

Generation of Three-phase Balanced Voltages

In the first lesson (No. 12) of the previous module, the generation of single-phase voltage, using a multi-turn coil placed inside a magnet, was described. It may be noted that, the scheme shown was a schematic one, whereas in a machine, the windings are distributed in number of slots. Same would be the case with a normal three-phase generator. Three windings, with equal no. of turns in each one, are used, so as to obtain equal voltage in magnitude in all three phases. Also to obtain a balanced three-phase voltage, the windings are to be placed at an electrical angle of 120° with each other, such that the voltages in each phase are also at an angle of 120° with each other, which will be described in the next section. The schematic diagram with multi-turn coils, as was shown earlier in Fig. 12.1 for a single-phase one, placed at angle of 120° with each other, in a 2-pole configuration, is shown in Fig. 18.1a. The waveforms in each of the three windings (R, Y & B), are also shown in Fig. 18.1b. The windings are in the stator, with the poles shown in the rotor, which is rotating at a synchronous speed of N_s (r/min, or rpm), to obtain a frequency of $f = ((p \cdot N_s) / 120)$ (Hz), p being no. of poles [$p = 2$] (see lesson no. 12).

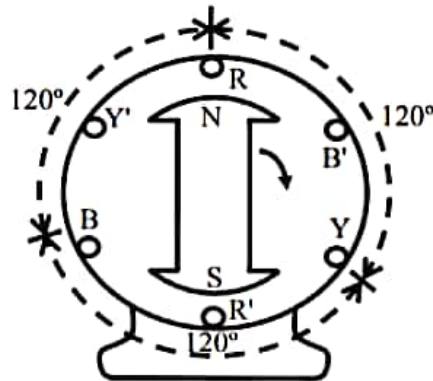


Fig. 18.1 (a) Schematic diagram of three windings of stator for the generation of three phased balanced voltage (2-pole rotor).

Three-phase Voltages for Star Connection

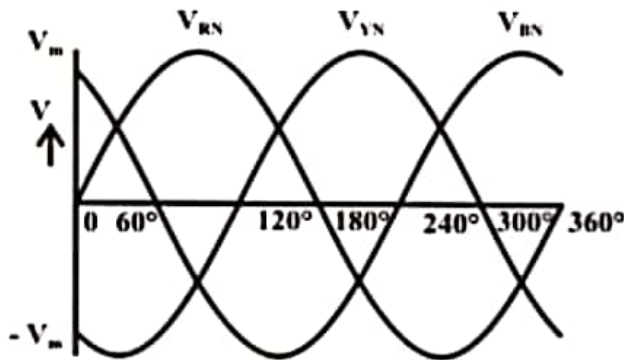


Fig. 18.1 (b) Three-phase balanced voltage waveforms with the source star-connected (the phase sequence, R-Y-B)

The connection diagram of a star (Y)-connected three-phase system is shown in Fig. 18.2a, along with phasor representation of the voltages (Fig. 18.2b). These are in continuation of the figures 18.1a-b. Three windings for three phases are R (+) & R'(-), Y (+) & Y'(-), and B (+) & Y'(-). Taking the winding of one phase, say phase R as an example, then R with sign (+) is taken as start, and R' with sign (-) is taken as finish. Same is the case with two other phases. For making star (Y)-connection, R', Y' & B' are connected together, and the point is taken as neutral, N. Three phase voltages are:

$$e_{RN} = E_m \sin \theta; \quad e_{YN} = E_m \sin (\theta - 120^\circ);$$

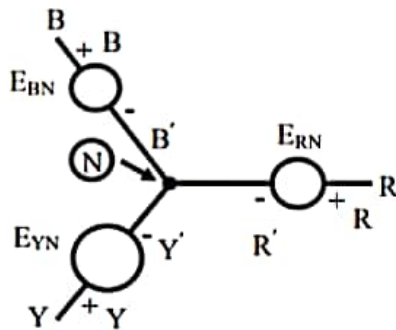
$$e_{BN} = E_m \sin (\theta - 240^\circ) = E_m \sin (\theta + 120^\circ)$$

It may be noted that, if the voltage in phase R (e_{RN}) is taken as reference as stated earlier, then the voltage in phase Y (e_{YN}) lags e_{RN} by 120° , and the voltage in phase B (e_{BN}) lags e_{YN} by 120° , or leads e_{RN} by 120° . The phasors are given as:

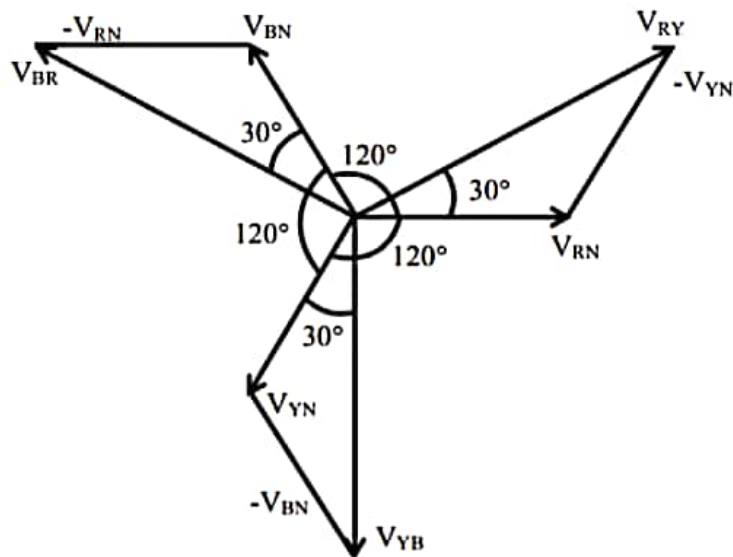
$$E_{RN} \angle 0^\circ = E(1.0 + j0.0):$$

$$E_{YN} \angle -120^\circ = E(-0.5 - j0.866);$$

$$E_{BN} \angle +120^\circ = E(-0.5 + j0.866).$$



(a)



(b)

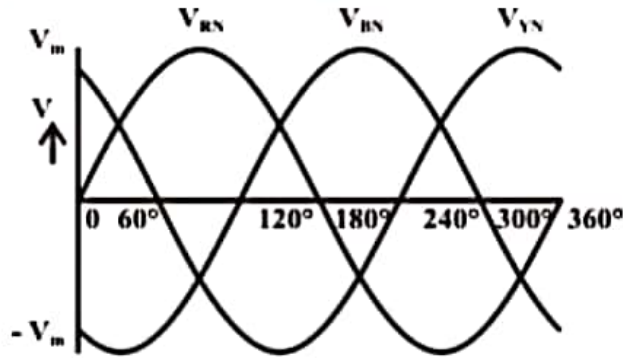
Fig. 18.2 (a) Three-phase balanced voltages, with the source star-connected (the phase sequence, R-Y-B)
(b) Phasor diagram of the line and phase voltages

The phase voltages are all equal in magnitude, but only differ in phase. This is also shown in Fig. 18.2b. The relationship between E and E_m is $E = E_m / \sqrt{2}$. The phase sequence is R-Y-B. It can be observed from Fig. 18.1b that the voltage in phase Y attains the maximum value, after $\theta = \omega \cdot t = 120^\circ$ from the time or angle, after the voltage in

phase R attains the maximum value, and then the voltage in phase B attains the maximum value. The angle of lag or lead from the reference phase, R is stated earlier.

Reversal of phase sequence from R-Y-B to R-B-Y

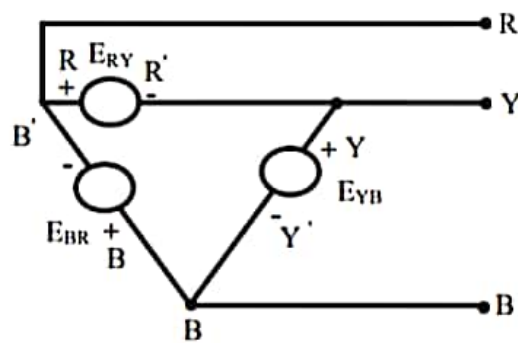
If the phase sequence is reversed from R-Y-B to R-B-Y, the waveforms and the corresponding phasor diagram are shown in figures 18.3 (a-b) respectively. It can be observed from Fig. 18.3a that the voltage in phase B attains the maximum value, after $\theta = 120^\circ$ from the time (or angle), after the voltage in phase R attains the maximum value, and then the voltage in phase Y attains the maximum value. The angle of lag or lead from the reference phase, R is stated earlier. The same sequence is observed in the phasor diagram (Fig. 18.3b), when the phase sequence is reversed to R-B-Y.



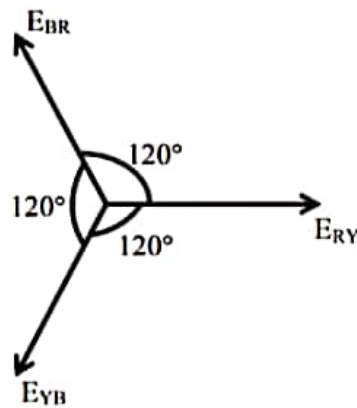
(a)

Relation between the Phase and Line Voltages for Delta Connection

The connection diagram of a delta (Δ)-connected three-phase system is shown in Fig. 18.4a, along with phasor representation of the voltages (Fig. 18.4b). For making delta (Δ)-connection, the start of one winding is connected to the finish of the next one in sequence, for instance, starting from phase R, R' is connected to Y, and then Y' to B, and so on (Fig. 18.4a). The line and phase voltages are the same in this case, and are given as



(a)



(b)

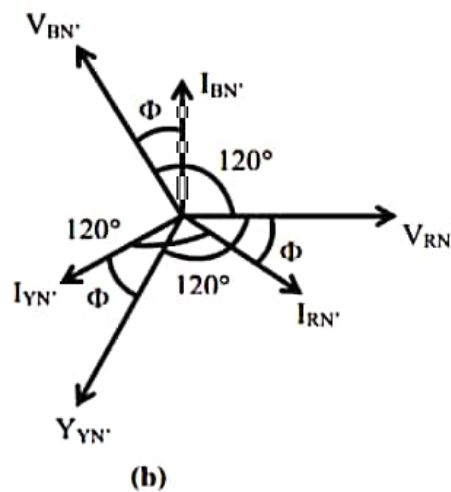
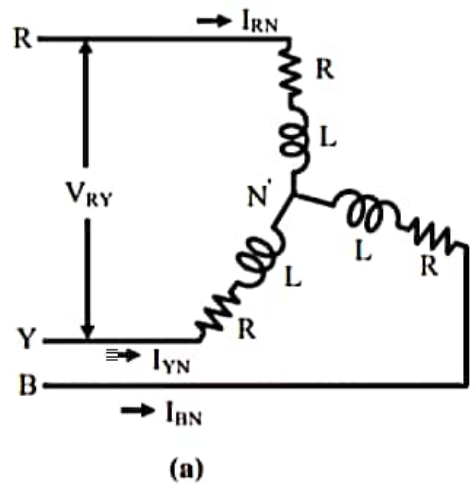
Fig. 18.4 (a) Three-phase balanced voltages, with the source delta-connected (the phase sequence, R-Y-B)

(b) Phasor diagram of the line and phase voltages

$$E_{RY} = E \angle 0^\circ; \quad E_{YB} = E \angle -120^\circ; \quad E_{BR} = E \angle +120^\circ$$

If the phasor sum of the above three phase (or line) voltages are taken, the result is zero (0). The phase or line voltages form a balanced one, with their magnitudes being equal, and the phase being displaced from each other in sequence by 120° .

Currents for Circuit with Balanced Load (Star-connected)



**Fig. 18.5 (a) Circuit diagram for a three-phase balanced star-connected load
(b) Phasor diagram of the phase voltages, and the line & phase currents**

A three-phase star (Y)-connected balanced load (Fig. 18.5a) is fed from a balanced three-phase supply, which may be a three-wire one. A balanced load means that, the magnitude of the impedance per phase, is same, i.e., $|Z_p| = |Z_{RN'}| = |Z_{YN'}| = |Z_{BN'}|$, and their angle is also same, as $\phi_p = \phi_{RN'} = \phi_{YN'} = \phi_{BN'}$. In other words, if the impedance per phase is given as, $Z_p \angle \phi_p = R_p + j X_p$, then $R_p = R_{RN'} = R_{YN'} = R_{BN'}$, and also

$X_p = X_{RN'} = X_{YN'} = X_{BN'}$. The magnitude and phase angle of the impedance per phase are: $Z_p = \sqrt{R_p^2 + X_p^2}$, and $\phi_p = \tan^{-1}(X_p / R_p)$. For balanced load, the magnitudes of the phase voltages, $|V_p| = |V_{RN'}| = |V_{YN'}| = |V_{BN'}|$ are same, as those of the source voltages per phase $|V_{RN}| = |V_{YN}| = |V_{BN}|$, if it is connected in star, as given earlier. So, this means that, the point N' , star point on the load side is same as the star point, N of the load side. The phase currents (Fig. 18.5b) are obtained as,

$$I_{RN'} \angle -\phi_p = \frac{V_{RN} \angle 0^\circ}{Z_{RN'} \angle \phi_p} = \frac{V_{RN}}{Z_{RN'}} \angle -\phi_p$$

$$I_{YN'} \angle -(120^\circ + \phi_p) = \frac{V_{YN} \angle -120^\circ}{Z_{YN'} \angle \phi_p} = \frac{V_{YN}}{Z_{YN'}} \angle -(120^\circ + \phi_p)$$

$$I_{BN'} \angle (120^\circ - \phi_p) = \frac{V_{BN} \angle +120^\circ}{Z_{BN'} \angle \phi_p} = \frac{V_{BN}}{Z_{BN'}} \angle (120^\circ - \phi_p)$$

In this case, the phase voltage, V_{RN} is taken as reference. This shows that the phase currents are equal in magnitude, i.e., ($|I_p| = |I_{RN'}| = |I_{YN'}| = |I_{BN'}|$), as the magnitudes of the voltage and load impedance, per phase, are same, with their phase angles displaced from each other in sequence by 120° . The magnitude of the phase currents, is expressed as $|I_p| = (|V_p| / Z_p)$. These phase currents are also line currents ($|I_L| = |I_R| = |I_Y| = |I_B|$), in this case.

Total Power Consumed in the Circuit (Star-connected)

In the lesson No. 14 of the previous module, the power consumed in a circuit fed from a single-phase supply was presented. Using the same expression for the above star-connected balanced circuit, fed from three-phase supply (Fig. 18.4a-b), the power consumed per phase is given by

$$W_p = V_p \cdot I_p \cdot \cos \phi_p = V_p \cdot I_p \cdot \cos (\angle V_p, I_p)$$

It has been shown earlier that the magnitude of the phase voltage is given by $|V_p| = |V_L| / \sqrt{3}$, where the magnitude of the line voltage is $|V_L|$. The magnitudes of the phase and line current are same, i.e., $|I_p| = |I_L|$. Substituting the two expressions, the total power consumed is obtained as

$$W = 3 \cdot (V_L / \sqrt{3}) \cdot I_L \cdot \cos \phi_p = \sqrt{3} V_L \cdot I_L \cdot \cos \phi_p$$

Please note that the phase angle, ϕ_p is the angle between the phase voltage V_p , and the phase current, I_p .

Before taking up an example, the formulas for conversion from delta-connected circuit to its star equivalent and vice versa (conversion from star to delta connection) using impedances, and also ideal inductances/capacitances, are presented here, starting with circuits with resistances, as derived in lesson #6 on dc circuits.

Delta(Δ)-Star(Y) conversion and Star-Delta conversion

Before taking up the examples, the formula for Delta(Δ)-Star(Y) conversion and also Star-Delta conversion, using impedances as needed, instead of resistance as elements, which is given in lesson #6 in the module on DC circuit, are presented. The formulas for delta-star conversion, using resistances (Fig. 18.6), are,

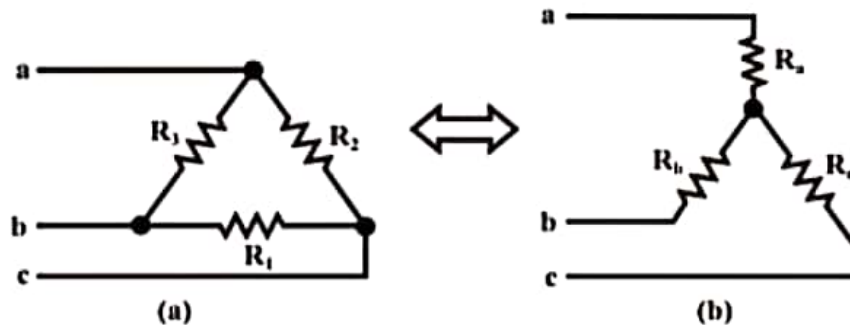


Fig. 18.6: Resistances connected (a) in delta, and (b) in star configurations

$$R_a = \frac{R_2 R_3}{R_1 + R_2 + R_3} \quad R_b = \frac{R_3 R_1}{R_1 + R_2 + R_3} \quad R_c = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

The formulas for delta-star conversion, using resistance, are,

$$R_1 = R_b + R_c + \frac{R_b R_c}{R_a} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_2 = R_c + R_a + \frac{R_c R_a}{R_b} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

$$R_3 = R_a + R_b + \frac{R_a R_b}{R_c} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

The derivation of these formulas is given in lesson #6. If three equal resistances ($R_1 = R_2 = R_3 = R$) connected in delta, are converted into its equivalent star, the resistances obtained are equal, its value being $R_a = R_b = R_c = (R/3) = R'$, which is derived using formulas given earlier. Similarly, if three equal resistances connected in star, are converted into its equivalent delta, the resultant resistances, using formulas, are equal ($R_1 = R_2 = R_3 = 3 \cdot R' = 3 \cdot (R/3) = R$).

The formula for the above conversions using impedances, instead of resistances, are same, replacing resistances by impedances, as the formula for series and parallel combination using impedances, instead of resistances, remain same as shown in the previous module on ac single phase circuits.

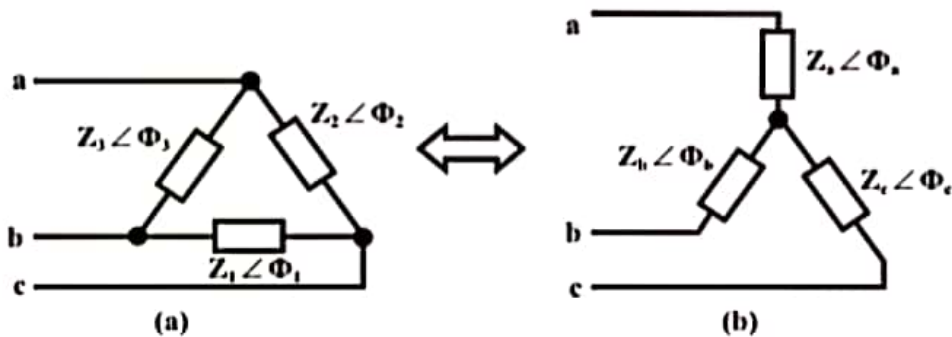


Fig. 18.7: Impedances connected (a) in delta, and (b) in star configurations

The formulas for delta-star conversion, using impedances (Fig. 18.7), are,

$$Z_a = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3} \quad Z_b = \frac{Z_3 Z_1}{Z_1 + Z_2 + Z_3} \quad Z_c = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

The formulas for delta-star conversion, using impedance, are,

$$Z_1 = Z_b + Z_c + \frac{Z_b Z_c}{Z_a} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a}$$

$$Z_2 = Z_c + Z_a + \frac{Z_c Z_a}{Z_b} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b}$$

$$Z_3 = Z_a + Z_b + \frac{Z_a Z_b}{Z_c} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}$$

Please note that all the impedances used in the formula given here are complex quantities, like $Z_1 \angle \phi_1, \dots, Z_a \angle \phi_a, \dots$, having both magnitude and angle as given. The formulas can be derived by the same procedure as given in lesson #6.

An example is taken up, when three equal impedances connected in delta are to be converted into its equivalent star. The impedances are equal, both in magnitude and angle, such that $|Z_1| = |Z_2| = |Z_3| = |Z|$, and $\phi_1 = \phi_2 = \phi_3 = \phi$. The impedances connected in delta are of the form $Z \angle \phi = R \pm jX$. Using the formula given here, the impedances of the star equivalent are also equal, having the magnitude as $|Z_a| = |Z_b| = |Z_c| = (|Z|/3) = |Z'|$ and angle as $\phi_a = \phi_b = \phi_c = \phi$.

The angles of the equivalent impedance connected in star are equal to the angles of the impedances connected in delta. The impedances connected in delta are also equal, both in magnitude and angle, and are of the form $Z' \angle \phi = (Z/3) \angle \phi = (R/3) \pm j(X/3)$.

Similarly, if three equal impedances connected in star are converted into its equivalent delta, the magnitude and angle of the impedances using the formulas given here, are

$|Z_1| = |Z_2| = |Z_3| = (3 \cdot |Z'|) = |Z|$ and $\phi_1 = \phi_2 = \phi_3 = \phi$ respectively. This shows that three impedances are equal, both in magnitude and angle, with its value being

$$Z \angle \phi = (3 \cdot Z') \angle \phi = [3 \cdot (R/3)] \pm j[3 \cdot (X/3)] = R \pm jX$$

which can also be obtained simply from the result given earlier.

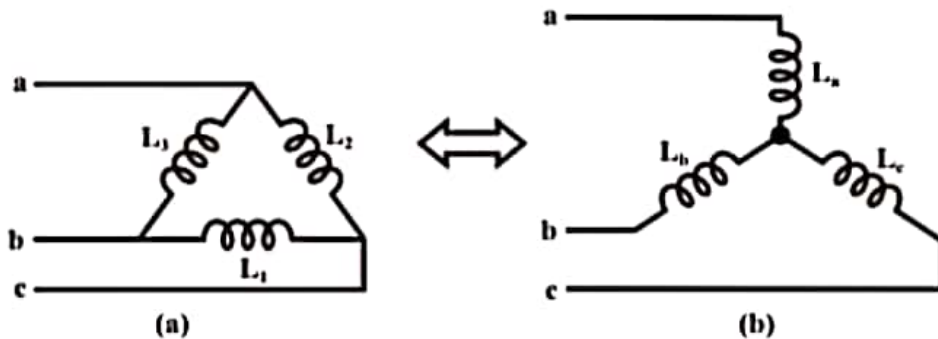


Fig. 18.8: Inductances (ideal) connected (a) in delta, and (b) in star configurations

Now, let us use the above formula for the circuits (Fig. 18.8), using inductances only. The symbols used for the inductances are same (L_1, \dots, L_n, \dots). The impedances of the inductances connected in delta, are computed as $Z_1 \angle \phi_1 = 0.0 + j\omega L_1 = X_1 \angle 90^\circ$, the angles in three cases are 90° . The magnitudes of the impedances are proportional to the respective inductances as $|Z_1| = X_1 \propto L_1$. Converting the combination into its equivalent star, the inductances using the formulas given here, are

$$L_a = \frac{L_2 L_3}{L_1 + L_2 + L_3} \quad L_b = \frac{L_3 L_1}{L_1 + L_2 + L_3} \quad L_c = \frac{L_1 L_2}{L_1 + L_2 + L_3}$$

These relations can also be derived. Further, these are of the same form, as has been earlier obtained for resistances. It may be observed here that the formulas for series and parallel combination using inductances, instead of resistances, remain same, as shown in the previous module on ac single phase circuits, and also can be derived from first principles, such as relationship of induced emf in terms of inductance, as compared with Ohm's law for resistance. The inductances are all ideal, i.e. lossless, having no resistive component. The formulas for star-delta conversion using inductances (conversion of star-connected inductances into its equivalent delta) are,

$$L_1 = L_b + L_c + \frac{L_b L_c}{L_a} = \frac{L_a L_b + L_b L_c + L_c L_a}{L_a}$$

$$L_2 = L_c + L_a + \frac{L_c L_a}{L_b} = \frac{L_a L_b + L_b L_c + L_c L_a}{L_b}$$

$$L_3 = L_a + L_b + \frac{L_a L_b}{L_c} = \frac{L_a L_b + L_b L_c + L_c L_a}{L_c}$$

These are of the same form as derived for circuits with resistances.

If three equal inductances ($L_1 = L_2 = L_3 = L$) connected in delta, are converted into its equivalent star, the inductances obtained are equal, its value being

$L_a = L_b = L_c = (L/3) = L'$, which is derived using formulas given earlier. Similarly, if three equal inductances connected in star, are converted into its equivalent delta, the resultant inductances, using formulas, are equal ($L_1 = L_2 = L_3 = 3 \cdot L' = 3 \cdot (L/3) = L$).

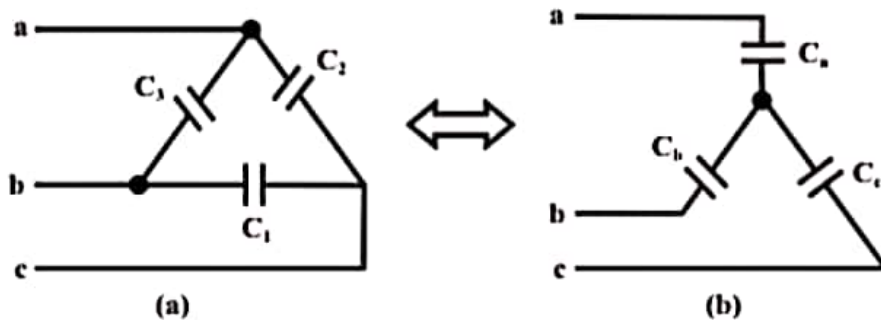


Fig. 18.9: Capacitances connected (a) in delta, and (b) in star configurations

The formulas for the circuits (Fig. 18.9) using capacitances are derived here. The symbols used for the capacitances are same (C_1, \dots, C_a, \dots). The impedances of the inductances connected in delta, are computed as $Z_1 \angle \phi_1 = 0.0 - jX_c = X_1 \angle -90^\circ$, the angles in three cases are (-90°). The magnitudes of the impedances are inversely proportional to the respective capacitances as, $|Z_1| = X_c = X_1 = (1/\omega C) \propto (1/C_1)$. Converting the combination into its equivalent star, the resultant capacitances using the formulas given here, are

$$1/C_a = \frac{(1/C_2)(1/C_3)}{1/C_1 + 1/C_2 + 1/C_3}$$

$$\text{or } C_a = \frac{C_1 C_2 + C_2 C_3 + C_3 C_1}{C_1} = C_2 + C_3 + \frac{C_2 C_3}{C_1}$$

Similarly,

$$C_b = C_3 + C_1 + \frac{C_3 C_1}{C_2} = \frac{C_1 C_2 + C_2 C_3 + C_3 C_1}{C_2}$$

$$C_c = C_1 + C_2 + \frac{C_1 C_2}{C_3} = \frac{C_1 C_2 + C_2 C_3 + C_3 C_1}{C_3}$$

The capacitances in this case are all ideal, without any loss, specially at power frequency, which is true in nearly all cases, except otherwise stated. The formulas for star-delta conversion using capacitances (conversion of star-connected capacitances into its equivalent delta) are,

$$1/C_1 = \frac{(1/C_a)(1/C_b) + (1/C_b)(1/C_c) + (1/C_c)(1/C_a)}{1/C_a}$$

$$\text{or } C_1 = \frac{C_b C_c}{C_a + C_b + C_c}$$

Similarly,

$$C_2 = \frac{C_c C_a}{C_a + C_b + C_c} \quad C_3 = \frac{C_a C_b}{C_a + C_b + C_c}$$

If three equal capacitances ($C_1 = C_2 = C_3 = C$) connected in delta, are converted into its equivalent star, the capacitances obtained are equal, its value being

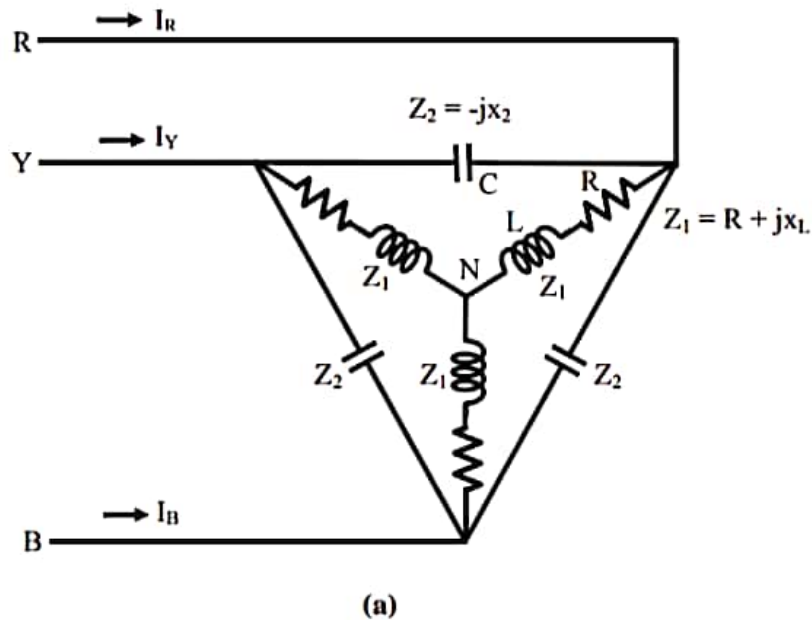
$C_a = C_b = C_c = (3 \cdot C) = C'$, which is derived using formulas given earlier. Similarly, if three equal capacitances connected in star, are converted into its equivalent delta, the resultant capacitances, using formulas, are equal ($C_1 = C_2 = C_3 = C'/3 = (3 \cdot C)/3 = C$).

The formulas for conversion of three equal inductances/capacitances connected in delta into its equivalent star and vice versa (star-delta conversion) can also be obtained from the formulas using impedances as shown earlier, only by replacing inductance with impedance, and for capacitance by replacing it reciprocal of impedance (in both cases using magnitude of impedance only, as the angles are equal (90° for inductance and -90° for capacitance)). Another point to note is left for observation by the reader. Please have a close look at the formulas needed for delta-star conversion and vice versa (star-delta conversion) for capacitances, including those with equal values of capacitances, and then compare them with the formulas needed for such conversion using resistances/inductances (may be impedances also). The rules for conversion of capacitances in series/parallel into its equivalent one can be compared to the rules for conversion of resistances/inductances in series/parallel into its equivalent one.

The reader is referred to the comments given after the example 18.1.

Example 18.1

The star-connected load consists of a resistance of 15Ω , in series with a coil having resistance of 5Ω , and inductance of 0.2 H , per phase. It is connected in parallel with the delta-connected load having capacitance of $90 \mu\text{F}$ per phase (Fig. 18.10a). Both the loads being balanced, and fed from a three-phase, 400 V , 50 Hz , balanced supply, with the phase sequence as R-Y-B. Find the line current, power factor, total power & reactive VA, and also total volt-amperes (VA).



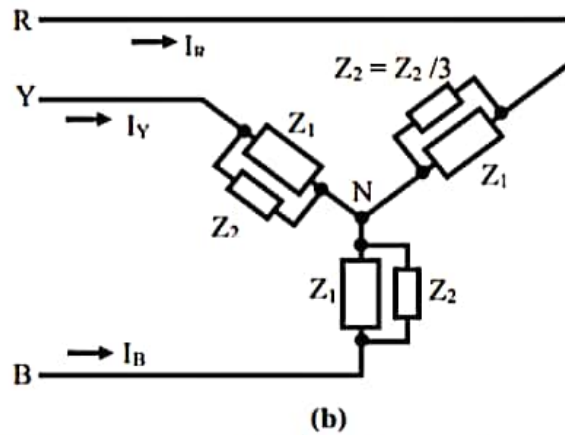


Fig. 18.10 (a) Circuit diagram (Example 18.1)
 (b) Equivalent balanced star-connected circuit

Solution

$f = 50 \text{ Hz}$ $\omega = 2\pi f = 2 \times \pi \times 50 = 314.16 \text{ rad/s}$
 For the balanced star-connected load, $R = 15 \Omega$
 For the inductance coil, $r = 5 \Omega$ $X_L = \omega L = 314.16 \times 0.2 = 62.83 \Omega$
 with the above values taken per phase.
 The impedance per phase is,
 $Z_1 = R + (r + jX_L) = 15 + (5 + j62.83) = (20 + j62.83) = 65.94 \angle 72.34^\circ \Omega$
 For the balanced delta-connected load, $C = 90 \mu\text{F}$
 Converting the above load into its equivalent star, $C_1 = C/3 = 90/3 = 30 \mu\text{F}$
 $X_{C_1} = 1/\omega C_1 = 1/(314.16 \times 30 \times 10^{-6}) = 106.1 \Omega$
 The impedance per phase is $Z'_2 = -j106.1 = 106.1 \angle -90^\circ$

In the equivalent circuit for the load (Fig. 18.10b), the two impedances, Z_1 & Z'_2 are in parallel. So, the total admittance per phase is,

$$Y_p = Y_1 + Y'_2 = \frac{1}{Z_1} + \frac{1}{Z'_2} = \frac{1}{65.94 \angle +72.34^\circ} + \frac{1}{106.1 \angle -90^\circ}$$

$$= 0.01517 \angle -72.34^\circ + 0.009425 \angle +90^\circ = [(4.6 - j14.46) + j9.425] \times 10^{-3}$$

$$= (4.6 - j5.03) \times 10^{-3} = 0.006816 \angle -47.56^\circ \Omega^{-1}$$

The total impedance per phase is,
 $Z_p = 1/Y_p = 1/(0.006816 \angle -47.56^\circ) = 146.71 \angle +47.56^\circ = (99.0 + j108.27) \Omega$

The phasor diagram is shown in Fig. 18.10c.

Taking the phase voltage, V_{RN} as reference,

$$|V_{RN}| = |V_p| = |V_L|/\sqrt{3} = 400/\sqrt{3} = 231.0 \text{ V}$$

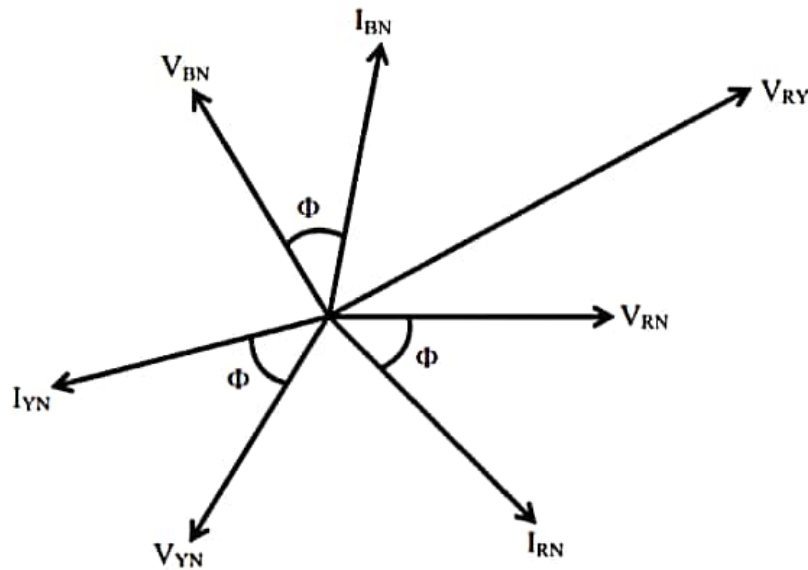


Fig. 18.10 (c) Phasor diagram

The phase voltages are,

$$V_{RN} = 231.0 \angle 0^\circ ; \quad V_{YN} = 231.0 \angle -120^\circ ; \quad V_{BN} = 231.0 \angle +120^\circ$$

So, the phase current, I_{RN} is,

$$I_{RN} = \frac{V_{RN}}{Z_p} = \frac{231.0 \angle 0^\circ}{146.71 \angle +47.56^\circ} = 1.575 \angle -47.56^\circ = (1.0625 - j1.162) \text{ A}$$

The two other phase currents are,

$$I_{YN} = 1.575 \angle -167.56^\circ ; \quad I_{BN} = 1.575 \angle +72.44^\circ$$

As the total circuit (Fig. 18.5b) is taken as star-connected, the line and phase currents are same, i.e., $|I_L| = |I_p| = 1.575 \text{ A}$

Also, the phase angle of the total impedance is positive.

So, the power factor is $\cos \phi_p = \cos 47.56^\circ = 0.675 \text{ lag}$

The total volt-amperes is $S = 3 \cdot V_p \cdot I_p = 3 \times 231 \times 1.575 = 1.0915 \text{ kVA}$

The total VA is also obtained as $S = \sqrt{3} \cdot V_L \cdot I_L = \sqrt{3} \times 400 \times 1.575 = 1.0915 \text{ kVA}$

The total power is $P = 3 \cdot V_p \cdot I_p \cdot \cos \phi_p = 3 \times 231 \times 1.575 \times 0.675 = 737 \text{ W}$

The total reactive volt-amperes is,

$$Q = 3 \cdot V_p \cdot I_p \cdot \sin \phi_p = 3 \times 231 \times 1.575 \times \sin 47.56^\circ = 805 \text{ VAR}$$

This example can be solved by converting the star-connected part into its equivalent delta, as shown in Example 19.1 (next lesson). A simple example (20.1) of a balanced star-connected load is also given in the last lesson (#20) of this module.

After starting with the generation of three-phase balanced voltage system, the phase and line voltages, both being balanced, first for star-connection, and then for delta-connection (both on source side), are discussed. The currents (both phase and line) for

balanced star-connected load, along with total power consumed, are also described in this lesson. An example is given in detail. In the next lesson, the currents (both phase and line) for balanced delta-connected load will be presented.