. TILT DISTORTION OR TILT DISPLACEMENT

If a terrain is photographed, once with a tilted photograph and then with a vertical photograph, both taken at the same flight altitude and with the same focal length, the two photographs will match at the axis of tilt only. The image of any other point, not on the axis of tilt, will be *displaced* either outward or inward with respect to its corresponding position on a vertical photograph.

Tilt distortion or tilt displacement is defined as the difference between the distance of the image of a point on the tilted photograph from the isocentre and the distance of the image of the same point on the photograph from the isocentre if there had been no tilt.

Fig. . / shows a vertical photograph and tilted photograph of the same terrain, intersecting each other in a line which is the axis of tilt. *n* is the nadir point of the tilted photograph, and serves as the principal point of the vertical photograph. *k* is the principal point of the tilted photograph. The portion of the tilted photo above the axis of tilt is known as the *upper part* while the portion below it is known as the *lower part* of the photograph.

Let us consider two ground points *A* and *B* photographed both on the vertical photograph as well as on the tilted photograph. *a* and *b* are their images on the tilted photo while *a'* and *b'* are the corresponding images on the vertical photograph. If the vertical photograph is now rotated about the axis of tilt until it is in the plane of

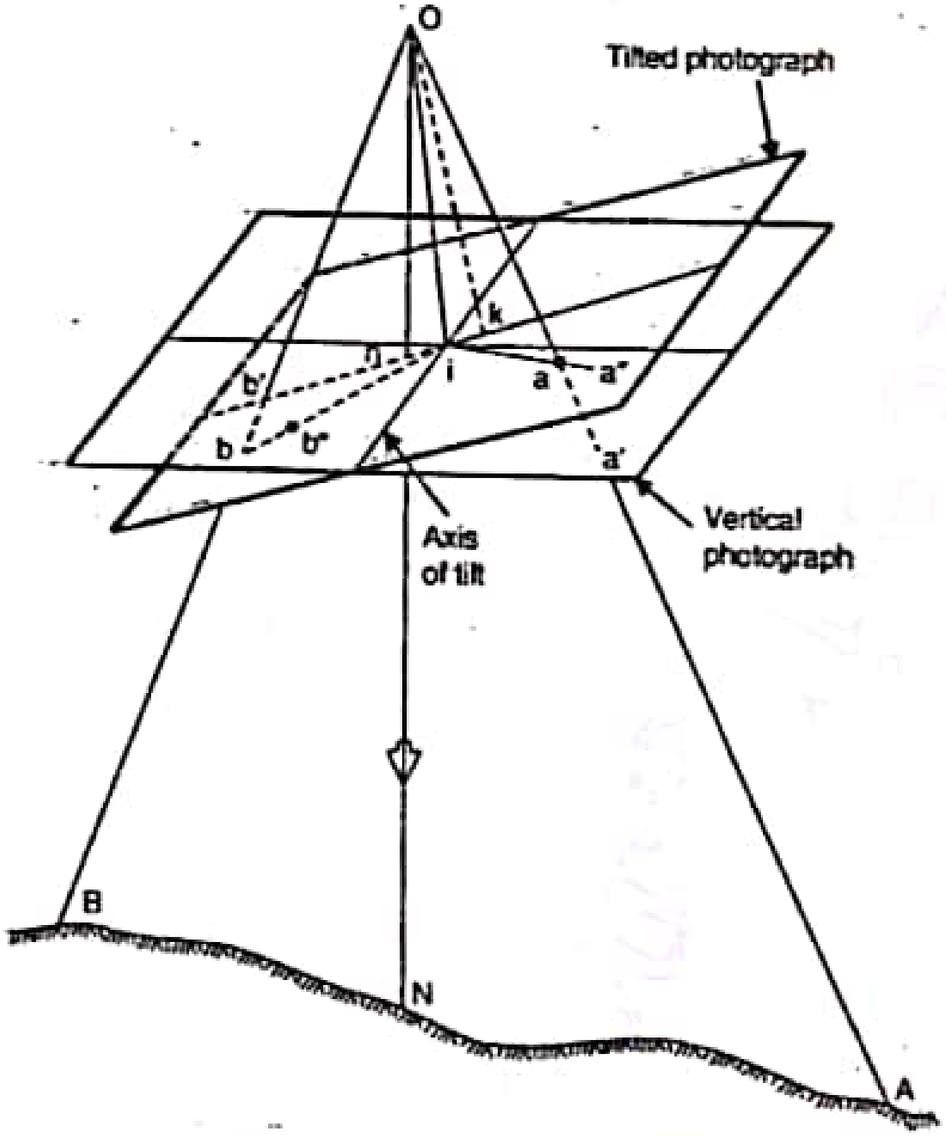

FIG. TILT DISTORTION.

the tilted photograph, point a' would fall at a'' while point b' would fall at b''. The tilt displacement of points a and b are therefore aa'' and bb''. *It is to be noted that these displacement occur along lines which radiate from the isocentre.*

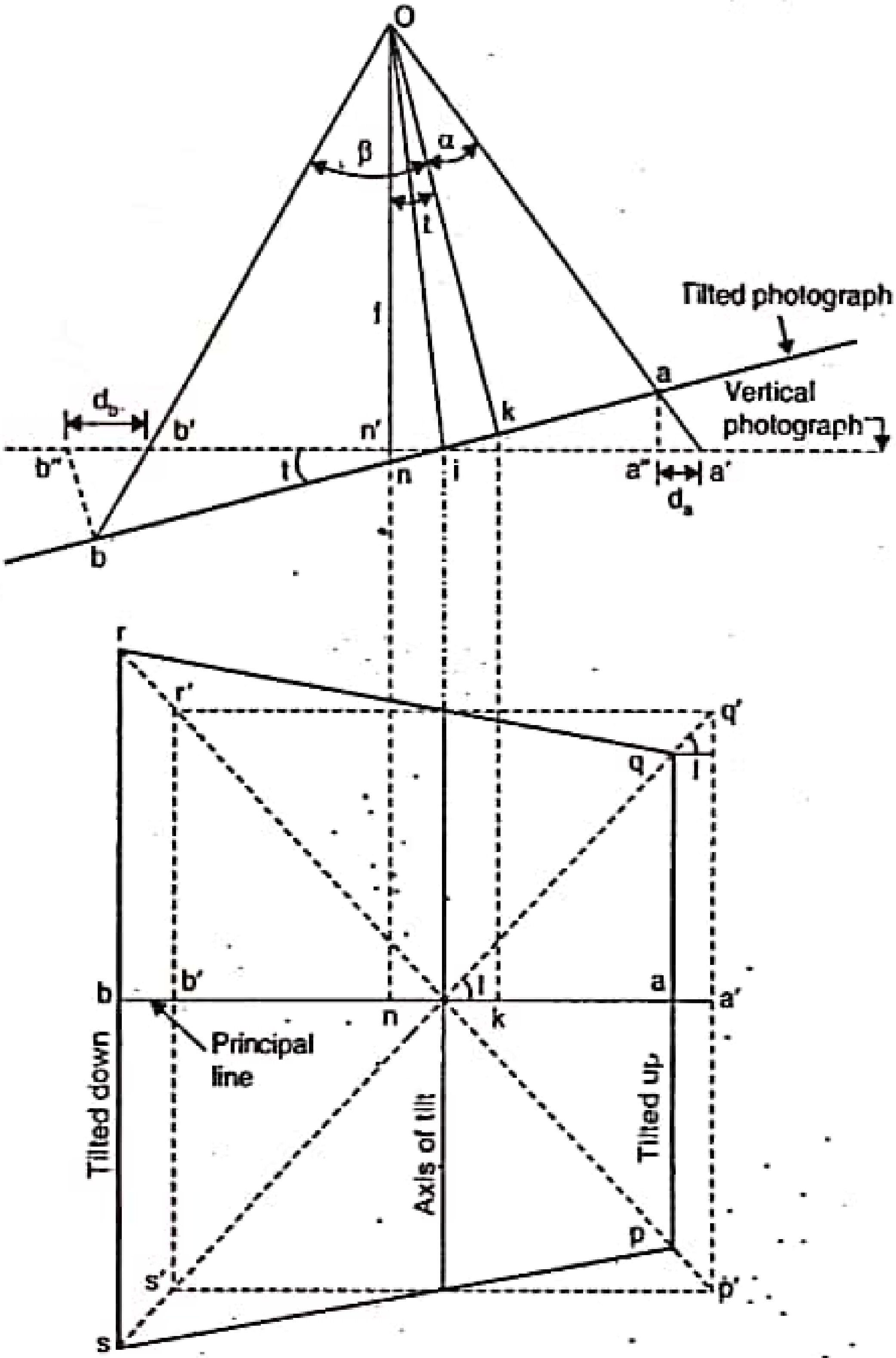

FIG. 1· 5. DISTORTION OF A SQUARE.

To calculate the amount of tilt distortion or displacement, consider Fig. which shows the effect of tilt along a line perpendicular to the axis of tilt. The line ab represents the principal line of the tilted photograph and $a'b'$ represents the principal line of the vertical photograph. The principal point for the tilted photograph is at k while that of the vertical photograph is at n'. The axis of tilt (perpendicular to the plane of the paper in the sectional view) is at i. O is the exposure station which is common for both the photographs. a

and b are the images of two points on the tilted photograph, along its principal line, while a' and b' are the corresponding positions on the vertical photograph. Since i is the point of rotation, d_a and d_b represent the displacements of the points a and b with respect to a' and b' respectively. Let α be the inclination of the ray Oa with Ok. Similarly, β is the inclination of the ray Ob to Ok.

Thus d_a = tilt displacement of a with respect to a'

or $d_a = ia' - ia$

But $ia' = n'a' - n'i = f \tan(t + \alpha) - f \tan t/2$ and $ia = ka + ki = f \tan \alpha + f \tan t/2$

Hence $d_a = f \tan(t + \alpha) - f \tan t/2 - f \tan \alpha - f \tan t/2$

or $d_a = f[\tan(t + \alpha) - \tan \alpha - 2 \tan t/2]$

Similarly, $d_b = ib - ib'$

$ib = kb - ki = f \tan \beta - f \tan t/2$; $ib' = n'b' + n'i = f \tan(\beta - t) + f \tan t/2$

$\therefore$ $d_b = f \tan \beta - f \tan t/2 - f \tan(\beta - t) - f \tan t/2$

or $d_b = f[\tan \beta - \tan(\beta - t) - 2 \tan t/2]$

In the above expressions, the angles α and β can be found by the relations :

$$\tan \alpha = \frac{ka}{f}, \text{ and } \tan \beta = \frac{kb}{f}.$$

It can be shown that equations can be represented by the approximate formula

$$d = \frac{(ia)^2 \sin t}{f}$$

It is quite clear from the figure that the tilt displacement of a point on the upward half of a tilted photograph is *inward* (such as for point a) while the tilt displacement of a point on the downward or nadir point half is *outward* (such as for b)

Equations give the tilt displacements for the points on the principal line. The tilt displacement of a point not lying on the principal line is greater than that of a corresponding point on the principal line.

Let I = angle measured at the isocentre from the principal line to the point.

d_u = displacement of the point on the upward half of the tilted photograph.

d_d = displacement of the point on the downward half of the tilted photograph.

In Fig. (plan), the point q is not on the principal line while point a is on the principal line. qq' is the displacement of q while aa' is the displacement of point a. Since both q and a are equidistant from the axis of tilt, we have

$$qq' = aa' \sec I$$

where I is the angle at the isocentre from the principal line to the point q.

Hence the ratio of the tilt displacement of a point not on the principal line to that of a point on the principal line is equal to the secant of the angle at the isocentre from the principal line to the point.