Reciprocating compressor

1.A single stage reciprocating compressor takes 1m^3 of air per minute at 1.013 bar and 15^{o}C and delivers it at 7 bar. Assuming that the law of compression is $P_v^{1.35}$= constant, and the clearance is negligible, calculate the indicated power?

Solution

Volume of air taken in, $V_1 = 1\ m^3/\min$

Intake pressure, $p_1 = 1.013$ bar

Initial temperature, $T_1 = 15 + 273 = 288$ K

Delivery pressure, $P_2 = 7$ bar

Law of compression: $Pv^{1.35}$ = constant

Indicated power I.P.:

Mass of air delivered per min.,

$$m = \frac{p_1 v_1}{RT_1} = \frac{1.013 \times 10^5 \times 1}{287 \times 288} = 1.266 kg/\min$$

Delivery temperature, $T_2 = T_1 \left(\frac{p2}{p1}\right)^{(n-1/n)}$

$$= 288\left(\frac{7}{1.013}\right)^{(1.35-1)/1.35} = 475.2K$$

Indicated work $= \frac{n}{n-1} mR(T_2 - T_1) kJ/\min$

$$= \frac{1.35}{1.35-1} \times 1.226 \times 0.287(475.2 - 288 = 254 kJ/\min$$

i.e., Indicated power I.P $= \frac{254}{60} = 4.23kW.(Ans)$

2.An air compressor cylinder has 150mm bore and 150mm stroke and the clearance is 15%. It operates between 1 bar, 27°C and 5 bar. Take polytrophic exponent n=1.3 for compression and expansion processes find?

i. **Cylinder volume at the various salient points of in cycle.**
ii. **Flow rate in m^3/min at 720 rpm and .**
iii. **The deal volumetric efficiency.**

Given

$D = 150\times10^{-3}$m	$P_2 = 5\times10^5$ N/m^2
$L = 150\times10^{-3}$ m	$T_1 = 27+273 = 300$K
$V_c = 0.15V_s$	N = 720rpm
$P_1 = 1\times10^5$ N/m^2	$pv^n = C$ n=1.3

Find

i. V_1, V_2, V_3, V_4
ii. FAD (V_a)
iii. η_v

Solution

$V_1 = V_c + V_s$

$$V_s = \frac{\pi}{4}D^2L.N = \frac{\pi}{4}(0.15)^2\times.0.15\times720 = 1.9085m^3/\min$$

$V_c = 0.15\ V_s$

$=0.15\times1.9085$

$V_c = 0.2862$ m^3/min

$V_1 = V_c + V_s$

$= 0.2862 + 1.9085$

$V_1 = 2.1948$ m^3/min

$$P_1V_1^n = P_2V_2^n$$

$$V_2 = V_1\left(\frac{P_1}{P_2}\right)^{1/n}$$

$$= 2.1948\left(\frac{1\times10^5}{5\times10_5}\right)^{1/1.3}$$

$$\boxed{V_2 = 0.6366\ m^3/min}$$

$$\boxed{V_3 = 0.2862\ m^3/min} = V_c$$

$V_c = V_3 \quad \therefore$

$$P_3V_3^n = P_4V_4^n$$

$$V_4 = V_3\left(\frac{P_3}{P_4}\right)^{1/4}$$

WKT

$$P_2 = P_3$$

$$P_1 = P_4$$

$$\therefore V_4 = V_3\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

$$= 0.2862\left[\frac{5\times10^5}{1\times10^5}\right]^{1/1.3}$$

$$\boxed{V_4 = 0.98674\ m^3/min}$$

$\therefore$ Volumetric efficiency $(\eta_v) = 1+k-k\left(\frac{P^2}{P_1}\right)^{1/n}$

k= Clearance Raito $= \frac{V_c}{V_s} = \frac{0.2862}{1.9085}$

$$\boxed{K = 0.1499}$$

$$\therefore \eta_v = 1+0.1499 - 0.1499\left[\frac{5}{1}\right]^{1/1.3}$$

$\eta_v = 0.633 = 63.3\%$

$$\boxed{\eta_v = 63.3\%}$$

$\therefore$ **WKT**

$$\eta_v = \frac{FAD}{V_s}$$

$\therefore FAD = \eta_v \times V_s$

$=0.633 \times 1.9085$

FAD = 1.2083 m^3/min

3.Calcute the diameter and stroke for a double acting single stage reciprocating air compressor of 50kW having induction pressure 100 kN/m^2 and temperature 150°C. The law of compression is $PV^{1.2}$ = C and delivery pressure is 500 kN/ m^2. The revolution/sec =1.5 and mean piston speed in 150 m/min. Clearance is neglected.

Given:

Double acting single stage

Compressor

IP = 50kW

$P_1 = 100 \times 10^3$ N/m^2

$T_1 = 15 + 273 = 288K$

$PV^{1.2} = C$ $\therefore n=1.2$

$P2 = 500 \times 10^3$ N/m^2

N = 1.5 rps = 1.5×60 rpm

2LN = 150m/min (Double acting)

Find

i. D and L

Solution

For double acting compressor average piston speed = 2LN

$\therefore$ 2LN = 150 m/min

$$\therefore L = \frac{150}{2 \times 1.5 \times 60} = 0.833m$$

L = 0.833 m

To Find D

$IP = W.N_w$

where

N_w = Number of working stroke

For Double acting $N_w = 2N$

For single acting $N_w = N$

$\therefore N_w = 2\times1.5\times60 = 180$ rpm

$$\therefore \text{W.D/cycle} = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.2}{1.2-1} \times 100 \times 10^3 \left(\frac{\pi}{4} D^2 \times 0.833 \right) \times \left[\left(\frac{(500)}{100} \right)^{\frac{0.2}{1.2}} - 1 \right]$$

W= 120764.2D^2

N-m

$$\therefore IP = \frac{W.N_w}{60}$$

$$50\times10^3 = \frac{1207642 D^2 \times 180}{60}$$

$D^2 = 0.1380$

D=0.371 m

4.A single acting reciprocating air compressor has cylinder diameter and stroke of 200 mm and 300 mm respectively. The compressor sucks air at 1 bar and 27°C and delivers at 8 bar while running at 100 r.p.m Find: 1. Indicated power of the compressor; 2. Mass of air delivered by the compressor per minute; and 3. temperature of the delivered by the compressor. The compression follows the law $P_V^{1.25}$ = C Take R as 287 J/kg K.

Solution.

Given

D=200 mm = 0.2m;

L = 300 mm = 0.3m;

$p_1 = 1$ bar $= 1\times10^5$ N/m^2;

$T_1 = 27^oC = 27 + 273 = 300K$;

$p_2 = 8$ bar;

N = 100 r.p.m

n=1.25;

R= 287 J/kg K

1.Indicated power of the compressor

$$V_1 = \frac{\pi}{4} D^2 L = 0.0094 m^3$$

We know that workdone by the compressor for polytropic compression of air as

$$W = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

$$= \frac{1.25}{1.25-1} \times 1\times10^5 \times 0.0094 \left[\left(\frac{8}{1} \right)^{\frac{1.25}{1.25-1}} - 1 \right] N-m$$

=4700(1.516-1) = 2425 N-m

Since the compressor is single acting, therefore number of working strokes per minute,

$$N_w = N = 100$$

$\therefore$ Indicated power of the compressor

$$= \frac{W \times N_w}{60} = \frac{2425 \times 100}{60} = \mathbf{4042kW}$$

2.Mass of air delivered by the compressor per minute

Let m= Mass of air delivered by the compressor per stroke.

We know that $P_1V_1 = mRT_1$

$$\therefore \quad m = \frac{P_1V_1}{RT_1} = \frac{1\times10^5 \times 0.0094}{287 \times 300} = 0.0109kg \text{ per stroke}$$

and mass delivered minute $= m\times N_w = 0.0109\times100 =$ **1.09kg**

3. Temperature of air delivery by the compressor

Let T_2= Temperature of air delivered by the compressor.

We know that $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = \left(\frac{8}{1}\right)^{\frac{1.25-1}{1.25}} = 8^{0.2} = 1.516$$

$$T_2 = 1.516 \times T_1 = 1.516 \times 300 = 454.8K = \mathbf{181.8^oK}$$

5.A single –stage double –acting air compressor is required to deliver 14 m³ of air per minute measured at 1.013 bar and 150°C. The delivery pressure is 7 bar and the speed 300 r.p.m. Take the clearance volume as 5% of the swept volume with the compression and expansion index of η = 1.3 Calculate:

i. Swept volume of the cylinder;

ii. The delivery temperature;

iii. Indicated power.

Solution

Quantity of air to be delivered		=	14 m^3/min
Intake pressure and temperature	p_1	=	1.0.13 bar,
	T_1	=	15 + 273 = 288 K
Delivery pressure	p_2	=	7 bar
Compressor speed,			N = 300 r.p.m
Clearance volume,			$V_c = 0.05\ V_s$
Compression and expansion index			n=1.3

Swept volume of the cylinder, V_s:

Swept volume $V_s = V_1 - V_3 = V_1 - V_c = V_1 - 0.05\ V_s$

$$V_1 - V_4 = \frac{FAD}{N_w}$$

and $$V_1 - V_4 = \frac{14}{300 \times 2} = 0.0233\ m^3$$

Now, $V_1 = 1.05\ V_8$ and $$\frac{V_4}{V_3} = \left(\frac{p_2}{p_1}\right)^{1/n} = \left(\frac{7}{1.013}\right)^{1/1.3} = 4.423$$

i.e., $V_1 - V_3 = 4.423\ V_3 = 4.423 \times 0.05\ V_s = 0.221\ V_s$

∴ $(V_1 - V_4) = 1.05\ V_s - 0.221\ \ \ V_s = 0.221\ V_s$

$$\therefore \quad V_8 \frac{0.0233}{1.05-0.221} = 0.0281 m^3$$

i.e Swept volume of the cylinder = 0.0281 m³ .

ii. The delivery temperature, T_2

Using the relation $$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

$$\therefore \quad T_2 = T_1 \times \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}} = 288 \times \left(\frac{7}{1.013}\right)^{\frac{1.3-1}{1.3}} = 450K$$

$\therefore$ Delivery temperature = 450 – 273 = 177°C

iii Indicated power:

Indicated power $$= \frac{n}{n-1} p_1 (V_1 - V_4) \left\{ \left(\frac{P_2}{p_1}\right)^{\frac{n-1}{n}} - 1 \right\}$$

$$= \frac{1.3}{1.3-1} \times \frac{1.013 \times 10^5 \times 14}{10^3 \times 6} \left\{ \left(\frac{7}{1.013}\right)^{\frac{1.3-1}{1.3}} - 1 \right\}$$

= 57.56 kW

Indicated power = 57.56 kW

6. A single stage single acting air compressor delivers 0.6 kg of air per minute at 6 bar . The temperature and pressure at the end of suction stroke are 30°C and 1 bar. The bore and stroke of the compressor are 100 mm and 150 mm respectively. The clearance is 3% of the swept volume Assuming the index of compression and expansion to be 1.3. find:

i. Volumetric efficiency of the compressor

ii. Power required if the mechanical efficiency is 85%, and

iii. Speed of the compressor (r.p.m)

Given

Mass of air delivered, m=0.6kg/min

Delivery Pressure, p_2 =6 bar

Induction Pressure, p_1 =6 bar

Induction temperature, $T_1 = 30 + 273 = 303$ K

Bore, D= 100mm = 0.1m

Stroke length , L=150mm = 0.15 m

Clearance volume, $V_c = 0.03\ V_s$

Mechanical efficiency η_{mech} =85%

i. Volumetric efficiency of the compressor, η_{vol}:

$$\eta_{vol} = 1 + k - k\left(\frac{P_2}{p_1}\right)^{\frac{1}{n}}$$

Where $k = \frac{V_c}{V_s} = 0.03$

$$\therefore\ \eta_{vol} = 1 + 0.03 - 0.03\left(\frac{6}{1}\right)^{\frac{1}{1.3}} = 0.91096 \text{ or } 91.096\%$$

ii. Power required if the mechanical efficiency is 85%, and

$$\text{Indicated power} == \frac{n}{n-1} mRT_1\left[\left(\frac{P_2}{p_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$\frac{1.3}{1.3-1} \times \frac{0.6}{60} \times 0.287 \times 303\left[\left(\frac{6}{1}\right)^{\frac{1.3-1}{1.3}} - 1\right] = 1.93kW$$

$$\therefore \text{ Power required to drive the compressor:} = \frac{1.93}{\eta_{mech}} = \frac{1.93}{0.85} = 2.27kW$$

iii. Speed of the compressor (r.p.m)

Free air delivery, F.A.D $= \frac{mRT_1}{p_1} \times \frac{0.6 \times 0.287 \times 1000 \times 303}{1 \times 10^5} = 0.5218\ m^3/min$

Displacement volume $= \frac{F.A.D}{\eta_{vol}} = \frac{0.5218}{0.91096} = 0.5728\ m^3/min$

Also $0.5728 = \frac{\pi}{4} D^2 L \times N$ (for single – acting compressor)

or $0.5728 = \frac{\pi}{4} 0.1^2 \times 0.15 \times N$

$\therefore$ Speed of compressor N= $\frac{0.5728\times4}{\pi\times0.1^2\times0.15}$ = **486.2 r.p.m**

7.A single stage , single air compressor running at 1000 r.p.m delivers air at 25 bar . For this purpose the induction and free air conditions can be taken as 1.013 bar and 150°C and the free air delivery as 0.25 m^3/min. The clearance volume is 3% of the swept volume and the stroke bore ratio is 1:2:1 Take the index of compression and expansion as 1.3. calculate also the indicated power and the isothermal efficiency

Given data:-

With clearance volume

Single stage single acting air compressor

N=1000 rpm

p_2= 25 bar = 25×10^5 N/m^2

$T_1 = 15^o = 273+15 = 288K$

$V_c = 0.03V_s$

$$\frac{V_c}{V_s} = k = 0.03$$

$$\frac{Stroke(D)}{Bore(L)} = \frac{1}{2}$$

L = 2D

n = 1.3

$PV^n = C$ (polytropic process) To find

i. η_v , D and L
ii. (I.P) and η_{Iso}

i. $$\eta_v = 1+k-k\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

$$\eta_v = 1+0.03-0.03\left(\frac{25\times10^5}{1.013\times10^5}\right)^{\frac{1}{1.3}}$$

$$\eta_v = 0.6766$$

ii. $IP = \frac{Wpoly \times N}{60}$

Inductor power =I.P

$$Wpoly = \frac{n}{n-1} p(V_1 - V_4)\left[\left(\frac{P_2}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$$\therefore \eta_v = \frac{FAD/V_a}{V_S}$$

$$V_s = \frac{V_a}{\eta_v} = \frac{0.25}{0.6766}$$

$V_s = 0.369\ m^3/min$

$$V_s = \frac{\pi}{4} \times D^2 \times L$$

$$0.369 = \frac{\pi}{4} \times D^2 \times 2D = \frac{\pi D^3}{2}$$

D =0.617 m = 617 mm

L = 2D = 2 × 0.617

L = 1234 mm

$V_c = 0.011\ m^3/min = V_3$

$V_1 = V_c + V_s$

$V_1 = 0.011 + 0.369$

3-4 is expansion

$pv^n = C$

$$\frac{P_4}{P_3} = \left(\frac{v_3}{v_4}\right)^n$$

$$\frac{v_4}{v_3} = \left(\frac{P_3}{P_4}\right)^{\frac{1}{\eta}} = \left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$$

$$V_4 = V_3\left(\frac{P_2}{P_1}\right)^{\frac{1}{\eta}}$$

$V_4 = 0.129\ m^3/min$

$$Wpoly = \left(\frac{n}{n-1}\right)p(V_1 - V_4)\left[\left(\frac{P_2}{p_1}\right)^{\frac{n-1}{n}}\right]$$

$$= \frac{1.3}{0.3}\times 1.013\times 10^5(0.38-0.129)\left[\left(\frac{2.5}{1.013}\right)^{\frac{0.3}{1.3}}-1\right]$$

$W.D_{poly} = 120713.193N\text{-}min$

$$I.P = \frac{W.Dpoly\times N}{60}$$

$$I.P = \frac{12713\times 1000}{60}$$

$$\eta_{iso} = \frac{W_{iso}}{W_{poly}}$$

$$W_{iso}\ \ p_1(V_1 - V_2)\ln\left(\frac{p_2}{p_1}\right)$$

$$\eta_{iso} = \frac{p_1(V_1 - V_2)\ln\left(\frac{p_2}{p_1}\right)}{\left(\frac{n}{n-1}\right)p_1(V_1 - V_4)\left(\frac{p_2}{p_1}\right)^{\frac{\eta-1}{\eta}} - 1}$$

$$\eta_{iso} = \frac{1.013\times 10^5\times(0.0380-0.129)\ln\left(\frac{25}{1.013}\right)}{120713.19}$$

$\eta_{iso} = 0.675$

$\eta_{iso} = 67.5\%$

8.A two stage air compressor air from 1 bar and 20°C to 42 bar. If the law of compression is $p_v^{1.35}$ = constant and the intercooling is complete to 20°C, find per kg of air:1. The work done is compressing; and 2. The mass of water necessary for abstracting the heat in the intercooler, if the temperature rise of the cooling water is 250°C

p_1= 1 bar = 1×10^5 N/m^2

T_1 = 20° C= 20+273 =293K

p_3= 42 bar = 42×10^5 N/m^2

n=1.35

T_3 = 20° C= 20+273 =293K; m= 1 kg;

Rise in temperature of cooling water =25° C;

R=287J/kg K c_p= 1 kJ /kg K

we know that for complete intercooling. the intercooler pressure

$$p_2=\sqrt{p_1 p_3}=\sqrt{1\times 42}=6.48\ bar$$

and volume of air admitted for compression

$$V_1\frac{mRT_1}{p_1}=\frac{1\times287\times293}{1\times10^5}=0.84m^3/kg \text{ of air}$$

1. Work done compressing the air

$$W=\left(\frac{n}{n-1}\right)\times p_1 v_1\left[\left(\frac{P_2}{p_1}\right)^{\frac{n-1}{n}}+\left(\frac{P_2}{p_1}\right)^{\frac{n-1}{n}}-2\right]$$

$$=\frac{1.35}{1.35-1}\times1\times10^5\times0.84\left[\left(\frac{6.48}{1}\right)^{\frac{1.35-1}{1.35}}+\left(\frac{42}{6.48}\right)^{\frac{1.35-1}{1.35}}-2\right]N-m$$

$=3.24\times10^5$ (1.62 +1.62 -2) = **4.017×10^5 N-m**

2.Mass of water necessary for abstracting the heat in the intercooler.

Let m_w= Mass of water necessary /kg of air ,and

T_2 = Temperature of the air entering the intercooler.

$$\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}=\left(\frac{6.48}{1}\right)^{\frac{1.35-1}{1.35}}=1.622$$

$T_2 = T_1 \times 1.622 = 293 \times 1.622 = 475.6$ K

We know that heat gained by water

= Heat lost by air

$\therefore m_w \times c_w \times$ Rise in temperature

$= mc_p(T_2 - T_3)$

$m_w \times 4.2 \times 25 = 1 \times 1(475.6 - 293) = 182.6$

$\mathbf{m_w = 1.74kg}$

9.A two- stage acting reciprocating compressor takes in air at the rate of 0.2 m^3/s. The intake pressure and temperature of air 0.1MP_a and 16°C. The air is compressed to a final pressure of .7Mp_a. The intermediate pressure is ideal and intercooling is perfect. The compression index in both the stages is 1.25 and the compressor runs at 600 r.p.m. Neglecting clearance determine:

i. The intermediate pressure
ii. The total volume of each cylinder,
iii. The power required to drive the compressor and
iv. The rate of heat rejection in the intercooler.

Take c_p= 1.005 kJ/kg K and R =0.287 kJ/kg K

Solution .

Intake volume	$V_1 = 0.2\ m^3/s$
Intake pressure	$p_1 = 0.1MP_a$,
Intake temperature	$T_1 = 16 + 273 = 289$ K
Final pressure	$p_3 = 0.7MP_a$
Compression index in both stages,	$n_1 = n_2\ n = 1.25$
Speed of the compressor	N = 600 r.p.m
	c_p = 1.005kJ/kg K; R = 0.287 kJ/kg K

i. The power required to drive the compressor, P_2:

$$p_2 = \sqrt{p_1 p_3} = \sqrt{0.1 \times 0.7} = 0.2646\ MP_a$$

ii. The total volume of each cylinder , V_{s1}, V_{s2}:

We know that $V_{s1} \times \frac{N}{60} = V_1$ or $V_{s1} \times \frac{600}{60} = 0.2$

$\therefore$ V_{s1} (Volume of L.P cylinder) $= \dfrac{600 \times 0.2}{60} = 0.02 m_3$ (Ans).

Also $\quad p_1 V_{s1} = p_1 V_{s2}$ or $\quad V_{s2} = \dfrac{p_1 V_{s1}}{p_2}$

V_{S2} (Volume of H.P. Cylinder) $= \dfrac{0.1 \times 0.02}{0.2646} = 0.00756 m^3$ (Ans)

iii. The rate of heat rejection in the intercooler:

Mass of air handled, $m = \dfrac{p_1 V_1}{RT_1} = \dfrac{(0.1 \times 10^3) \times 0.2}{0.287 \times 289} = 0.241 kg/s$

Also, $\dfrac{T_2}{T_1} = \left(\dfrac{p_2}{p_1}\right)^{\frac{n-1}{n}}$ or $\quad \dfrac{T_2}{289} = \left(\dfrac{0.2646}{0.1}\right)^{\frac{1.25-1}{1.25}}$ or T_2=351.1K

$\therefore$ Heat rejected in the intercooler = $m \times c_p \times (T_2 - T_1)$

= 0.241×1.005× (351.1-289) = **15.04 kJ/s or 15.04 kW.(Ans)**

10. A single acting reciprocating air compressor has a swept volume of 2000 cm^3 and runs at 800 rpm. It operates with pressure ratio of 8 and clearance of 5% of the swept volume. Inlet pressure and temperature are 1.013 bar, and 15°C respectively. Assume n=1.25 for both compression and expansion. Find

- **i. Indicated power**
- **ii. Volumetric efficiency**
- **iii. Mass flow rate**
- **iv. FAD**
- **v. Isothermal efficiency**
- **vi. Actual Power required to drive the compressor if η_{mech}=85%**

Given

Single acting reciprocating compressor

V_s = 2000 cm^3 = 0.002 m^3

N = 800 rpm

$\dfrac{P_2}{P_1} = \dfrac{P_3}{P_4} = 8$

V_c = 5% V_s

= 0.05×0.002

V_c = $0.0001\ m^3$

p_1 = $1.013 \times 10^5\ N/m^2$

T_1 = $15+273 = 288\ K$

Pv^n = C n = 1.25

Find: i)IP ii)η_v iii)m iv) FAD(V_a) v) η_{iso} vi) P_{act} if η_m=85%

Solution

WKT

Clearance ratio (k) = $\frac{V_c}{V_s} = \frac{0.0001}{0.002}$

k	=	0.05

$\therefore$ Volumetric efficiency (η_v) = $1+k-k\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}}$

= $1+0.05 - 0.05\,(8)^{1/1.25}$

η_v = 78.61%

WKT

$\eta_v = \frac{FAD}{V_s}$

FAD = $\eta_v \times V_s$

= 0.7861×0.002

= $1.5722 \times 10^{-3}\ m^3$

$\frac{FAD}{min} = \frac{FAD}{Stroke} \times Speed$

= $1.5722 \times 10^{-3} \times 800$

$\therefore$ FAD = $1.2578\ m^3/min$

To find mass flow rate

Pv = mRT

m = $\frac{pV}{RT} = \frac{1.013 \times 10^5 \times 1.2578}{287 \times 288}$

m = 1.542 kg/min

To find – Indicated power

IP = $\frac{W.D \times N_\omega}{\text{sec}}$

N_w = N (single acting) = 800 rpm.

W.D = $\frac{n}{n-1} mR(T_2 - T_1)$ kJ/min.

T_2 = $T_1 \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}$

= $288(8)^{\frac{1.25-1}{1.25}}$

T_2 = 436.53 K

∴ W.D = $\frac{1.25}{0.25} \times 1.542 \times 0.287(436.53 - 2880$

W.D = 328.66 kJ/min

∴ IP = $\frac{W.D}{s} = \frac{328.66}{60}$

IP = 5.48 kW

To find isothermal efficiency

W_{iso} = $P_1 V_1 \ln\left(\frac{P_2}{P_1}\right)$

= $mRT_1 \ln\left(\frac{P_2}{P_1}\right)$

= 1.542×0.287×288×ln (8)

$(W.D)_{iso}$ = 265.04 kJ/min

η_{iso} = $\frac{(W.D)_{iso}}{(W.D)_{act}}$

∴ Actual work= $\eta_{mech} \times (W.D)_{theoretical}$

= 0.85×328.66

$(W.D)_{act}$ = 279.361 kJ/min

$$\therefore \eta_{iso} = \frac{265.04}{279.361} \times 100$$

η_{iso} = 94.87%

11. An air compressor takes in air at 1 bar and 20°C and compresses it according to law $pv^{1.2}$=constant. It is then delivered to a receiver at a constant pressure of 10 bar. R=0.287 kJ/kg K. Determine

(i) Temperature at the end of compression

(ii) Workdone and heat transferred during compression per kg of air.

Solution

T_1=20+273=293 K; P_1=1 bar; P_2=10 bar

Law of compression : $pv^{1.2}=C;\ R=287$ J/kgK

(i) Temperature at the end of compression T_2

For compression process 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} = \left(\frac{10}{1}\right)^{\frac{1.2-1}{1.2}} = 1.468$$

$T_2=T_1\times1.468 = 293\times1.468 = 430$ K or 157°C

(ii) Workdone and heat transferred during compression per kg of air:

$$\text{Workdone, } W = mRT_1 \frac{n}{n-1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$$

$$= 1\times0.287\times293\times\left(\frac{1.2}{1.2-1}\right)\left(\frac{10}{1}\right)^{\frac{1.2-1}{1.2}} - 1 = 236.13 kJ/kg\ of air$$

Heat transferred during compression,

$$Q = W + \Delta U$$

$$= \frac{p_1v_1 - p_2v_2}{n-1} + c_v(T_2 - T_1) = (T_2 - T_1)\left[c_v - \frac{R}{n-1}\right]$$

$$= (430-293)\left[0.718 - \frac{0.287}{1.2-1}\right] = -98.23 \text{ kJ/kg}$$

Negative sign indicates heat rejection.