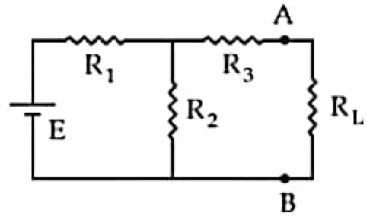
NETWORK THEOREMS

1. Thevenin's theorem

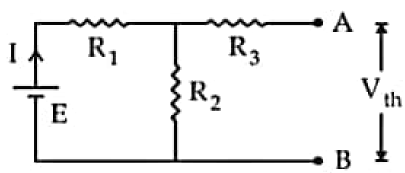
Statement: *A linear network consisting of a number of voltage sources and resistances can be replaced by an equivalent network having a single voltage source called Thevenin's voltage* (V_{Th}) *and a single resistance called Thevenin's resistance* (R_{Th}).

Explanation:



Consider a network or a circuit as shown. Let E be the emf of the cell having its internal resistance $r = 0$. $R_L \rightarrow$ load resistance across AB.

To find V_{Th}:

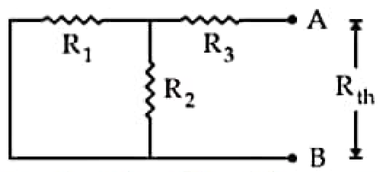


The load resistance R_L is removed. The current I in the circuit is $I = \dfrac{E}{R_1 + R_2}$.

The voltage across $AB =$ Thevenin's voltage V_{Th}.

$$V_{Th} = I R_2 \quad \Rightarrow \quad \boxed{V_{Th} = \frac{E R_2}{R_1 + R_2}}$$

To find R_{Th}:



The load resistance R_L is removed. The cell is disconnected and the wires are short as shown.

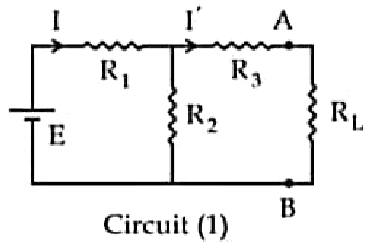
The effective resistance across $AB =$ Thevenin's resistance R_{Th}.

$\boxed{R_{Th} = R_3 + \dfrac{R_1 R_2}{R_1 + R_2}}$ [R_1 is parallel to R_2 and this combination in series with R_3]

If the cell has internal resistance r, then $V_{Th} = \dfrac{E R_2}{R_1 + R_2 + r}$ and $R_{Th} = R_3 + \dfrac{(R_1 + r) R_2}{R_1 + r + R_2}$.

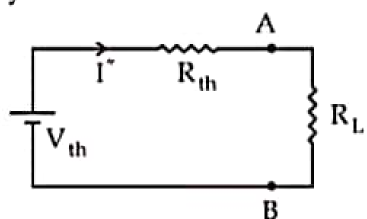
Proof of Thevenin's theorem:

Consider the network as shown below



Circuit (1)

The equivalent circuit is given by



Circuit (2)

The effective resistance of the network in (1) is R_3 and R_L in series and this combination is parallel to R_2 which in turn is in series with R_1.

Thus, $R_{eff} = R_1 + \dfrac{R_2(R_3+R_L)}{R_2+R_3+R_L}$ -------------- (1)

The current I in the circuit is $I = \dfrac{E}{R_{eff}} = \dfrac{E}{R_1 + \dfrac{R_2(R_3+R_L)}{R_2+R_3+R_L}}$

or $I = \dfrac{E(R_2+R_3+R_L)}{R_1R_2+R_1R_3+R_1R_L+R_2R_3+R_2R_L}$ ----------- (2)

The current through the load resistance (I') is found using branch current method.

$I' = \dfrac{I\,R_2}{R_2+R_3+R_L}$ ------------ (3)

Substituting for I from (2) in (3)

$I' = \dfrac{E(R_2+R_3+R_L)R_2}{(R_2+R_3+R_L)(R_1R_2+R_1R_3+R_1R_L+R_2R_3+R_2R_L)}$

or $I' = \dfrac{ER_2}{R_1R_2+R_1R_3+R_1R_L+R_2R_3+R_2R_L}$ ------------ (4)

Thevenin's voltage $V_{Th} = \dfrac{ER_2}{R_1+R_2}$ ----------- (5)

Thevenin's resistance $R_{Th} = R_3 + \dfrac{R_1R_2}{R_1+R_2}$ ---------- (6)

Consider the equivalent circuit (circuit (2))

The current I'' in the equivalent circuit is $I'' = \dfrac{V_{Th}}{R_{Th}+R_L}$ ----------- (7)

Substituting for V_{Th} and R_{Th} from (5) and (6) in (7)

$$I'' = \frac{ER_2}{R_1 + R_2} \times \frac{1}{R_3 + \frac{R_1 R_2}{R_1 + R_2} + R_L} = \frac{ER_2}{(R_1 + R_2)\frac{R_3R_1 + R_3R_2 + R_1R_L + R_2R_L + R_1R_2}{(R_1 + R_2)}}$$

or $$I'' = \frac{ER_2}{R_1R_2 + R_1R_3 + R_1R_L + R_2R_3 + R_2R_L} \quad \text{-------------} \ (8)$$

From equations (4) and (8), it is observed that $I' = I''$.

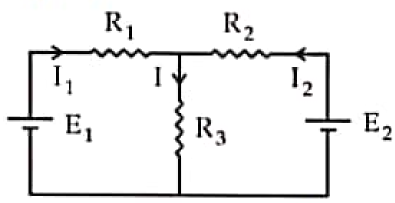
Hence Thevenin's theorem is verified.

3. Superposition theorem

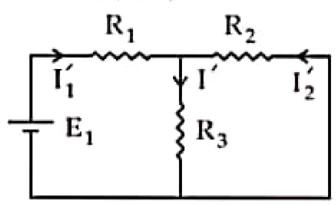
Statement: *In a linear network having number of voltage or current sources and resistances, the current through any branch of the network is the algebraic sum of the currents due to each of the sources when acting independently.*

Explanation: By mesh current analysis.

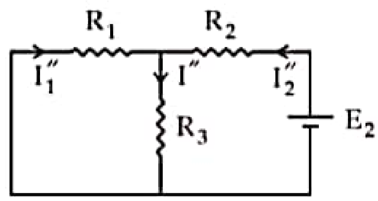
1. Consider the network as shown. The currents in different branches of the network are I_1, I_2 and I as shown. Also $I_1 + I_2 = I$.



2. [Let the internal resistance r of the cells be negligible].
   The cell E_2 is removed and the terminals are short as shown. Now the currents in the branches are I_1', I_2' and I'. Also $I' = I_1' + I_2'$.



3. The E_1 is removed and the terminals are short as shown. The currents are I_1'', I_2'' and I''. Also $I'' = I_1'' + I_2''$.

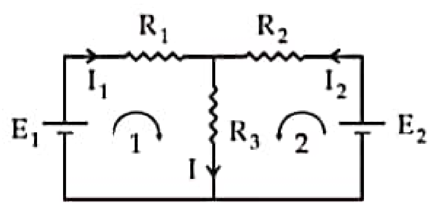


According to superposition theorem $I_1 = I_1' + I_1''$. $I_2 = I_2' + I_2''$ and $I = I' + I''$

$\boxed{I = I_1 + I_2}$

Verification of superposition theorem:

1. Consider the network shown. Applying Kirchhoff's voltage to the loop 1.



$I_1R_1 + I_1R_3 + I_2R_3 = E_1$ $\quad [\because I_1R_1 + I(R_3) = E_1 \quad I = I_1 + I_2]$

or $I_1 = \dfrac{E_1 - I_2R_3}{R_1 + R_3}$ ----------- (1)

Considering loop 2, $I_2R_2 + I\,R_3 = E_2$.

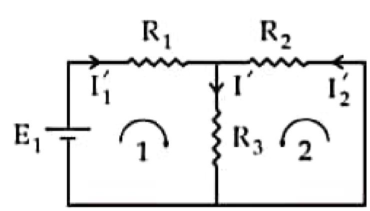
$I_2R_2 + I_1R_3 + I_2R_3 = E_2$

$I_2 = \dfrac{E_2 - I_1R_3}{R_2 + R_3}$ ---------- (2)

Thus, $I = I_1 + I_2$

$I = \dfrac{E_1 - I_2R_3}{R_1 + R_3} + \dfrac{E_2 - I_1R_3}{R_2 + R_3}$ ----------- (3)

2. Consider the circuit shown with E_2 removed and terminals short. Applying Kirchhoff's law to loop 1.



$I_1'R_1 + I'R_3 = E_1$ $\qquad$ As $\quad I' = I_1' + I_2'$,

$I_1R_1 + I_1'R_3 + I_2'R_3 = E_1 \Rightarrow \quad I_1' = \dfrac{E_1 - I_2'R_1}{R_1 + R_3}$ ---------- (4)

Similarly for loop 2,

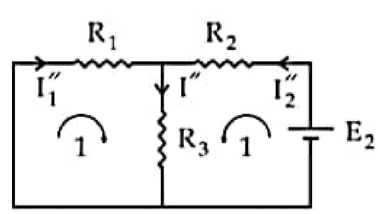
$I_2'R_2 + I'R_3 = 0 \quad \Rightarrow \qquad I_2'R_2 + I_1'R_3 + I_2'R_3 = 0$

$I_2' = -\dfrac{I_1'R_3}{R_2 + R_3}$ ----------- (5)

$I' = I_1' + I_2' = \dfrac{E_1 - I_2'R_3}{R_1 + R_3} - \dfrac{I_1'R_3}{R_2 + R_3}$ ....(6)

3. Consider the circuit with E_1 removed and terminals short.

For loop (1) $I_1''R_1 + I''R_3 = 0$



As $I'' = I_1'' + I_2''$

$$I_1''R_1 + I_1''R_3 + I_2''R_3 = 0 \quad \Rightarrow \quad I_1'' = \frac{-I_2''R_3}{R_1 + R_3} \text{ ------------ (7)}$$

For loop (2)

$$I_2''R_2 + I''R_3 = E_2 \quad \Rightarrow \quad I_2''R_2 + I_1''R_3 + I_2''R_3 = E_2$$

or $I_2'' = \dfrac{E_2 - I_1''R_3}{R_2 + R_3}$ ------------- (8)

$$I'' = I_1'' + I_2'' = \frac{-I_2''R_3}{R_1 + R_3} + \frac{E_2 - I_1''R_3}{R_2 + R_3} \text{ -------------- (9)}$$

Adding equations (6) and (9)

$$I' + I'' = \frac{E_1 - I_2'R_3}{R_1 + R_3} - \frac{I_1'R_3}{R_2 + R_3} - \frac{I_2''R_3}{R_1 + R_3} + \frac{E_2 - I_1''R_3}{R_2 + R_3}$$

$$= \frac{1}{R_1 + R_3}\left[E_1 - I_2'R_3 - I_2''R_3\right] + \frac{1}{R_2 + R_3}\left[E_2 - I_1''R_3 - I_1'R_3\right]$$

$$I' + I'' = \frac{1}{R_1 + R_3}\left[E_1 - R_3\left(I_2' + I_2''\right)\right] + \frac{1}{R_2 + R_3}\left[E_2 - R_3\left(I_1' + I_1''\right)\right] \text{ ....(10)}$$

Comparing equations (3) and (10) it is observed that

$$I_1 = I_1' + I_1''$$

$$I_2 = I_2' + I_2''$$

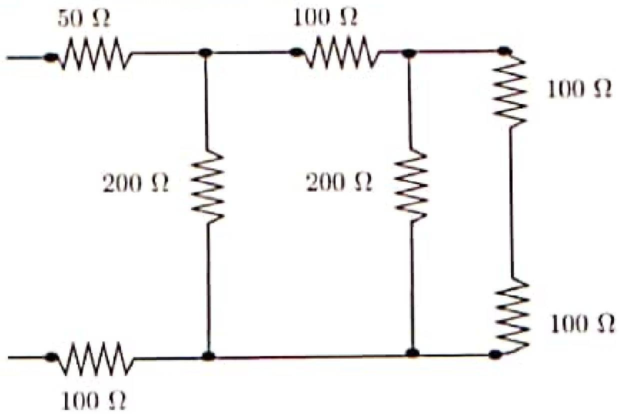
$$I = I' + I''$$

Hence the proof of the theorem.

1. Circle T (true) or F (false) for each of these Boolean equations.

(a).	$\boxed{T}$	F	Negative power indicates power delivered
(b).	T	$\boxed{F}$	The unit of charge is the amp
(c).	$\boxed{T}$	F	0.005 V = 5 mV
(d).	$\boxed{T}$	F	Ohm's Law states that $v = iR$
(e).	$\boxed{T}$	F	Current is the rate of flow of charge

2. **Equivalent Resistance:** Find the equivalent resistance R_{eq} of the resistive circuit below.



$R_{eq} = 50\,\Omega + 200\,\Omega \| (100 + 200 \| \underbrace{(100+100)}_{=200}) + 100\,\Omega$

$200\,\Omega \| 200\,\Omega = \frac{40000}{400} = 100\,\Omega$ ← note that $R_1 \| R_1 = \frac{R_1}{2}$

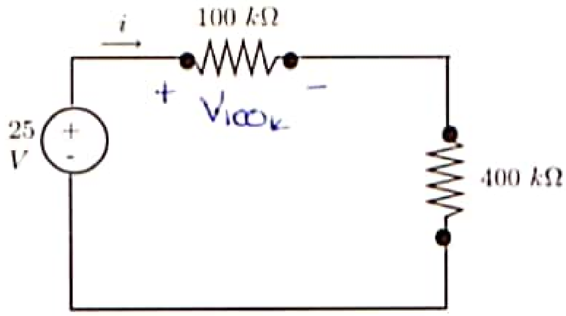
$R_{eq} = 50 + 200 \| (100+100) + 100 = 50 + 200 \| 200 + 100\,\Omega$

as above, $200 \| 200 = 100\,\Omega$

$R_{eq} = 50 + 100 + 100\,\Omega = \underline{250\,\Omega}$

$\boxed{R_{eq} = 250\Omega}$

3. Ohm's Law and Power: Given the simple resistive circuit below:



3(a). Find the current i flowing through the 400 $k\Omega$ resistor.

By KVL: $-25\text{ V} + 100\times10^3 i + 400\times10^3 i = 0$

$500\times10^3 i = 25$

$i = \frac{25}{500\times10^3} = \frac{1}{20}\times10^{-3}\text{ A} = .05\text{ mA} = 50\,\mu\text{A} = i$

$\boxed{i = 0.05 \text{ mA} = 50\ \mu\text{A}}$

Is this the same current that flows through the 100 $k\Omega$ resistor?
(circle one): $\boxed{\text{Yes}}$ No

3(b). Find the power absorbed or supplied by the 100 $k\Omega$ resistor. Negative power indicates power absorbed and positive power indicates power supplied.

• i goes into + terminal of $V_{100k\Omega}$, so use $P = +iv$

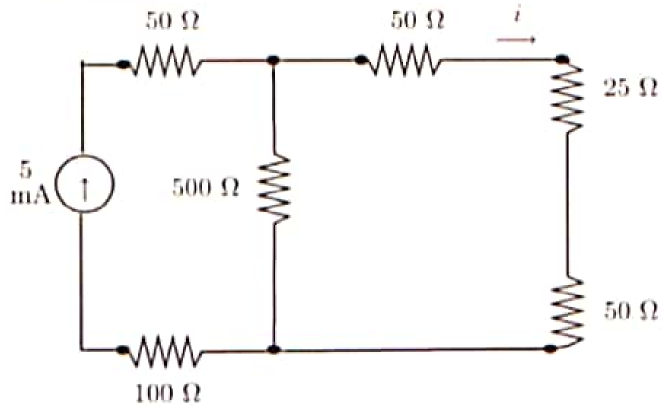
$P = +iV_{100k} = +i^2R = (5\times10^{-5})^2(1\times10^5) = 25\times10^{-10}(1\times10^5)$

$P = 25\times10^{-5}\,\omega = 250\,\mu\text{W} = .25\text{ mW}$

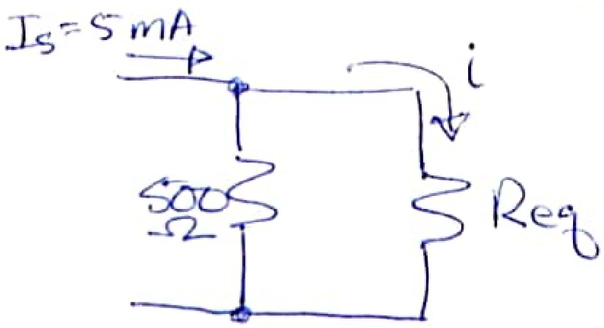
$\boxed{P = 0.25 \text{ mW}} = 250\,\mu\text{W}$

Power is (circle one): $\boxed{\text{absorbed}}$ supplied

5. Current Division: Use current division to find the current i flowing through the 25 Ω resistor.



For current division, reduce R in branch with 25Ω resistor to a single R_{eq}.



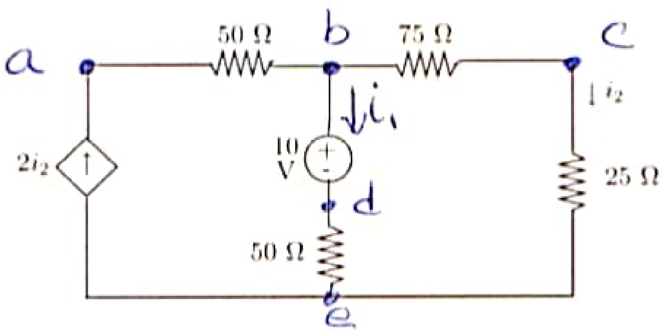
$R_{eq} = 50 + 25 + 50 = 125\ \Omega$

By current division, $i = \frac{I_s(500)}{500 + R_{eq}} = \frac{(5\text{ mA})(500)}{500 + 125}$

$i = \frac{500}{625}(5\text{mA}) = \frac{100}{125}(5\text{mA}) = \frac{4}{5}(5\text{mA}) = \underline{4\text{ mA}}$

$\boxed{i = 4\text{ mA}}$

6. KVL, KCL and Dependent Current Source: Use Kirchhoff's Voltage Law (KVL) and Kirchhoff's Current Law (KCL) to find the current flowing through the 25 Ω resistor, i_2.



By KCL at node b, $-2i_2 + i_1 + i_2 = 0$ so $i_1 = i_2$

By KVL from $e \to d \to b \to c \to e$,

$-V_{de} - 10 + V_{bc} + V_{ce} = 0$

$-V_{de} = -i_1 \cdot 50$; $V_{bc} = 75 i_2$; $V_{ce} = 25 i_2$

$-50i_1 - 10 + 75i_2 + 25i_2 = -50i_1 - 10 + 100i_2 = 0$

From KCL, $i_1 = i_2$ so $-50i_2 - 10 + 100i_2 = 0$

or $50i_2 = 10$

$i_2 = 10/50 = 1/5\ A = .2A = 200mA = i_2$

$\boxed{i_2 = 0.2\ \text{A}} = 200mA$

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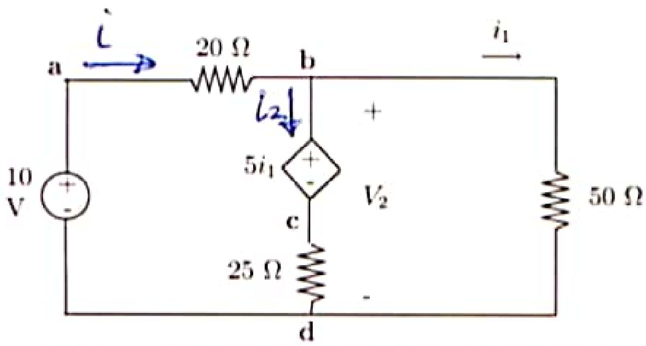
7. Nodal Analysis (Node-Voltage Method): Find equations for the voltage V_2 and the current i_1 in the circuit below, using nodal analysis (node-voltage method):



7(a). First, label your nodes. Which nodes are essential nodes? Which node is your reference node? Indicate this below and on the circuit diagram.

Essential nodes: b and d | Reference node: d

7(b). Use KVL to find an equation for V_2 that includes the unknown current i_1.

• By Ohm's Law, $V_2 = 50 i_1$ (or by KVL: d→c→d→d, $-V_2 + i_1 \cdot 50 = 0$)

• Also by KVL at nodes d→c→b→d, $-25 i_2 - 5 i_1 + V_2 = 0$

$V_2 = 5 i_1 + 25 i_2$ ← either eqn is fine   dependent voltage source

• Also by KVL at nodes d→a→b→d, $-10 + 20 i + V_2 = 0$

$\boxed{V_2 = 50 i_1 \text{ Volts}}$ or $\boxed{V_2 = 5 i_1 + 25 i_2 \text{ Volts}}$ so $20 i = 10 - V_2$ (use this in part 7(c))

7(c). Now use KCL at the appropriate essential node to find an equation for i_1 in terms of V_2.

KCL at node b: $-i + i_2 + i_1 = 0$ $i_1 = i - i_2$

From 7(b), $i = \frac{10 - V_2}{20}$ & $i_2 = \frac{V_2 - 5 i_1}{25}$, plug these into above eqn

$i_1 = \frac{10 - V_2}{20} - \frac{V_2}{25} + \frac{5 i_1}{25}$; $i_1 (1 - 5/25) = \frac{10}{20} - V_2\left(\frac{1}{20} + \frac{1}{25}\right)$

$i_1 (4/5) = \frac{1}{2} - \frac{9 V_2}{100}$

$\boxed{i_1 = \frac{5}{8} - \frac{9V_2}{80} = 0.625 - 0.1125 V_2 \text{ Amps}}$ $\boxed{i_1 = 5/8 - 9V_2/80 \text{ Amps}}$

Note: if instead of $20i = 10 - V_2$ in 7(b), you use $20i = 10 - 50 i_1$ (since $V_2 = 50 i_1$), your i_1 eqn will be different: $i_1 = 5/33 - \frac{2V_2}{165}$ Amps
How? i_1 & V_2 are dependent. Both eqns result in the same (i_1, V_2) solution.

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7(d). **Extra:** Given your two equations above for V_2 and i_1, solve for both unknowns.

$V_2 = 50 i_1$ & $i_1 = 5/8 - 9V_2/80$

plug eqn for V_2 into eqn for i_1:

$$i_1 = 5/8 - \frac{9(50 i_1)}{80} = 5/8 - \frac{450 i_1}{80} = \frac{5}{8} - \frac{45 i_1}{8}$$

$i_1(1 + 45/8) = 5/8$; $\frac{53}{8} i_1 = 5/8$ or $\underline{i_1 = 5/53 \text{ A}}$

$V_2 = 50 i_1 = \frac{50(5)}{53} \text{ V} = \underline{\frac{250}{53} \text{ V}}$

$\boxed{V_2 = \frac{250}{53} = 4.717 \text{ Volts}}$ $\boxed{i_1 = \frac{5}{53} = 0.09434 \text{ Amps}}$