

# Mach's number

## ➤ Mach's Number, M:

- It is the ratio of inertia force to the elastic force of flowing fluid.

$$\begin{aligned} M &= \sqrt{\frac{F_i}{F_e}} = \sqrt{\frac{\text{Mass.} \frac{\text{Velocity}}{\text{Time}}}{\text{Elastic Stress. Area}}} = \sqrt{\frac{\rho \frac{\text{Volume}}{\text{Time}} \cdot \text{Velocity}}{\text{Elastic Stress. Area}}} \\ &= \sqrt{\frac{\rho Q.V}{K.A}} = \sqrt{\frac{\rho AV.V}{K.A}} = \sqrt{\frac{\rho L^2 V^2}{KL^2}} = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C} \end{aligned}$$

$$\text{Where : } C = \sqrt{K/\rho}$$

# Weber's Number

- The Weber Number is a dimensionless value useful for analyzing fluid flows where there is an interface between two different fluids.
- The Weber Number is the ratio between the inertial force and the surface tension force and the Weber number indicates whether the kinetic or the surface tension energy is dominant. It can be expressed as

$$We = \rho v^2 l / \sigma$$

where

$We$  = Weber number (dimensionless)

$\rho$  = density of fluid ( $\text{kg/m}^3$ ,  $\text{lb/ft}^3$ )

$v$  = velocity of fluid ( $\text{m/s}$ ,  $\text{ft/s}$ )

$l$  = characteristic length ( $\text{m}$ ,  $\text{ft}$ )

$\sigma$  = surface tension ( $\text{N/m}$ )

Since the Weber Number represents an index of the *inertial force* to the *surface tension force* acting on a fluid element, it can be useful analyzing thin films flows and the formation of droplets and bubbles.

# Example - Calculating Reynolds Number

- ❖ A Newtonian fluid with a dynamic or absolute viscosity of  $0.38 \text{ Ns/m}^2$  and a specific gravity of  $0.91$  flows through a  $25 \text{ mm}$  diameter pipe with a velocity of  $2.6 \text{ m/s}$ .

The density can be calculated using the specific gravity like

$$\begin{aligned}\rho &= 0.91 (1000 \text{ kg/m}^3) \\ &= \underline{910} \text{ kg/m}^3\end{aligned}$$

The Reynolds Number can then be calculated using equation (1) like

$$\begin{aligned}Re &= (910 \text{ kg/m}^3) (2.6 \text{ m/s}) (25 \text{ mm}) (10^{-3} \text{ m/mm}) / (0.38 \text{ Ns/m}^2) \\ &= 156 (\text{kg m} / \text{s}^2) / \text{N} \\ &= \underline{156} \sim \text{Laminar flow}\end{aligned}$$

$$(1 \text{ N} = 1 \text{ kg m} / \text{s}^2)$$

# Example - Calculating Froude's Number

- ▶ **Question 1:** Find the Froude number if the length of the boat is 2m and velocity is 10 m/s.

Solution:

- ▶ Given: length  $l = 2\text{m}$ , velocity  $v = 10\text{ m/s}$

- ▶ The Froude number is given by,

- ▶ 
$$Fr = \frac{v}{\sqrt{gl}}$$

- ▶ 
$$Fr = \frac{10\text{ m/s}}{\sqrt{9.8\text{ m/s}^2 \times 2\text{ m}}}$$

- ▶ 
$$Fr = 0.451$$

Therefore, the Froude number of the boat is **0.451**.

- ▶ **Question 2:** Calculate the velocity of the moving fish in the water if its froude number is 0.72 and length 0.5 m.

**Solution:**

Given: length  $l = 0.5\text{m}$ , froude number  $Fr = 0.72$

- ▶ The froude number is given by,
- ▶  $Fr = v(gl)^{1/2} / v(gl)^{1/2}$ .
- ▶ The velocity of the moving fish is,
- ▶  $v = Fr \times (gl)^{1/2} / (gl)^{1/2}$
- ▶  $v = 0.72 \times (9.8 \times 0.5)^{1/2} / (9.8 \times 0.5)^{1/2}$
- ▶  $v = 1.59 \text{ m/s}$ .