A vertical curve is used to join two intersecting grade lines of rail-roads, highways or other routes to smooth out the changes in vertical motion. An abrupt change in the rate of the grade could otherwise subject a vehicle passing over it to an impact that would be either injurious or dangerous. The vertical curve, thus, contributes to the safety, comfort and appearance. Either a circular arc or a parabola may be used for this purpose, but for simplicity of calculation work, the latter is preferred and is invariably used. The parabolic curve also produces the best riding qualities, since the rate of change in grade is uniform throughout in a parabola having a vertical axis. This is proved as under.

The general equation of a parabola with a vertical axis can be written as

$$y = ax^2 + bx$$

The slope of this curve at any point in given by $\frac{dy}{dx} = 2ax + b$

The rate of change of slope or rate of change of grade (r) is given by

$$\frac{d^2y}{dx^2} = r = 2a = \text{constant}$$

Thus, the grade changes *uniformly throughout the curve,* which is a desired condition.

The Grade

The grade or gradient of a rail-road or highway is expressed in two ways :

(*i*) As a percentage : *e.g.* 2% or 3%

(*ii*) As 1 vertical in n horizontal (1 in n) : *e.g.* 1 in 100 or 1 in 400.

A grade is said to be *upgrade or* + ve grade when elevations along it increase, while it is said to be a downgrade or − ve grade when the elevations decrease along the direction of motion.

Rate of change of grade (r) : Equation (*ii*) gives the grade at any point on the curve. The gradient changes from point to point on the curve, but the rate of change of grade, given by equation, (*iii*) is constant in a parabola. For first class railways, the

TYPES OF VERTICAL CURVES :

Vertical curves may be of the following six types :

(1) An upgrade (+ g_1%) followed by a downgrade (− g_2%) (Fig.

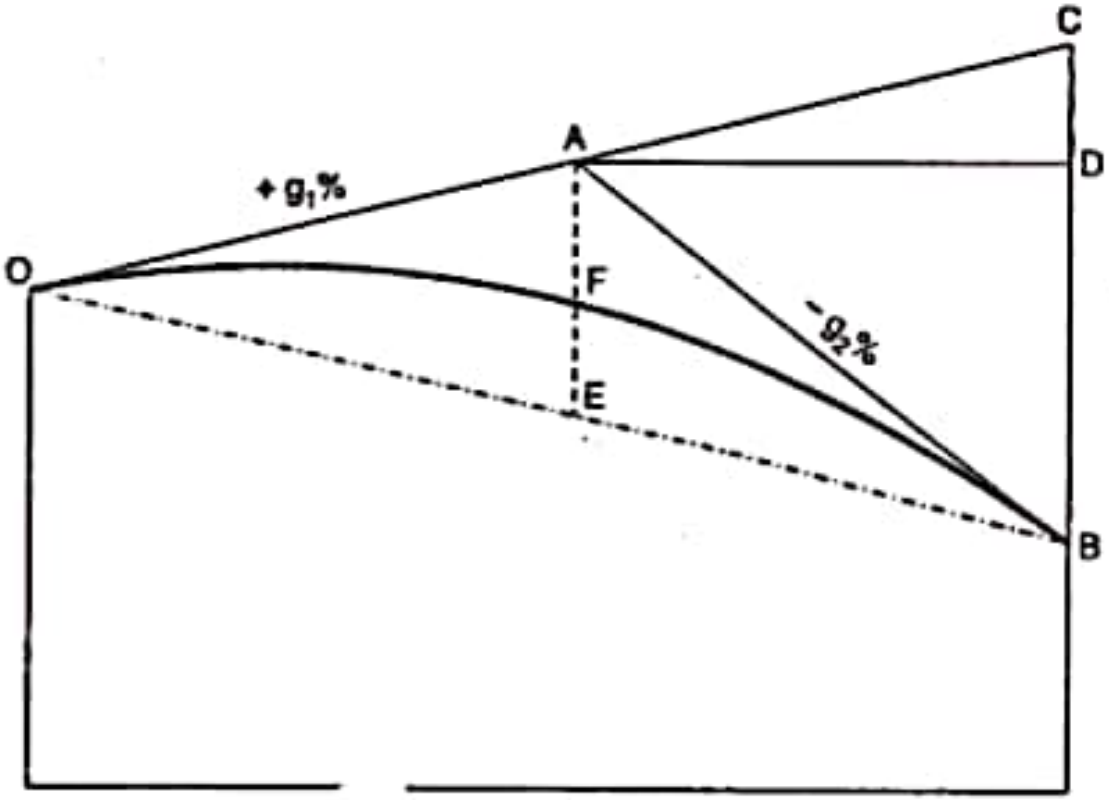


FIG. - SUMMIT OR CONVEX.

(2) A downgrade (− g_1%) followed by an upgrade (+ g_2%) (Fig.

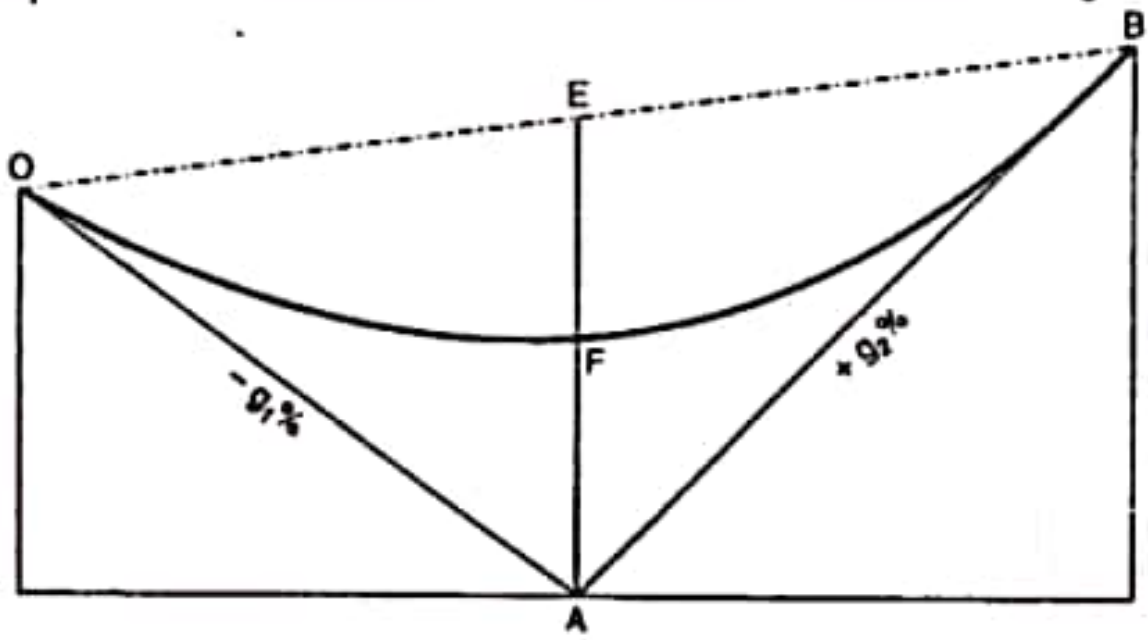


FIG. SAG OR CONCAVE ($g_2 > g_1$).

An upgrade (+ g_1%) followed by another upgrade (+ g_2%) $g_2 > g_1$

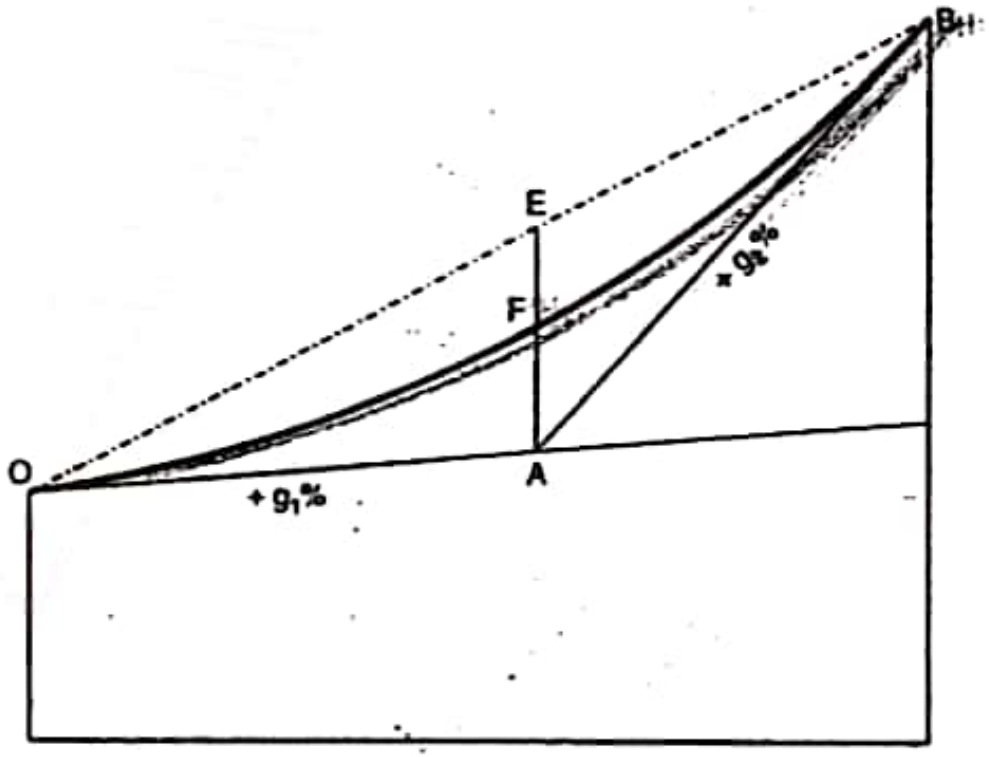
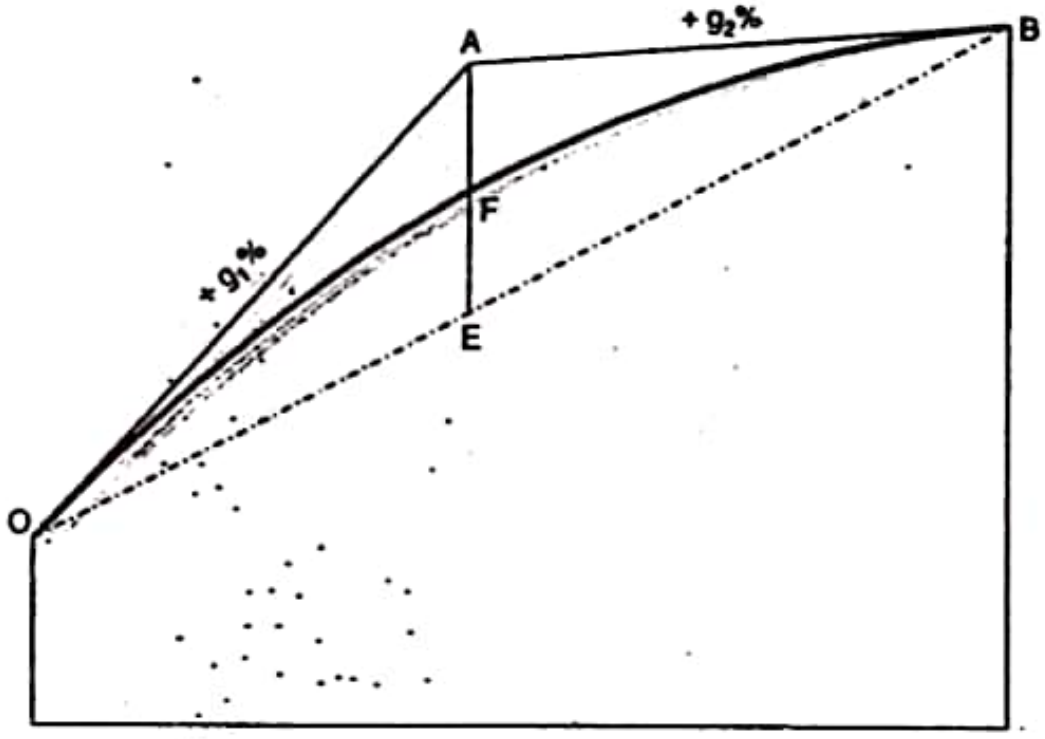
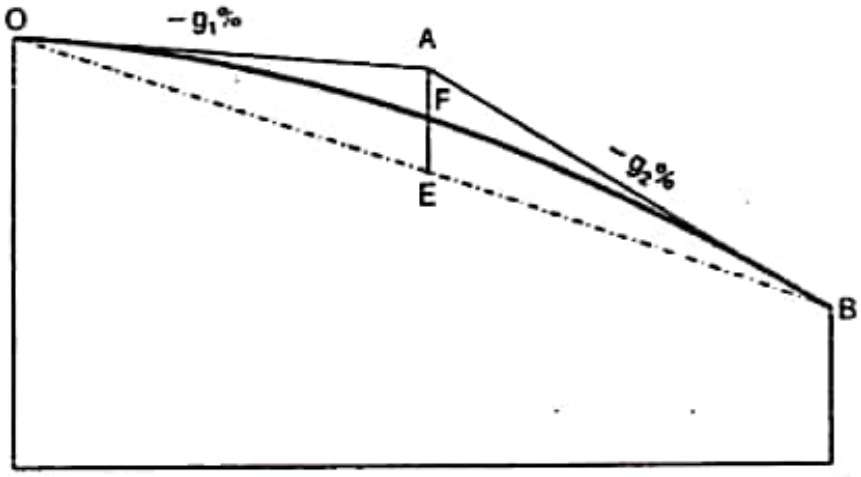


FIG. SAG OR CONCAVE ($g_2 > g_1$)

An upgrade (+ g_1%) followed by another upgrade (+ g_2%) $g_1 > g_2$

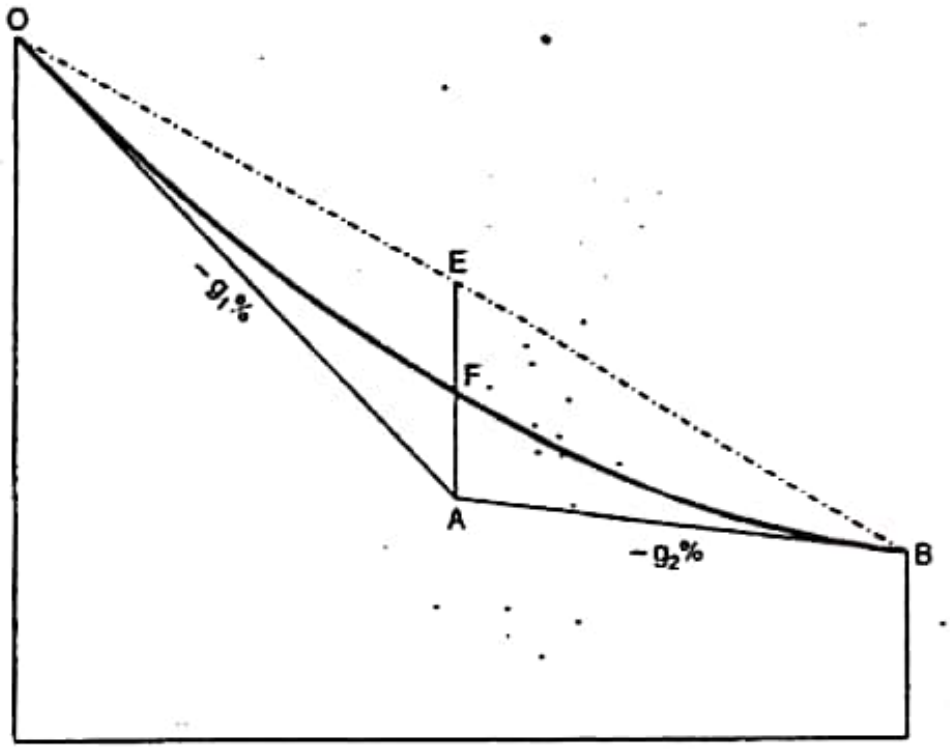


A downgrade ($-g_1\%$) followed by another downgrade ($-g_2\%$) : $g_2 > g_1$



SUMMIT OR CONVEX ($g_2 > g_1$).

A downgrade ($-g_1\%$) followed by another downgrade ($-g_2\%$) : $g_1 > g_2$



SAG OF CONCAVE ($g_1 > g_2$).

LENGTH OF VERTICAL CURVE

The length of the vertical curve can be obtained by dividing the algebraic difference of the two grades by the rate of change of grade, due regard being paid to the sign of the grade. Thus,

$$\text{Length of curve } (L) = \frac{\text{Total change of grade}}{\text{Rate of change of grade}} = \frac{g_1 - g_2}{r} \text{ chains}$$

where $g_1 - g_2$ = Algebraic difference of the two grades (%)

r = Rate of change of grade (%) per chain