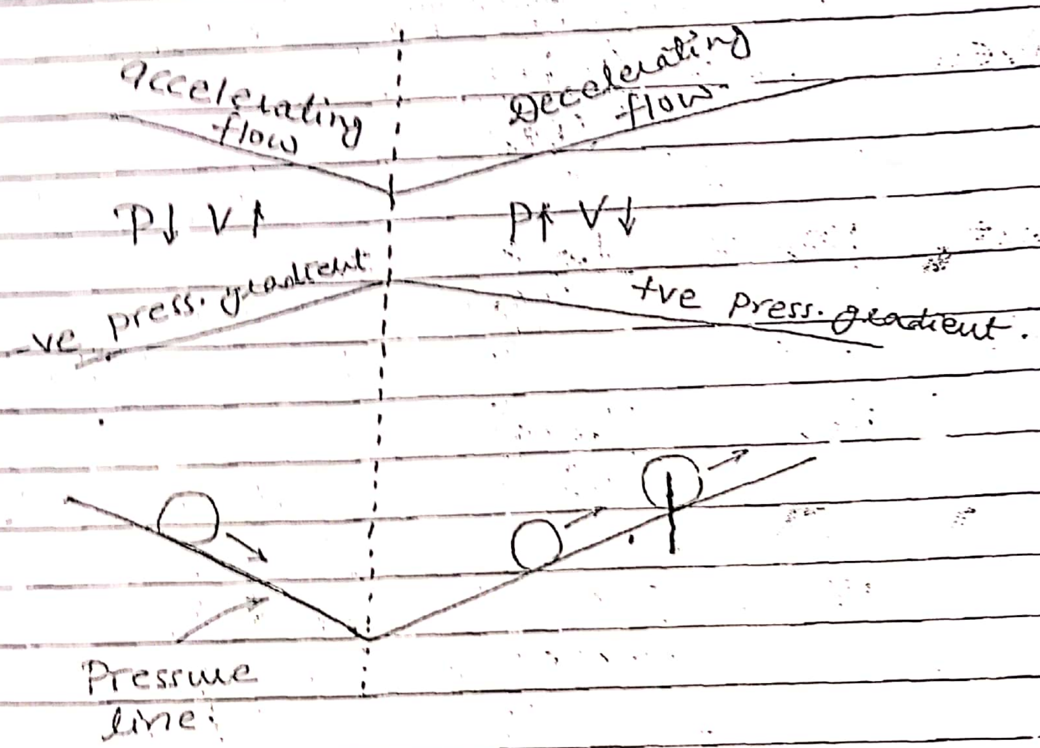
Imp

Boundary layer separation:-



In case of convergent flow the velocity increases & pressure decreases & the fluid slide's down the press. line when fluid flow in divergent passage the velocity decreases & pressure increases. i.e. The fluid move's under possitive pressure gradient & hence it has to climb press. hill if the velocity reduction is more momentum of particle may not be support to climb a pressure hill & hence the flow may reverse it's direction from boundary & this is known as boundary layer separation & reversal of flow & this occur's under +ve press. gradients & These press. gradient is also known as

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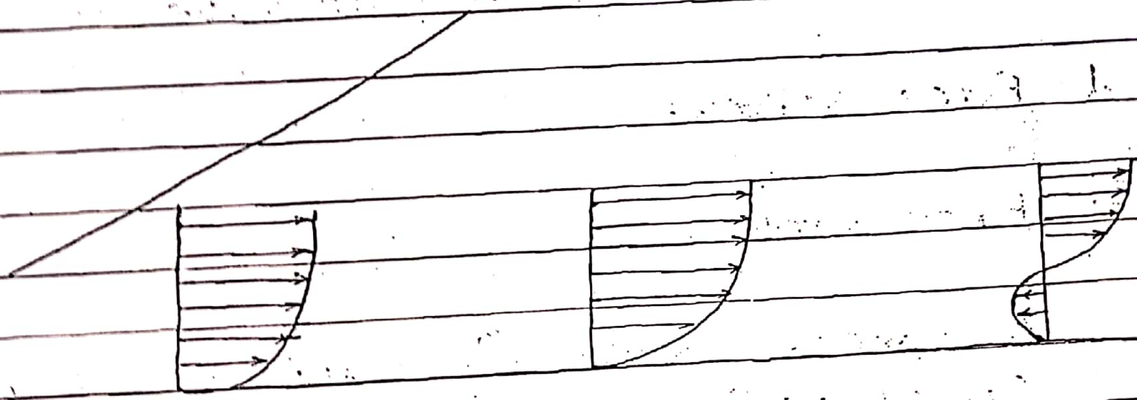
adverse press. gradient.

NOTE- For a fully developed flow the velocity profile remain's same.



$\left.\frac{du}{dy}\right|_{y=0} = +ve$ — No separation. (Attached flow)

$\left.\frac{du}{dy}\right|_{y=0} = 0$ — About to separate. Separation point. $\boxed{\tau = 0}$

$\left.\frac{du}{dy}\right|_{y=0} = -ve$ — separated flow, detached flow

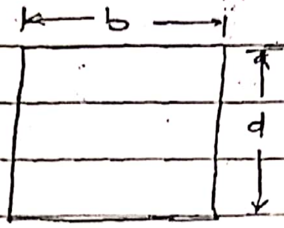
NOTE- The max. Thickness of boundary layer in pipe flow is equal to radius of pipe.

NOTE- Reynold's number in non-circular pipe's.

$$\boxed{Re = \frac{\rho V D_{eq.}}{\mu}}$$

where $D_{equivalent} = \frac{4A}{P}$ A = Area. P = Perimeter.

e.g. $D_{eq.} = \frac{4(bd)}{2(b+d)}$



$$\boxed{D_{eq.} = \frac{2bd}{b+d}}$$

Prob-105 For a velocity profile in laminar boundary layer –

$$\frac{u}{u_\infty} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$$

then find –

(i) Boundary layer thickness.
(ii) shear stress on the surface of the plate.
(iii) Drag force.
(iv) Average drag coefficient in term's of Reynold's number.

Soln:-

$$\frac{\tau_0}{\rho u_\infty^2} = \frac{d\theta}{dx}$$

$$\tau_0 = \rho u_\infty^2 \frac{d\theta}{dx}$$

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Momentum thickness,

$$\theta = \int_0^{\delta} \frac{u}{u_\infty}\left(1 - \frac{u}{u_\infty}\right) dy$$

$$\theta = \int_0^{\delta} \left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)\left\{1 - \left(\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}\right)\right\} dy$$

$$\theta = \frac{39\delta}{280}$$

then $\tau_0 = \rho u_\infty^2 \times \frac{d}{dx}\left(\frac{39\delta}{280}\right)$

$$\tau_0 = \frac{39}{280} \cdot \rho u_\infty^2 \frac{d\delta}{dx} \quad \text{——— (1)}$$

we know that,

$$\tau = \mu \frac{du}{dy}$$

$$\frac{u}{u_\infty} = \frac{3y}{2\delta} - \frac{y^3}{2\delta^3}$$

$$u = u_\infty \boxed{\frac{3y}{2\delta} - \frac{y^3}{2\delta^3}}$$

$$\frac{du}{dy} = u_\infty \left[\frac{3}{2\delta}(1) - \frac{3y^2}{2\delta^3}\right]$$

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$$\frac{du}{dy} = u_\infty \left[\frac{3}{2\delta} - \frac{3y^2}{2\delta^2} \right]$$

$$\left.\frac{du}{dy}\right|_{y=0} = \frac{3u_\infty}{2\delta}$$

$$\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0}$$

$$\tau_0 = \mu \times \frac{3u_\infty}{2\delta}$$

$$\tau_0 = \frac{3\mu u_\infty}{2\delta} \quad \text{——— (11)}$$

from eqn. (1) & (11),

$$\frac{39}{280} \rho u_\infty^2 \frac{d\delta}{dx} = \frac{3\mu u_\infty}{2\delta}$$

$$\frac{13}{140} \rho u_\infty \cdot \frac{d\delta}{dx} = \frac{\mu}{\delta}$$

$$\frac{13}{140} \frac{\rho u_\infty}{\mu} \cdot \delta \cdot d\delta = dx$$

Integrate above eqn.

$$\frac{13}{140} \frac{\rho u_\infty}{\mu} \cdot \frac{\delta^2}{2} = x + C$$

At $x = 0$, $\delta = 0$ $\Rightarrow$ $C = 0$.

$$\frac{13}{280} \frac{\rho u_\infty \delta^2}{\mu} = x$$

multiplied by x in both side.

$$\frac{13}{280} \frac{\rho u_\infty x \, \delta^2}{\mu} = x^2$$

$$\frac{13}{280} \times Re_x \cdot \delta^2 = x^2$$

$$\delta^2 = \frac{280 x^2}{13 \, Re_x}$$

$$\delta = \sqrt{\frac{280}{13}} \frac{x}{\sqrt{Re_x}}$$

$$\delta = \frac{4.64 x}{\sqrt{Re_x}}$$

$$\delta = \frac{4.64 x}{\sqrt{\frac{\rho u_\infty x}{\mu}}}$$

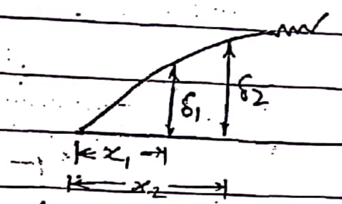
$$\delta \propto \frac{x}{\sqrt{x}}$$

$$\boxed{\delta \propto x^{1/2}}$$

δ i vary with parabolically.

$$\delta \propto \sqrt{x}$$

$$\boxed{\frac{\delta_2}{\delta_1} = \frac{\sqrt{x_2}}{\sqrt{x_1}}}$$



This is vailid only for laminar boundary layer.

(ii) from eqn. (11),

$$\tau_0 = \frac{3\mu u_\infty}{2\delta}$$

$$\tau_0 = \frac{3\mu u_\infty}{2\times \frac{4.64x}{\sqrt{Re_x}}}$$

$$\tau_0 = \frac{3}{2\times 4.64}\,\frac{\mu u_\infty \sqrt{Re_x}}{x}$$

$$\boxed{\tau_0 = \frac{0.323\,\mu u_\infty \sqrt{Re_x}}{x}}$$

$$\tau_0 \propto \frac{1}{x}\cdot\sqrt{x}$$

$$\tau_0 \propto \frac{1}{x^{1/2}}$$

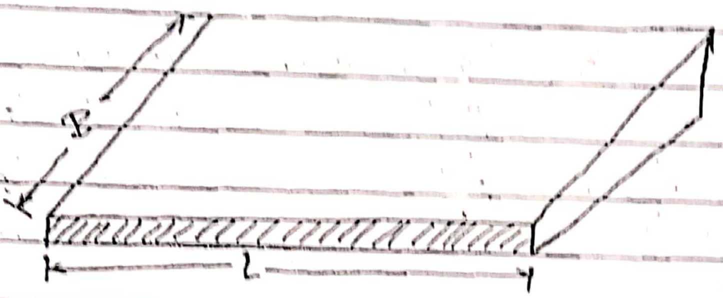
$$\tau_0 \propto x^{-1/2}$$

shear stresses decreases along length.

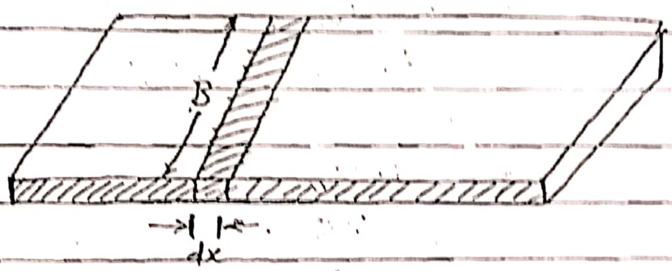
$$\boxed{\frac{\tau_{01}}{\tau_{02}} = \frac{\sqrt{x_2}}{\sqrt{x_1}}}$$

Shear stress vary with inversely proportional.

(iii)



As the shear stress τ_0 vary with length then take small element and find the drag force in small element & then integrate for finding total drag force.



The drag force on small element:

$$dF_D = \tau_0 \times B\,dx$$

Total drag force.

$$F_D = \int_0^l \tau_0 \times B\,dx$$

$$F_D = \int_0^l \frac{0.323\,\mu U_\infty \sqrt{Re_x}}{x}\,B\,dx$$

$$F_D = \int_0^l 0.323\,\frac{\mu U_\infty}{x}\sqrt{\frac{\rho U_\infty x}{\mu}}\,.\,B\,dx$$

$$F_D = 0.323\,\mu U_\infty B\,\frac{\sqrt{\rho U_\infty}}{\sqrt{\mu}}\int_0^l \frac{1}{\sqrt{x}}\,dx$$

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$$F_D = \frac{0.323\, \mu U_\infty B \sqrt{\rho U_\infty}}{\sqrt{\mu}} \left[2\sqrt{L}\right]$$

$$F_D = \frac{0.646\, \mu U_\infty B \cdot \sqrt{\rho}\, \sqrt{U_\infty}\, \sqrt{L}}{\sqrt{\mu}}$$

$$F_D = 0.646\, \mu U_\infty B \sqrt{\frac{\rho U_\infty L}{\mu}}$$

$$F_D = 0.646\, \mu U_\infty B \cdot \sqrt{Re_L}$$

$$C_D = \frac{F_D}{\frac{1}{2} \rho A U_\infty^2}$$

$$C_D = \frac{0.646\, \mu U_\infty B \sqrt{Re_L}}{\frac{1}{2} \rho A U_\infty^2}$$

$$C_D = \frac{1.292\, \mu \sqrt{Re_L}}{\rho L U_\infty}$$

$$C_D = \frac{1.292 \sqrt{Re_L}}{\frac{\rho U_\infty L}{\mu}}$$

$$C_D = \frac{1.292 \sqrt{Re_L}}{Re_L}$$

$$\boxed{C_D = \frac{1.292}{\sqrt{Re_L}}}$$

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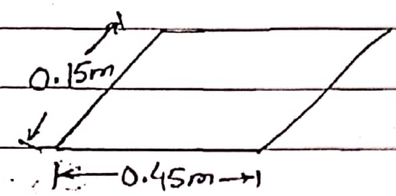
:- When velocity profile is not given use the following eqn. (Blausius equation)

Laminar	Turbulent.
$\delta = \frac{5x}{\sqrt{Re_x}}$	$\delta = \frac{0.576x}{(Re)^{1/5}}$
$C_{fx} = \frac{0.664}{\sqrt{Re_x}}$	$C_{fx} = \frac{0.059}{(Re)^{1/5}}$
$C_D = \frac{1.328}{\sqrt{Re_L}}$	$C_D = \frac{0.074}{(Re_L)^{1/5}}$

prob-106- Calculate the drag force on the plate 0.15m wide & 0.45m long placed in a stream of oil having a free stream velocity of 6m/s. also find the thickness of boundary layer & shear stress at the trailing edge. The density of oil has 925 Kg/m³ & Kinematic viscosity is equal to 0.9×10^{-4} m²/s.

Soln:-

$$Re_L = \frac{\rho u_\infty L}{\mu}$$



$$Re_L = \frac{u_\infty L}{\frac{\mu}{\rho}} = \frac{u_\infty L}{\nu}$$

$$Re_L = \frac{6\times0.45}{0.9\times10^{-4}} = 3\times10^{-4}$$

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As $Re_L < 10^5$

flow in the boundary layer is laminar.

(i)
$$C_D = \frac{F_D}{\frac{1}{2} \rho A U_\infty^2} = \frac{1.328}{\sqrt{Re_L}}$$

$$\frac{F_D}{\frac{1}{2} \times 925 \times 0.15 \times 0.45 \times 6^2} = \frac{1.328}{\sqrt{3 \times 10^4}}$$

$$\boxed{F_D = 8.6\ N} \text{ Ans.}$$

This is the drag force on one side of the plate.

(ii) Shear stress at the trailing edge.

$$C_{fx} = \frac{0.664}{\sqrt{Re_{x=L}}} = \frac{\tau_0}{\frac{1}{2} \rho U_\infty^2}$$

$$= \frac{0.664}{\sqrt{3 \times 10^4}} = \frac{\tau_0}{\frac{1}{2} \times 925 \times 6^2}$$

$$\boxed{\tau_0 = 63.8\ N/m^2} \text{ Ans.}$$

(iii) Boundary layer thickness.

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{5 \times 0.45}{\sqrt{3 \times 10^4}}$$

$$\boxed{\delta = 0.0129\ m} \text{ Ans.}$$

prob-107 - For an air flow over a flat plate the velocity u & boundary layer thickness δ can be expressed as:-

$$\frac{u}{u_\infty} = \frac{3y}{2\delta} \quad \frac{y^3}{2\delta^3}$$

& $\delta = \frac{4.64x}{\sqrt{Re_x}}$ If $u_\infty = 2 m/s$.

$$\nu_{air} = 1.5 \times 10^{-5} \ m^2/sec.$$
$$\rho = 1.32 \ Kg/m^3.$$

find the shear stress on the surface of plate at $x = 1m$.

Soln:- given,

$$\frac{u}{u_\infty} = \frac{3y}{2\delta} \quad \frac{y^3}{2\delta^3}.$$

$$u = u_\infty \left[\frac{3y}{2\delta} \quad \frac{y^3}{2\delta^3} \right]$$

$$\frac{du}{dy} = u_\infty \left[\frac{3(1)}{2\delta} - \frac{3y^2}{2\delta^3} \right]$$

$$\left.\frac{du}{dy}\right|_{y=0} = u_\infty \left[\frac{3}{2\delta} \right] \quad \text{---}$$

& $$\tau_0 = \mu \left.\frac{du}{dy}\right|_{y=0}.$$

$$\tau_0 = \frac{\mu \times 3 u_\infty}{2\delta} \quad \text{—— ①}$$

given $\delta = \frac{4.64 x}{\sqrt{Re_x}}$

but $Re_x = \frac{u_\infty \cdot x}{\mu/\rho}$

$$Re_x = \frac{u_\infty \cdot x}{\nu}$$

$$Re_x = \frac{2 \times 1}{1.5 \times 10^{-5}} = 1.33 \times 10^5$$

then $\delta = \frac{4.64 \times 1}{\sqrt{1.33 \times 10^5}} = 0.0127\ m.$

Now put the value in eqn. ①.

$$\tau_0 = \frac{3 \times 1.5 \times 10^{-5} \times 1.23 \times 2}{2 \times 0.0127}$$

$$\boxed{\tau_0 = 4.35 \times 10^{-3}\ N/m^2}\ \text{Ans.}$$
