Example . Two straights *AB* having gradient rising to the right at 1 in 60 and *BC* having gradient falling to the right at 1 in 50, are to be connected at a summit by a parabolic curve. The point *A*, reduced level 121.45 m, lies on *AB* at chainage 1964.00 m, and *C*, reduced level 120.05 m, lies on *BC* at chainage 2276.00 m. The vertical curve must pass through a point *M*, reduced level 122.88 at chainage 2088.00 m.

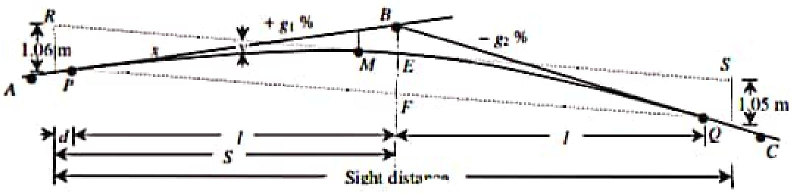
Design the curve, and determine the sight distance between two points 1.06 m above road level.

Solution (:

Given that

$$g_1 = + \frac{100}{60} = \frac{10}{6}\ \%$$

$$g_2 = + \frac{100}{50} = \frac{10}{5} = 2\%.$$



Fig

Let the horizontal distance $AB = x_1$.

$$\text{Level of } B = \text{Level of } A + \frac{x_1}{60}$$

$$= 121.45 + \frac{x_1}{60}$$

$$= \text{Level of } C + \frac{2276 - 1964 - x_1}{50}$$

$$= 120.05 + \frac{312 - x_1}{50}$$

Therefore

$$121.45 + \frac{x_1}{60} = 120.05 + \frac{312 - x_1}{50}$$

$$x_1 = \frac{50 \times 60}{110}\left(120.05 + \frac{312}{50} - 121.45\right) - 132.00 \text{ m.}$$

$$\text{Chainage of } B = \text{Chainage of } A + x_1$$

$$= 1964.00 + 132.00 = 2096.00 \text{ m.}$$

$$\text{Distance of } M \text{ from } B = x' = \text{Chainage of } B - \text{chainage of } M$$

$$= 2096.00 - 2088.00 = 8.00 \text{ m.}$$

$$\text{Distance of } M \text{ from } A = \text{Chainage of } M - \text{chainage of } A$$

$$= 2088 - 1964.00 = 124.00 \text{ m .}$$

$$\text{Grade level at the chainage of } M = 121.45 + \frac{124.00}{60} = 123.52 \text{ m.}$$

$$\text{Curve level at } M = 122.88 \text{ m}$$

$$\text{Offset at } M = 123.52 - 122.88 = 0.64 \text{ m.}$$

If the tangent length of the curve is l then the offset at M

$$= \frac{g_1 - g_2}{400l}(l - x')^2$$

or

$$0.64 = \frac{g_1 - g_2}{400l}(l - 8)^2$$

$$= \frac{\frac{10}{6} + 2}{400l}(l - 8)^2$$

$$l^2 - 85.818182\ l + 64 = 0$$

$$l = 85.066 \text{ m.}$$

The value of l can be taken as 85 m for the design purposes. Therefore

$$\text{chainage of } P = 2096.00 - 85 = 2011.00 \text{ m}$$

$$\text{chainage of } Q = 2011.00 + 2 \times 85 = 2181.00 \text{ m .}$$

Taking peg interval as 20 m, the values of x and chainage of the points are

x = 0.0 m	chainage of 0 (P)	= 2011.00 m
= 9.0 m	1	= 2020.00 m
= 29.0 m	2	= 2040.00 m
= 49.0 m	3	= 2060.00 m
= 69.0 m	4	= 2080.00 m
= 89.0 m	5	= 2100.00 m
= 109.0 m	6	= 2120.00 m
= 129.0 m	7	= 2140.00 m
= 149.0 m	8	= 2160.00 m
= 169.0 m	9	= 2180.00 m
= 170.0 m	10	= 2181.00 m

Grade levels

$$\text{Distance between } A \text{ and } P = 2011.00 - 1964.00 = 47.00 \text{ m}$$

$$\text{Grade level at } P = \frac{47}{60} + 121.45 = 122.23 \text{ m.}$$

The grade levels of other points can be obtained from 122.23 + $\frac{x}{60}$, and the offsets y from $\frac{g_1 - g_2}{400l}x^2$ by substituting the values of x. The curve levels can be calculated by subtracting the offsets from the corresponding grade levels. The calculated values of the design data are presented in Tabl

The level of a point on the curve above P is obtained by the expression

$$h = \frac{g_1}{100}x - \frac{g_1 - g_2}{400l}x^2$$

where x is the distance of the point P. The highest point on the curve is that point for which h is maximum. By differentiating the above expression and equating to zero, we get the value x_{max} at which $h = h_{max}$.

$$h = \frac{g_1 x}{100} - \frac{g_1 - g_2}{400l}x^2$$

$$\frac{dh}{dx} = \frac{g_1}{100} - 2\frac{(g_1 - g_2)}{400l}x = 0$$

$$x = \frac{400l\,g_1}{200(g_1 - g_2)}$$

$$= \frac{400 \times 85 \times \frac{10}{6}}{200 \times \left(\frac{10}{6} + 2\right)} = 77.27 \text{ m.}$$

$$= x_{max} \text{ for } h_{max}.$$

Table '. .

Point	Chainage (m)	Grade level (m)	x (m)	y (m)	Curve level (m)
0 (P)	2011.00	122.23	0.0	0.0	122.23
1	2020.00	122.38	9.00	0.01	122.37
2	2040.00	122.71	29.00	0.09	122.62
3	2060.00	123.05	49.00	0.26	122.79
4	2080.00	123.38	69.00	0.51	122.87
5	2100.00	123.71	89.00	0.85	122.86
6	2120.00	124.05	109.00	1.28	122.77
7	2140.00	124.38	129.00	1.79	122.59
8	2160.00	124.71	149.00	2.39	122.32
9	2180.00	125.05	169.00	3.08	121.97
10 (Q)	2181.00	125.06	170.00	3.11	121.95

The grade level at x_{max} = $122.23 + \frac{77.27}{60} = 123.52$ m

and the offset $= \frac{\left(\frac{10}{6}+2\right)}{400 \times 85} \times 77.27^2 = 0.64$ m.

Thus the reduced level of the highest point

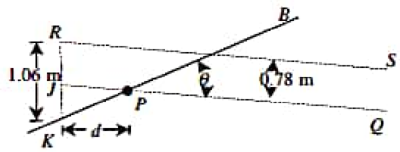
$= 123.52 - 0.64 = 122.88$ m.

The offset BE at B $= \frac{\left(\frac{10}{6}+2\right)}{400 \times 85} \times 85^2 = 0.78 \text{ m} = EF.$

The sight line can be taken as the tangent RS to the curve at E. RS is parallel to PQ which has a slope of $\frac{\frac{g_1 l}{100}+\frac{g_2 l}{100}}{2l} = \frac{g_1+g_2}{200}$ radians, and the slope of PB is $\frac{g_1}{100}$ radians. Thus the angle θ between PB and PQ as shown in Fig. 7.27,

$$= \frac{g_1}{100} - \frac{g_1+g_2}{200}$$

$$= \frac{g_1-g_2}{200}.$$



Distance $JK = 1.06 - 0.78 = 0.28$ m.

Fig.

Thus in ΔJPK, we have

$$\frac{JK}{d} = \theta$$

$$d = \frac{JK}{\theta}$$

$$= \frac{JK}{\frac{g_1-g_2}{200}} = \frac{0.28 \times 200}{\left(\frac{10}{6}+2\right)} = 15.27 \text{ m.}$$

Thus the total sight distance

$$RS = 2\ (d + l)$$

$$= 2 \times (15.27 + 85) = 200.54 \text{ m}$$

$= \textbf{200 m}$ (say)