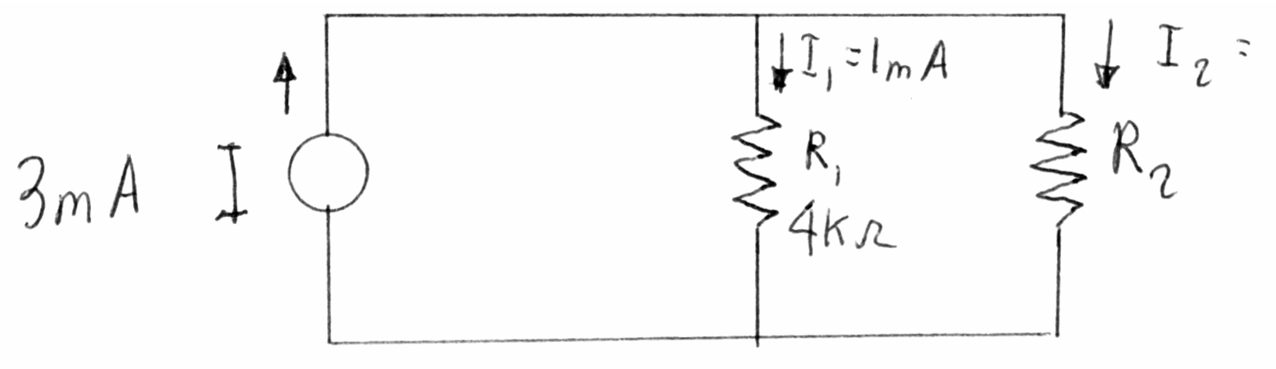
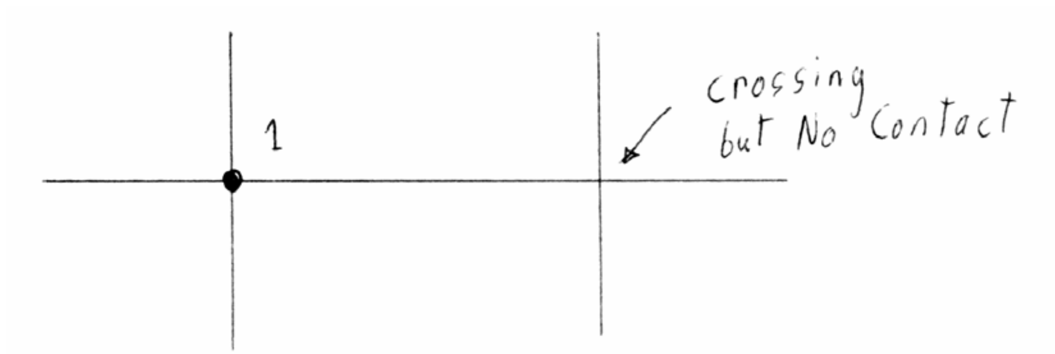
Kirchhoff's Laws and Circuit Analysis (EC 2)

- Circuit analysis: solving for I and V at each element
- Linear circuits: involve resistors, capacitors, inductors
- Initial analysis uses only resistors
- Power sources, constant voltage and current
- Solved using **Kirchhoff's Laws** (Current and Voltage)

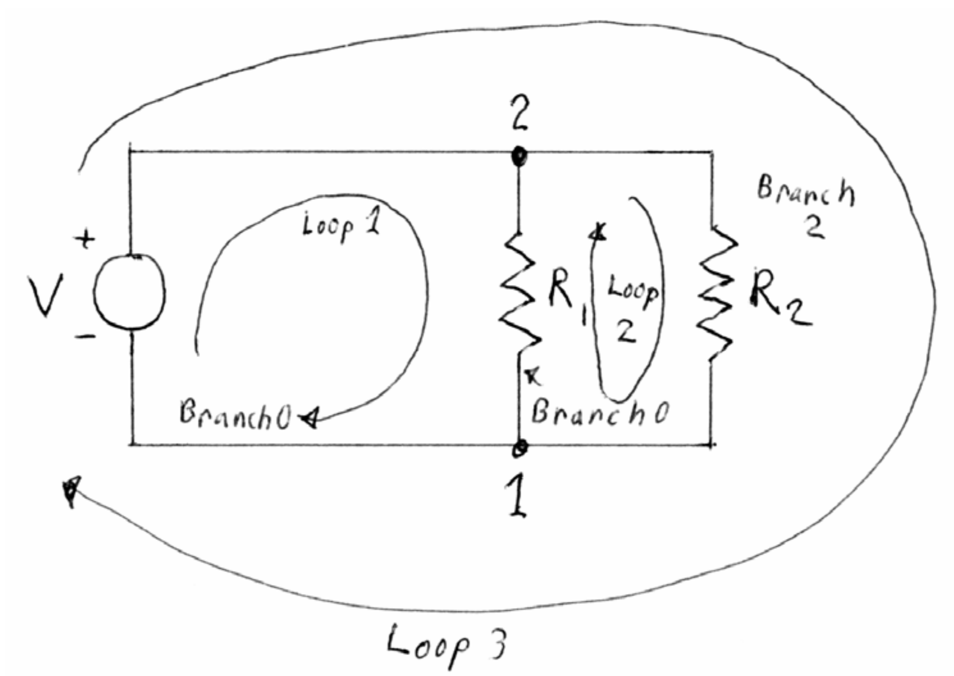


Circuit Nodes and Loops

- **Node**: a point where several wires electrically connect
- Symbolized by a dot or circle at the wire crossing
- If wires cross without a dot, then not connected
- Nodes also called junctions
- Typically give nodes a number or letter



- **Branches**: lines with devices connecting two nodes
- **Loop**: an independent closed path in a circuit
- There may be several possible closed paths



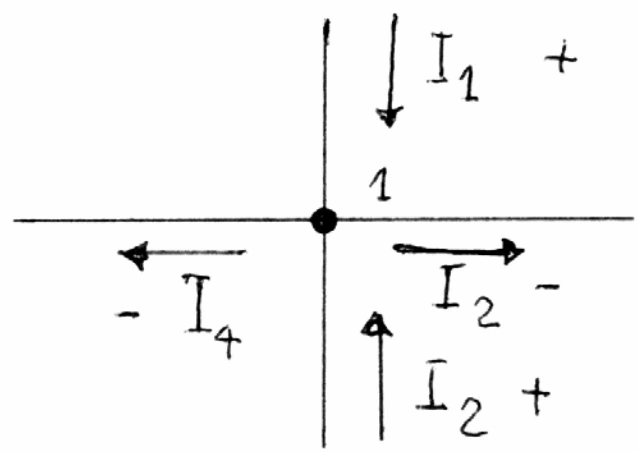
Kirchhoff's Current Law (KCL)

- Kirchhoff's Current Law (KCL)
- The algebraic sum of currents entering any node (junction) is zero

$$\sum_{j=1}^{N} I_j = 0$$

where N = number of lines entering the node

- NOTE: the sign convention:
- Currents are positive when they entering the node
- Currents negative when leaving
- Or the reverse of this.



KCL is called a **Continuity Equation**:
It says current is not created or destroyed at any node

Example of Kirchhoff's Current Law (KCL)

- Consider the simple parallel resistances below
- At node 1 define current positive into resistors
- Since V on R_1 = 5V the current is

$$I_1 = \frac{V}{R_1} = \frac{5}{1000} = 5\ mA$$

- Same V on R_2 = 5V the current is

$$I_2 = \frac{V}{R_2} = \frac{5}{5000} = 1\ mA$$

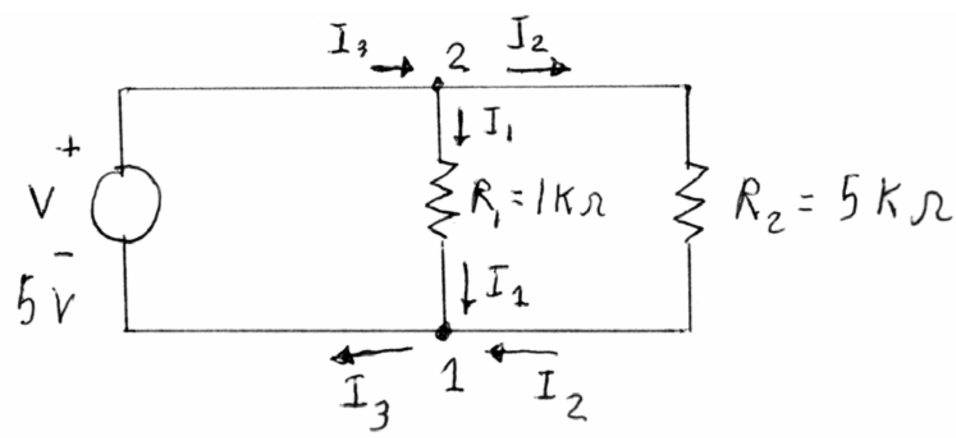
- Thus by KCL at node 1

$$I_1 + I_2 + I_3 = 0.005 + 0.001 + I_3 = 0$$

- Thus the total current is

$$I_3 = -I_1 - I_2 = -6\ mA$$

- Node 2 has the negatives of these values



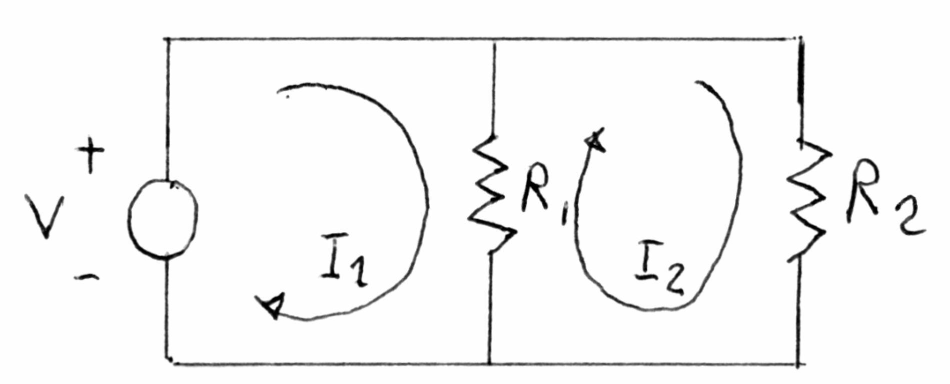
Kirchhoff's Voltage Law (KVL)

- Kirchhoff's Voltage Law (KVL)
- Algebraic sum of the voltage drops around any loop or circuit = 0

$$\sum_{j=1}^{N} V_j = 0$$

where N = number of voltage drops

- NOTE: the sign convention
- Voltage drops are positive in the direction of the set loop current
- Voltage drops negative when opposite loop current
- Voltage sources positive if current flows out of + side
- Voltage sources negative if current flows into + side



- A loop is an independent closed path in the circuit
- Define a "loop current" along that path
- Real currents may be made up of several loop currents

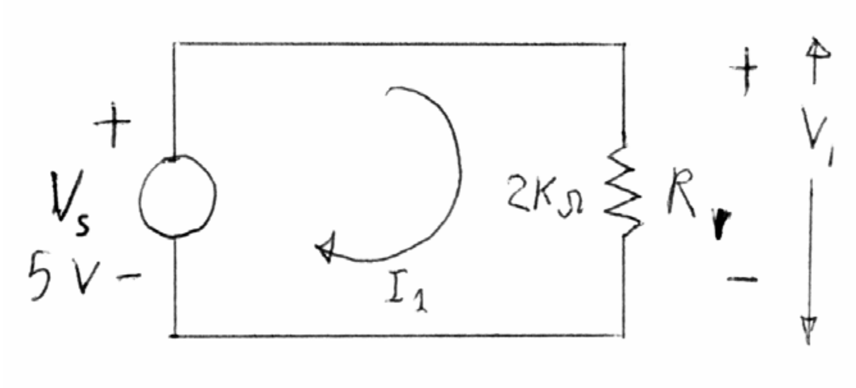
$$I_{R1} = I_1 - I_2$$

Example Kirchhoff's Voltage Law (KVL)

Consider a simple one loop circuit
Voltages are numbered by the element name
eg. V_1 or V_{R1} : voltage across R_1
Going around loop 1 in the loop direction
Recall by the rules:

- Voltage drops negative when opposite loop current.
- Voltage sources positive if current flows out of + side

$$V_s - V_1 = 0$$



Example Kirchhoff's Voltage Law (KVL) Continued

- Thus voltage directions are easily defined here:

$$V_s - V_1 = 0$$

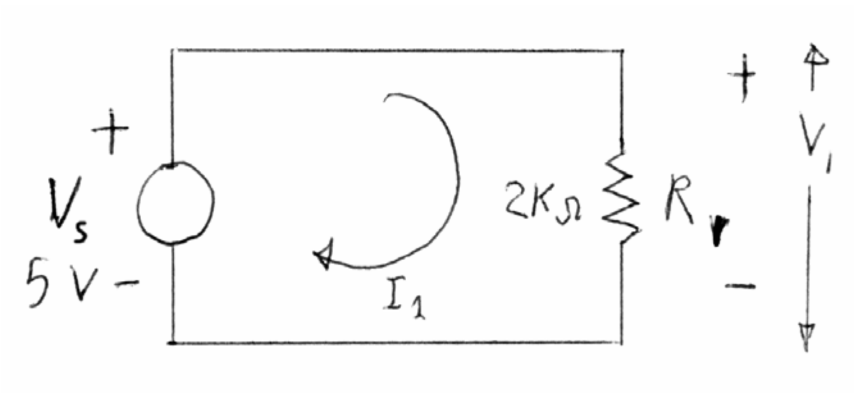
- Why negative V_1? Opposes current flow I_1
- Since

$$V_1 = I_1 R_1$$

$$V_s - I_1 R_1 = 0$$

- Thus this reduces to the Ohms law expression

$$I_1 = \frac{V_s}{R_1}$$



KVL Example Resistor Voltage Divider

- Consider a series of resistors and a voltage source
- Then using KVL

$$V - V_1 - V_2 = 0$$

- Since by Ohm's law

$$V_1 = I_1 R_1 \quad V_2 = I_1 R_2$$

- Then

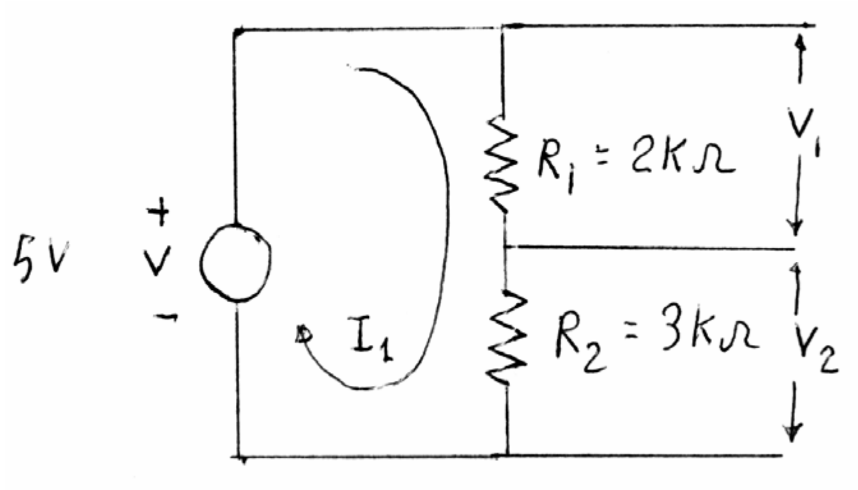
$$V - I_1 R_1 - I_1 R_2 = V - I_1 \left(R_1 + R_2 \right) = 0$$

- Thus

$$I_1 = \frac{V}{R_1 + R_2} = \frac{5}{2000 + 3000} = 1\, mA$$

- i.e. get the resistors in series formula

$$R_{total} = R_1 + R_2 = 5\ K\Omega$$



KVL Example Resistor Voltage Divider Continued

- What is the voltage across each resistor?
- Now we can relate V_1 and V_2 to the applied V
- With the substitution

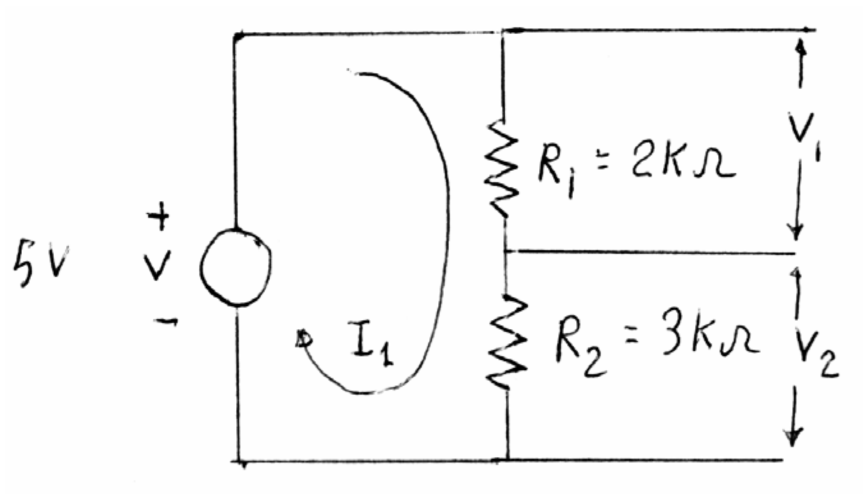
$$I_1 = \frac{V}{R_1 + R_2}$$

- Thus V_1

$$V_1 = I_1 R_1 = \frac{VR_1}{R_1 + R_2} = \frac{5(2000)}{2000 + 3000} = 2\,V$$

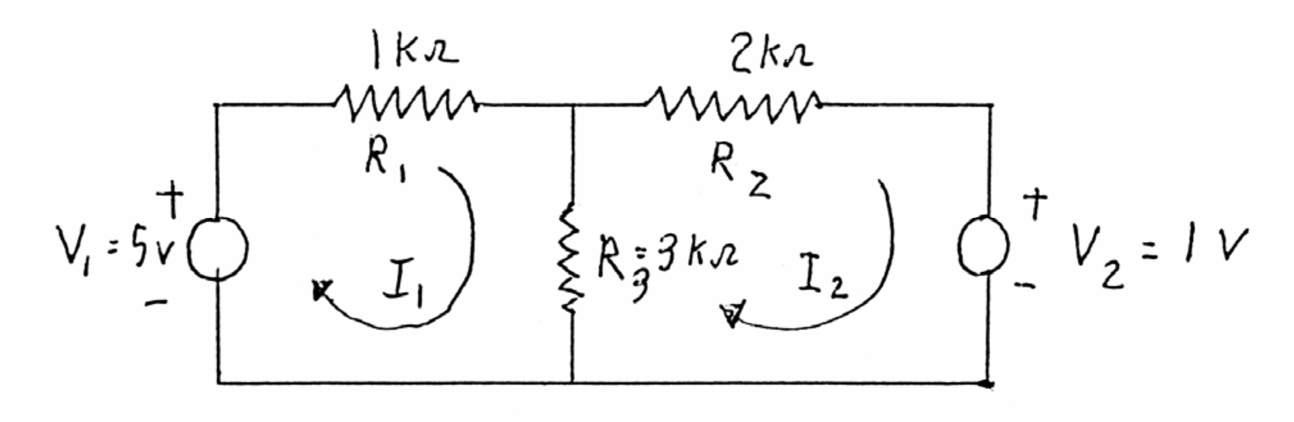
- Similarly for the V_2

$$V_1 = I_1 R_2 = \frac{VR_2}{R_1 + R_2} = \frac{5(3000)}{2000 + 3000} = 3\,V$$

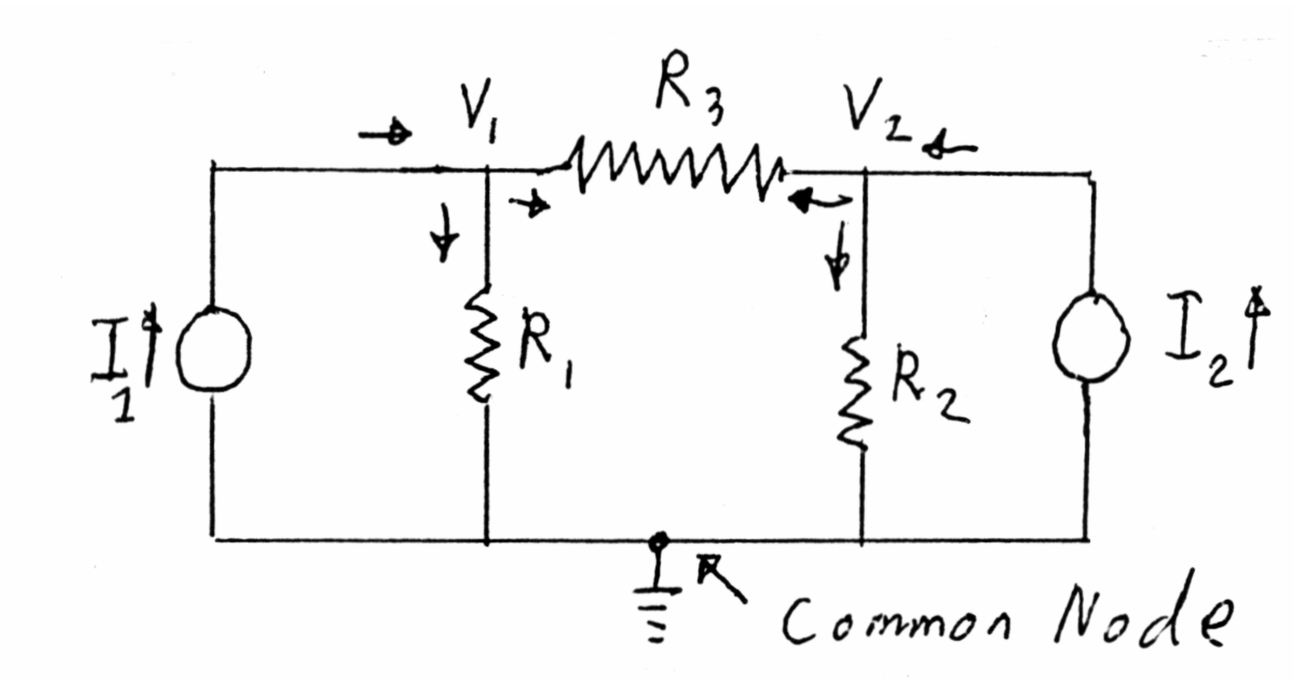


KVL and KCL for Different Circuits

- With multiple voltage sources best to use KVL
- Can write KVL equation for each loop



- With multiple current sources best to use KCL
- Can write KCL equations at each node.

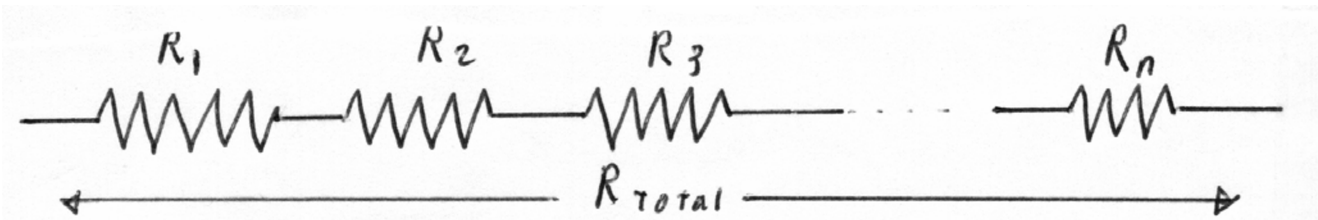


- In practice can solve whole circuit with either method

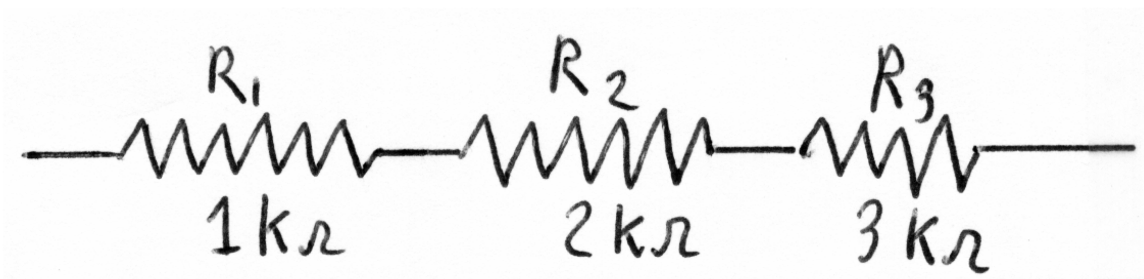
Resistors in Series (EC3)

- Resistors in series add to give the total resistance

$$R_{total} = \sum_{j=1}^{N} R_j$$



- Example: total of 1, 2, and 3 Kohm resistors in series



- Thus total is

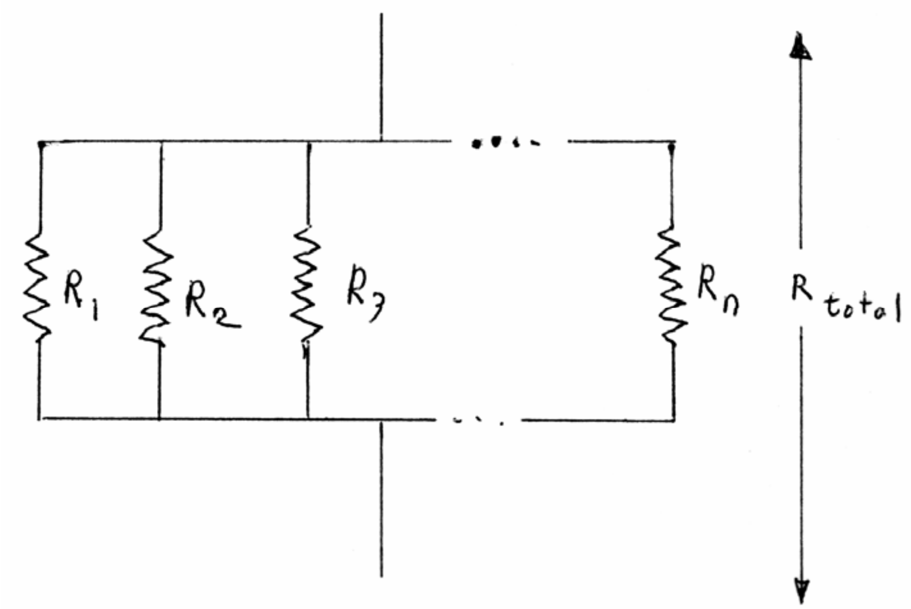
$$R_{total} = R_1 + R_2 + R_3 = 1000 + 2000 + 3000 = 6\ K\Omega$$

- Resistors in series law comes directly from KVL

Resistors in Parallel

- Resistors in parallel:
- Inverse of the total equals the sum of the inverses

$$\frac{1}{R_{total}} = \sum_{j=1}^{N} \frac{1}{R_j}$$



This comes directly from KCL at the node

$$I_{total} = \frac{V}{R_{total}} = \sum_{j=1}^{N} I_j = \sum_{j=1}^{N} \frac{V}{R_j}$$

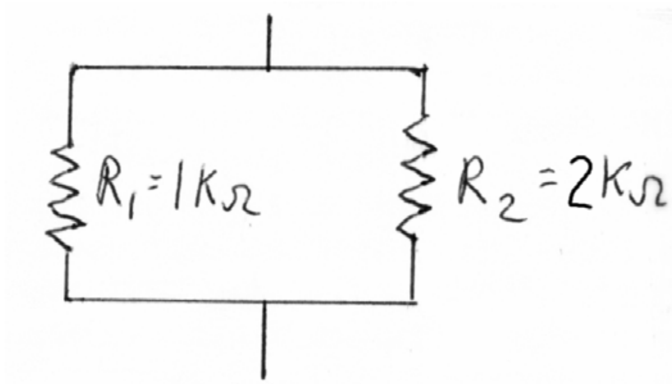
- NOTE: inverse of resistance called conductance (G)
- Units are mhos (ohms spelled backwards)

$$G_{total} = \sum_{j=1}^{N} G_j$$

- Thus when work in conductance change parallel to series equations

Example Parallel Resistors

Example 1K and 2K resistors in parallel

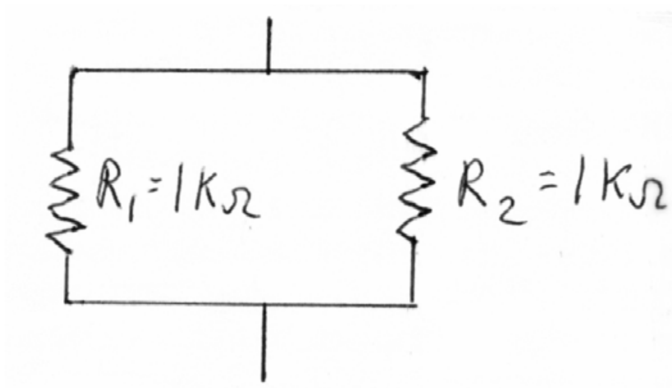


$$\frac{1}{R_{total}} = \sum_{j=1}^{N} \frac{1}{R_j} = \frac{1}{1000} + \frac{1}{2000} = \frac{3000}{2000000}$$

$$R_{total} = 666\ \Omega$$

Example Parallel Resistors

- Example: two 1 Kohm resistors in parallel



$$\frac{1}{R_{total}} = \sum_{j=1}^{N} \frac{1}{R_j} = \frac{1}{1000} + \frac{1}{1000} = \frac{2}{1000} \ldots or \ldots R_{total} = 500\ \Omega$$

- Thus adding N same resistors cuts get

$$R_{total} = \frac{R}{N}$$

- Good way to get lower R values