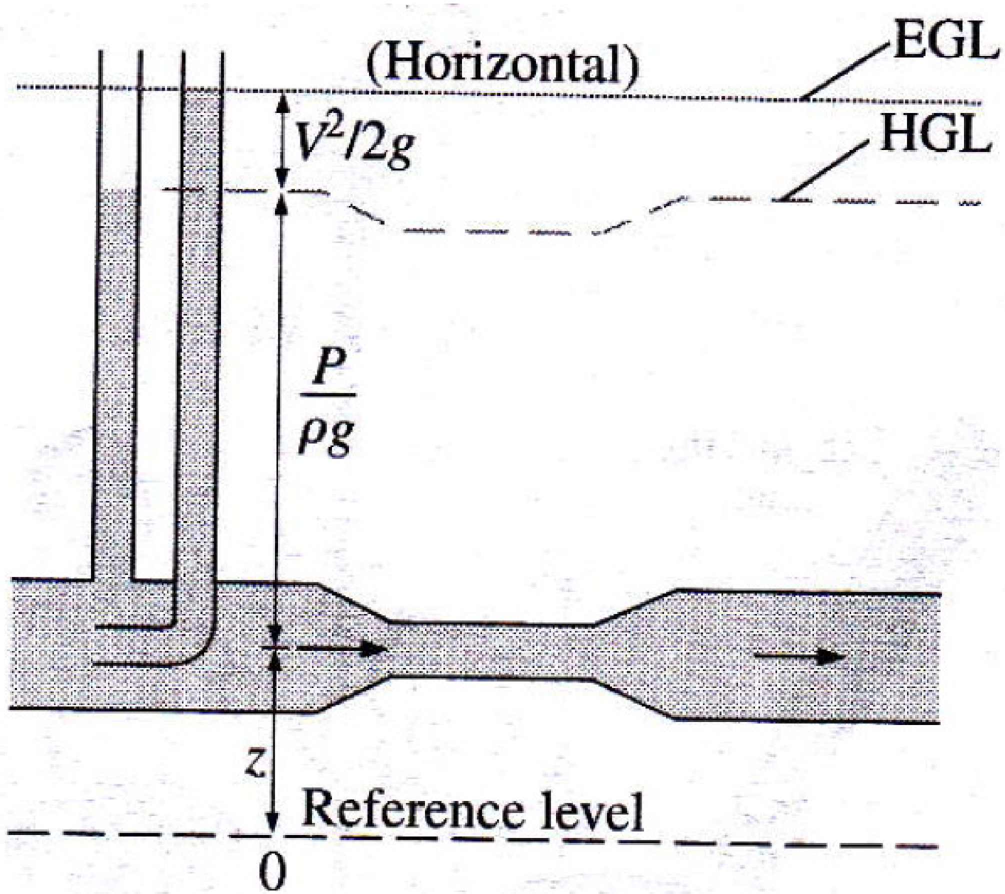
- Pitot tube: It measures sum of pressure and velocity heads i.e.,

$$\frac{P}{\gamma}+\frac{V^2}{2g}$$

What about measurement of elevation head !!

Energy Line and Hydraulic Grade line

- Energy line: It is line joining the total heads along a pipe line.
- HGL: It is line joining pressure head along a pipe line.



Energy Line and Hydraulic Grade line