COMPUTATIONS AND SETTING OUT A VERTICAL CURVE

In vertical curves, all distances along the curve are measured *horizontally* and all offsets from the tangents to the curve are measured *vertically*. The length of the curve is thus its horizontal projection, without appreciable error since the curve is quite flat.

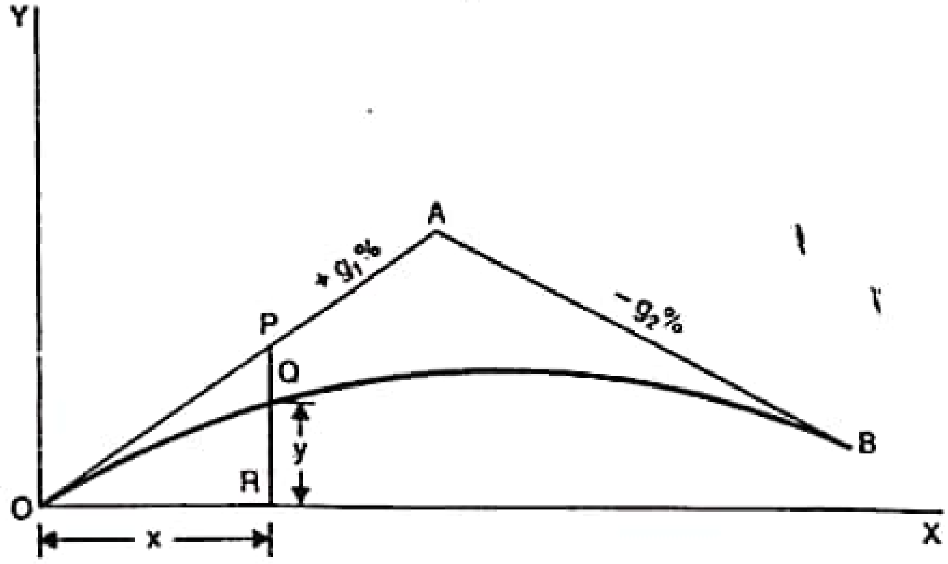


FIG. . THE PARABOLA.

In Fig. . ., let

OX and OY = The axes of the rectangular ordinates passing through the beginning (O) of the vertical curve

OA = Tangent having $+g_1\%$ slope

AB = Tangent having $-g_2\%$ slope

Q = Any point on the curve having co-ordinates (x, y)

Draw PQR, a vertical line through Q.

The equation of the parabola can be written as

$$y = ax^2 + bx \qquad \therefore \qquad \frac{dy}{dx} = 2ax + b$$

At $\quad x = 0, \ \frac{dy}{dx} = +g_1$

$\therefore \quad g_1 = 2a(0) + b \quad$ or $\quad b = g_1$

Hence, the equation of the parbola is

$$y = ax^2 + g_1x$$

Let $\quad PQ = h$ = vertical distance between the tangent and the corresponding point Q on the curve
= Tangent correction

$\therefore \quad PQ = PR - QR$

But $\quad PR = g_1 x$ and $QR = y$

$\therefore \quad PQ = h = g_1 x - y$

But $\quad g_1 x - y = -ax^2$, from equation

Hence $\quad h = g_1 x - y = -ax^2$

or $\quad h = C x^2$

or $\quad h = kN^2$

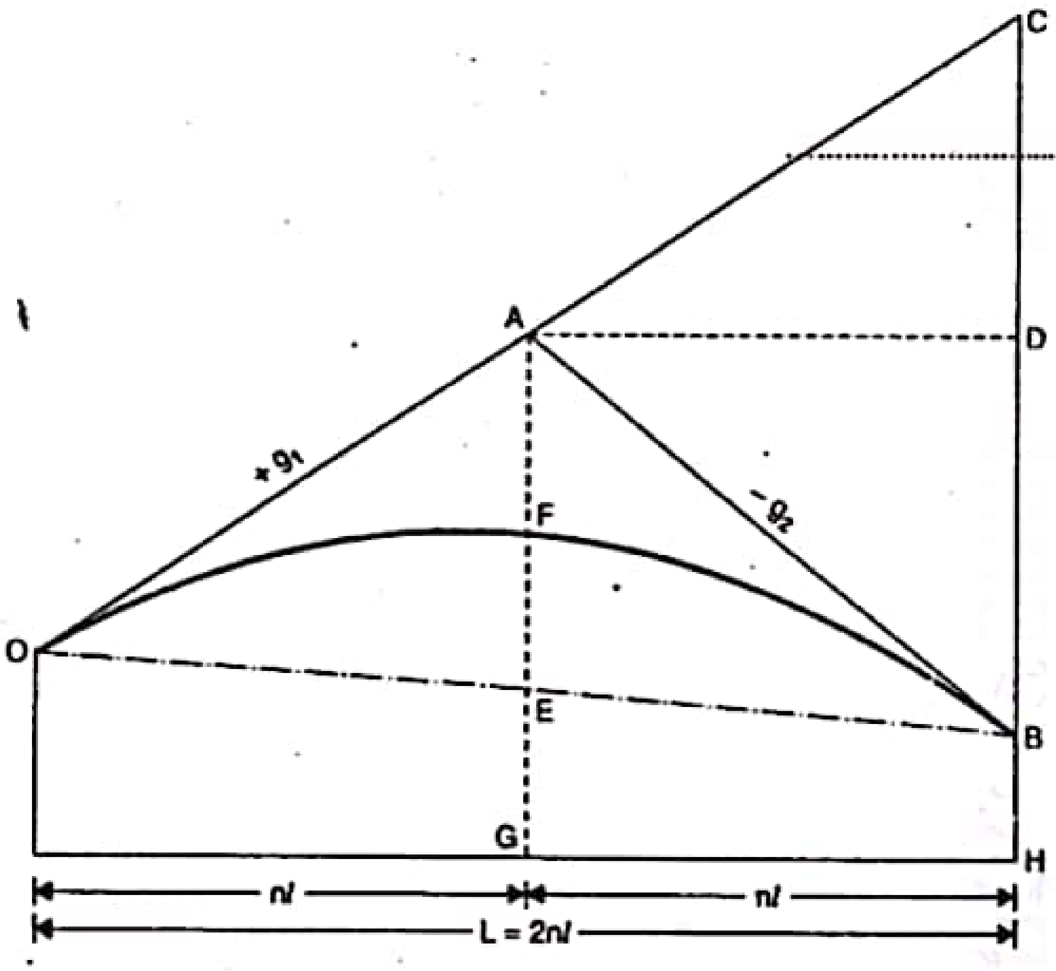
where N is counted from O at the beginning of the curve.

Thus, the difference in elevation between a vertical curve and a tangent to it varies as the square of its horizontal distance from the point of tangency. This difference in elevation is also known as the *tangent correction.*

The offsets are measured vertically downwards, though they should be measured parallel to the axis of the parabola for a true curve. Due to unequal values of g_1 and g_2, the axis is slightly tilted. Hence by making the offsets vertical (and not parallel to the tilted axis), the curve will be slightly distorted from its parabolic form. However, the distortion is negligible for all practical purposes.

The value of k in equation can be found by considering as follows.

In , produce OA to C, a point vertically above B. Through A, draw AD horizontal to meet BC in D.



Let $2n$ = Total number of equal chords, each of length l, on each side of the apex

g_1 and g_2 = Grades of the two tangents

e_1 and e_2 = Corresponding rises or falls *per chord length* l (figures plus or minus as they represent rises or falls)

$$OA = AC$$

$$CD = ne_1$$

and $$BD = -ne_2$$

$\therefore$ $$CB = CD + DB = n(e_1 - e_2), \text{ algebraically.}$$

From equation ,

$$CB = kN^2, \text{ where } N = 2n$$

$$= k(2n)^2$$

$\therefore$ $$4kn^2 = n(e_1 - e_2)$$

or $$k = \frac{e_1 - e_2}{4n}$$ ...

In the above equation, proper care for the signs must be taken while substituting the numerical values of e_1 and e_2.

Elevation by Tangent Correction

Knowing the value of k, the tangent corrections for various values of N can be calculated from equation) and the elevations of various points on the curve can be computed in the following steps :

(1) Let the elevation and chainage of the apex A be known.

Let the length of the curve on either side of the station be n chords of equal length l.

Then chainage of point of tangency (O) = Chainage of $A - nl$ and,

chainage of point of tangency (B) = Chainage of $A + nl$.

(2) Knowing the grades g_1 and g_2 and the elevation of the apex A, the elevation of O and B can be calculated as under :

Elevation of O = Elevation of $A \mp ne_1$ (use minus sign if e_1 is positive and plus sign if it is negative)

Elevation of B = Elevation of $A \pm ne_2$ (use plus sign if e_2 is positive and minus sign if it is negative)

If, however O is taken as the datum, elevation of F can be found as under :

Elevation of $A = ne_1$

Elevation of B = Elevation of $A + ne_2 = ne_1 + ne_2$

Elevation of $E = \frac{1}{2}$ (Elev. of O + Elev. of B) $= \frac{n}{2}(e_1 + e_2)$

Since $OE = EB$, AE is a diameter of the parabola,

and $AF = FE$.

Elevation of F = Elevation of $E + n^2k$

The elevation of F can also be found by subtracting algebraically n^2k from the elevation of A. Thus.

Elevation of F = Elevation of $A - n^2 k = ne_1 - n^2k$.

(3) Compute the tangent corrections from the expression

$$h = kN^2$$

Thus

$$h_1 = 1\ k$$
$$h_2 = 4\ k$$
$$h_3 = 9\ k$$
$$\ldots \quad \ldots$$
$$h_N = (2n)^2\ k.$$

(4) Compute the elevation of the corresponding stations on the tangent OAC. Thus:

Elevation of tangent at any station (n')=elevation of point of tangency (O) + n'e₁

where n' is the *number* of that station from O.

(5) Find the elevations of the corresponding stations on the curve by adding algebraically the tangent corrections to the elevations of the corresponding stations.

If the value of k is positive, the tangent corrections are to be subtracted from grade elevations ; if it is negative, tangent corrections are additive.

The result may be tabulated as under :

Station	*Chainage*	*Tangent or grade elevation*	*Tangent correction*	*Elevation of the curve*	*Remarks*
....					