Subject- Operations Management
Topic- Measures of Reliability

Module-2

Lecture Notes

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Mean time between failures (MTBF).

$$MTBF = \frac{TotalUptime}{NumberofFailures}$$

- MTBF is a measure of total uptime of the components(s) divided by the total number of failures within that time.
- Total Uptime is the measure of the total time a system or component is working, this is measured by taking the total time the machine should be operational, less the amount of time taken up by time to repair.
- This term is used for repairable systems.

Failure Rate

$$\lambda = \frac{1}{MTBF}$$

- Failure Rate is derived by taking the inverse of the mean time between failures.
- Failure Rate is a common tool to use when planning and designing systems.
- It allows to predict a component or systems performance. Failure rate is most commonly measured in number of failures per hour.

Mean time to repair (MTTR)

$$MTTR = \frac{Total\,Downtime}{Number of\,Failures}$$

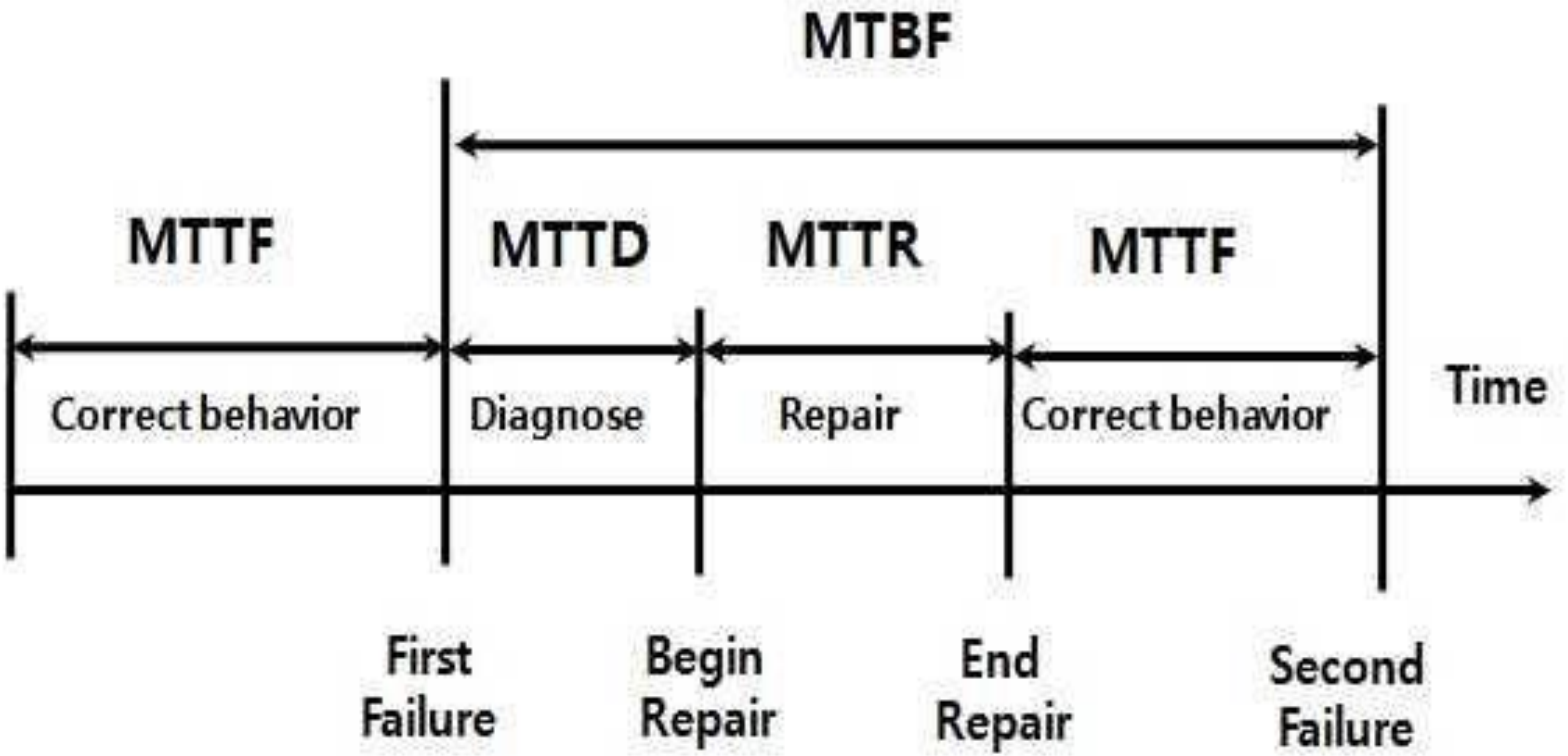
- Mean Time To Repair (MTTR) is a measure of the average downtime.
- MTTR is the ration of total downtime to number of failures.
- This takes the downtime of the system and divides it by the number of failures. again, be sure to check downtime periods match failures.

Mean time to failure (MTTF)

- This term is used for non repairable system.
- The expected time to failure for non repairable system is known as mean time to failure (MTTF).
- When MTTF is used as a measure, repair is not an option.

Mean time to diagnose (MTTD)

- **MTTD means the average time it takes to detect a problem inside an organization**.
- In other words, this indicator measures the elapsed time between the start of an issue (such as software malfunction or hardware failure) and its detection.


MTBF
MTTF
MTTD
MTTR
MTTF
Correct behavior
Diagnose
Repair
Correct behavior
Time
First
Failure
Begin
Repair
End
Repair
Second
Failure

Reliability

- Reliability is essentially the probability of a component or systems failure chances and is calculated as:
- if time is relatively small-

$$R = 1 - \lambda t$$

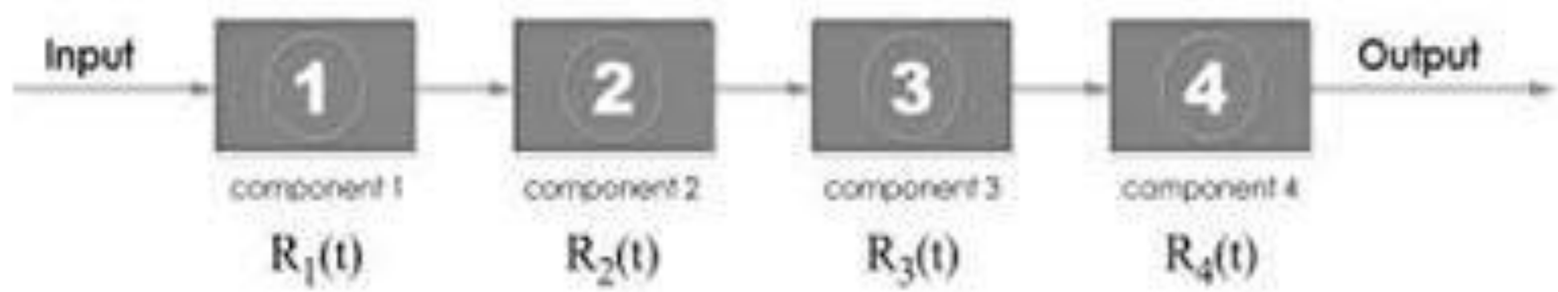
- Or if the time is large-

$$R = e^{-\lambda t}$$

- Calculating the reliability of a component allows you to design redundancy into a system. if a system exhibits high failure probability than placing an compnonent in parallel to increase total system reliability is termed as redundant component.

Reliability of the components in series

- This is a system in which all the components are in series and they all have to work for the system to work. If one component fails, the system fails.

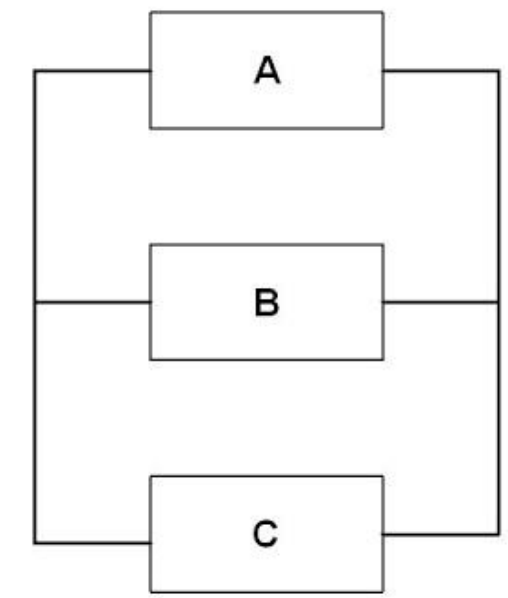


$$R_s(t) = R_1(t)R_2(t)R_3(t)\cdots R_n(t) = \prod_{i=1}^{n} R_i(t)$$

Reliability of components in parallel

- This is a system that will fail only if they all fail.

$$R_p = 1-(1-r_1)(1-r_2)\cdots(1-r_n)$$



Reliability of components in series- parallel

- This is a system where some of the components in series replicated in parallel.

Total system reliability

$$R_s = 1 - (1 - R)^n$$

Total System Reliability is a calculation which allows you to combine the reliabilities of several components to give a new value for system reliability.

A system consists of components which determine whether or not it will work. There are various types of configurations of the components in different systems

- Examples

1. A simple computer consists of a processor, a bus and a memory. The computer will work only if all three are functioning correctly. The probability that the processor is functioning is 0.99, that the bus is functioning 0.95, and that the memory is functioning is 0.99 .Processor Bus Memory

- input –→ Process or (0.99) → Bus (0.95) -→ Memor y (0.99) →output

The probability that the computer will work is:

Rel = .99 × .95 × .99 = 0.893475

So even though all the components have above 95% or more reliability, the overall Reliability of the computer is less that 90%.

2. A system consists of 5 components in series each having a reliability of 0.97. What is the reliability of the system?

$$\text{input} \longrightarrow \overset{C_1}{\boxed{0.97}} \longrightarrow \overset{C_2}{\boxed{0.97}} \longrightarrow \overset{C_3}{\boxed{0.97}} \longrightarrow \overset{C_4}{\boxed{0.97}} \longrightarrow \overset{C_5}{\boxed{0.97}} \longrightarrow$$

$$Rel = 0.97^5 = 0.86$$

With series systems, reliability decreases as the number of components increases.

With 6 components in series;

$$Rel = 0.97^6 = 0.832972$$

With 7 components; $Rel = 0.97^7 = 0.8079828$

3. An electronic product contains 100 integrated circuits. The probability that any integrated circuit is defective is .001 and the integrated circuits are independent. The product operates only if all the integrated circuits are operational. What is the probability that the product is operational?

Solution:

The probability that any component is functioning is .999. Since the product operates only if all 100 components are operational, the probability that the 100 components are functioning is:

$$Rel = .999^{100}$$

obtained in *R* with

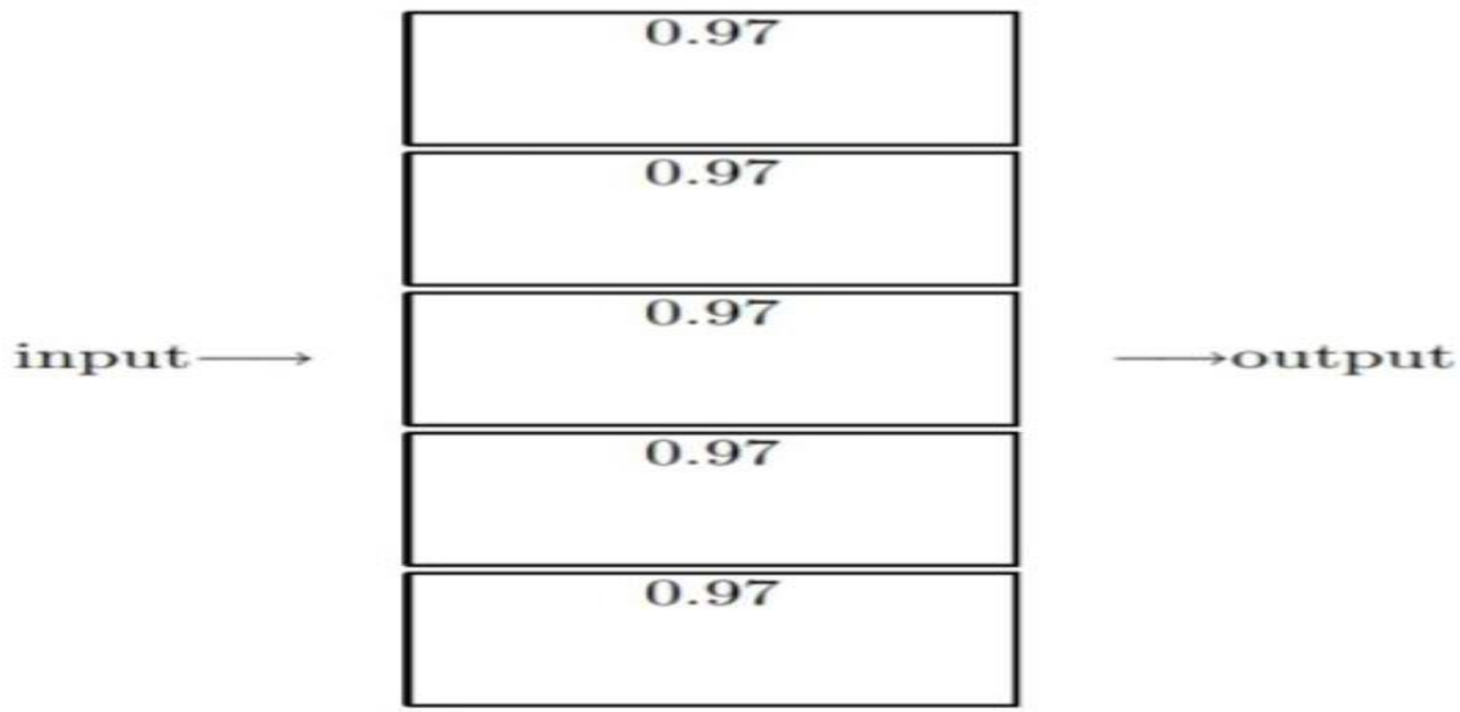
```
.999**100
[1] 0.9047921
```

The reliability is just over 90% even though each component has a reliability of 99.9%.

Bearing in mind that computing and electrical systems have hundreds or thousands of components, a series formation on its own will never be sufficiently reliable, no matter how high the individual component reliability is. The components need to be backed up in parallel.

Systems with parallel component

1. A system consists of 5 components in parallel. If each component has a reliability of 0.97, what is the overall reliability of the system?



System will function provided at least one of the 5 components works:

$$Rel = \text{P(At least one component is functioning)}$$

Taking the complementary approach,

P(at least one component functioning) $=$ 1- P(all components fail).

Therefore

$$Rel = 1 - (0.03)^5 \approx 1.00000$$

Series parallel system

1. Consider a system with 5 kinds of component, with reliabilities
   - component 1 : 0.95,
   - component 2 : 0.95,
   - component 3 : 0.70,
   - component 4 : 0.75,
   - component 5 : 0.90.

Because of the low reliability of the third and fourth components, they are replicated; the system contains 3 of the third component and 2 of the fourth component.

The System:

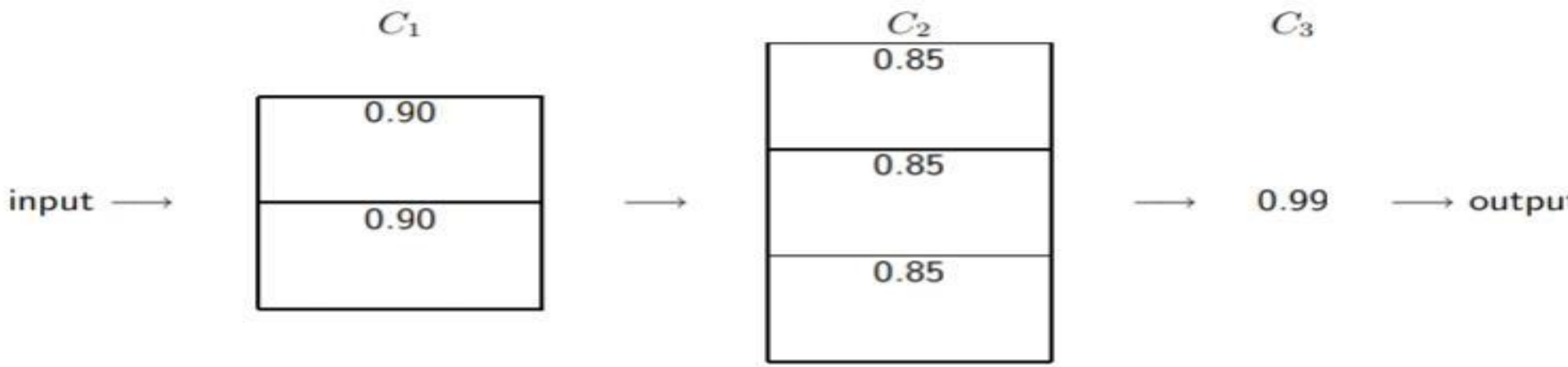
				0.70 0.70 0.70		0.75 0.75		
0.95	$\longrightarrow$	0.95	$\longrightarrow$		$\longrightarrow$		$\longrightarrow$	0.90
C_1	$\longrightarrow$	C_2	$\longrightarrow$	C_3	$\longrightarrow$	C_4	$\longrightarrow$	C_5
$P(C_1)$	$\times$	P(C_2)	$\times$	P(at least 1 C_3)	$\times$	P(at least 1 C_4)	$\times$	P(C_5)

$$Rel = .95 \times .95 \times (1 - .3^3) \times (1 - .25^2) \times .9$$

In *R*

```
.95*.95*(1-.3^3) * (1- .25^2) *.9
[1] 0.7409243
```

2. The following system operates only if there is a path of functional devices from left to right. The probability that each device functions is shown on the graph. Assume the devices fail independently.



Reliability: $P(\text{at least 1 of 2 } C_1) \quad \times \quad P(\text{at least 1 of 3 } C_2) \quad \times \quad P(C_3)$

$$1 - .1^2 \quad \times \quad 1 - .15^3 \quad \times \quad .99$$

In *R*

```
> (1-.2^2)* (1-.15^3)*.99
[1] 0.9471924
```