Example *A – 1.0 percent grade meets a + 2.0 percent grade at station 470 of elevation 328.605 metres. A vertical curve of length 120 metres is to be used. The peges are to be fixed at 10 metres interval. Calculate the elevations of the points on the curve by (a) tangent corrections and (b) by chord gradients.*

If the pege are to be driven with their tops at the formation of the curve, calculate the staff readings required. given that height of collimation is 330.890.

Solution.

(*a*) ***Tangent correction***

Total number of stations in 10 m unit $= \frac{120}{10} = 12$

Number of stations to each side of apex $= n = 6$

Change of elevation of first tangent per chord length of 10 m

$$\therefore \quad = e_1 = \frac{g_1}{100} \times 10 = \frac{-1.0}{100} \times 10 = -0.10 \text{ m}$$

Change of elevation of second tangent per chord length of 10 m

$$= e_2 = \frac{g_2}{100} \times 10 = \frac{+2.0}{100} \times 10 = +0.20 \text{ m}$$

Elevation of point of intersection $= 328.605$ m

Elevation of the beginning of curve $= 328.605 - ne_1$

$= 328.605 - (6)(-0.10)$

$= 329.205$ m

Elevation of the end of curve $= 328.605 + ne_2$

$= 328.605 + (6)(0.2)$

$= 329.805$ m

The tangent correction with respect to the first tangent is given by

$$h = kN^2$$

where $$k = \frac{e_1 - e_2}{4n} = \frac{(-0.10) - (0.20)}{4 \times 6} = \frac{-0.3}{24} = -\frac{1}{80}$$

Hence $$h = -\frac{N^2}{80}$$

Since the sign of k is negative, h will be additive to the tangent elevations to get the elevations on the curve.

For the first point, tangent elevation = elevation of the beginning of the curve + e_1

$$= 329.205 - 0.10 = 329.105$$

Tangent correction $= \frac{1}{80} = 0.0125 \text{ m} \approx 0.010 \text{ m}$

(Since the readings can be taken upto an accuracy of the multiples of 0.005 m).

Elevation of first point $= 329.105 + 0.010 = 329.115$ m

Similarly, for the second point, tangent elevation

$$= 329.205 - 0.2 = 329.005$$

Tangent correction $= \frac{(2)^2}{80} = 0.050$ m

Elevation of second point $= 329.005 + 0.050 = 329.055$ m

The values for other points along with the required staff reading are tabulated below. The required staff readings for the pegs are obtained by subtracting the elevations of the points from the height of collimation.

Station	*Chainage*	*Tangent elevation*	*Tangent correction (+ ve)*	*Curve elevation*	*Ht. of collim-ation*	*Staff reading*	*Remarks*
0	410	329.205	0	329.205	330.890	1.685	Beginning of the curve
1	420	329.105	0.010	329.115		1.775	
2	430	329.005	0.050	329.055		1.835	
3	440	328.905	0.115	329.020		1.870	
4	450	328.805	0.200	329.005		1.885	
5	460	328.705	0.315	329.020		1.870	
6	470	328.605	0.450	329.055		1.835	Vertex of the curve
7	480	328.505	0.615	329.120		1.770	
8	490	328.405	0.800	329.205		1.685	
9	500	328.305	1.015	329.320		1.570	
10	510	328.205	1.250	329.455		1.435	
11	520	328.105	1.515	329.620		1.270	
B	530	328.005	1.800	329.805		1.085	End of the curve

Check:

Elevation of mid-point of OB $= \frac{1}{2}(329.205 + 329.805) = 329.505$ m

Elevation of the vertex $= \frac{1}{2}(329.505 + 328.605) = 329.055$ m

(*b*) **Chords gradients**

The chord gradient for any point is given by equation 4.5 *i.e.*,

$$N\text{th chord gradient} = e_1 - (2N - 1)\,k$$

Here, $e_1 = -0.1$, $k = -\frac{1}{80}$

(1) For the first point, chord gradient

$$= -0.1 - (2 - 1)\left(-\frac{1}{80}\right) = -0.1 + \frac{1}{80} = -0.090.$$

Elevation of first point = elevation of O + chord gradient

$$= 329.205 - 0.090 = 329.115.$$

(2) For the second point, chord gradient

$$= -0.1 - (4 - 1)\left(-\frac{1}{80}\right) = -0.1 + \frac{3}{80} = -0.060$$

Elevation of second point = 329.115 − 0.060 = 329.055

(3) For the third point, chord gradient

$$= -0.1 - (6 - 1)\left(-\frac{1}{80}\right) = -0.1 + \frac{5}{80} = -0.040$$

Elevation of third point = 329.055 − 0.040 = 329.015.

(4) For the fourth point, chord gradient

$$= -0.1 - (8 - 1)\left(-\frac{1}{80}\right) = -0.1 + \frac{7}{80} = -0.010$$

Elevation of fourth point = 329.015 − 0.010 = 329.005.

(5) For the fifth point, chord gradient

$$= -0.1 - (10 - 1)\left(-\frac{1}{80}\right) = -0.1 + \frac{9}{80} = +0.015$$

Elevation of fifth point = 329.005 + 0.015 = 329.020

(6) For the sixth point chord gradient

$$= -0.1 - (12 - 1)\left(-\frac{1}{80}\right)$$

$$= -0.1 + \frac{11}{80} = +0.035$$

Elevation of sixth point = 329.020 + 0.035 = 329.055

(7) For the seventh point, chord gradient

$$= -0.1 - (14 - 1)\left(-\frac{1}{80}\right) = -0.1 + \frac{13}{80} = +0.065$$

Elevation of seventh point = 329.055 + 0.065 = 329.120

(8) For the eighth point, chord gradient

$$= -0.1 - (16 - 1)\left(-\frac{1}{80}\right) = -0.1 + \frac{15}{80} = +0.085$$

Elevation of eighth point = 329.120 + 0.085 = 329.205

(9) For the ninth point, chord gradient

$$= -0.1 - (18 - 1)\left(-\frac{1}{80}\right) = -0.1 + \frac{17}{80} = +0.115$$

Elevation of ninth point = 329.205 + 0.115 = 329.320

(10) For the tenth point, chord gradient

$$= -0.1 - (20 - 1)\left(-\frac{1}{80}\right) = -0.1 + \left(\frac{19}{80}\right) = +0.135$$

Elevation of tenth point = 329.320 + 0.135 = 329.455

(11) For the eleventh point, chord gradient

$$= -0.1 - (22 - 1)\left(-\frac{1}{80}\right) = -0.1 + \frac{21}{80} = +0.165$$

Elevation of eleventh point = 329.455 + 0.165 = 329.620

(12) For point B, chord gradient

$$= -0.1 - (24 - 1)\left(-\frac{1}{80}\right) = -0.1 + \frac{23}{80} = +0.185$$

Elevation of B = 329.620 + 0.185 = 329.805.

Knowing the elevations of the points on the curve, the staff readings for various pegs can be calculated and tabulated as done earlier.