Example *An instrument was set up at P and the angle of depression to a vane 2 m above the foot of the staff held at Q was 5° 36′. The horizontal distance between P and Q was known to be 3000 metres. Determine the R.L. of the staff station Q, given that staff reading on a B.M. of elevation 436.050 was 2.865 metres.*

Solution.

The difference in elevation between the vane and the instrument axis

$$= D \tan \alpha = 3000 \tan 5° 36' = 294.153$$

Combined correction due to curvature and refraction $= \frac{6}{7} \frac{D^2}{2R}$

or $C = 0.06728\, D^2$ metres, when D is in km $= 0.06728 \left(\frac{3000}{1000} \right)^2 = 0.606$ m.

Since the observed angle is negative, the combined correction due to curvature and refraction is subtractive.

Difference in elevation between the vane and the instrument axis $= 294.153 - 0.606$

$= 293.547 = h.$

R.L. of instrument axis $= 436.050 + 2.865 = 438.915$

$\therefore$ R.L. of the vane $=$ R.L. of instrument aixs. $- h$

$= 438.915 - 293.547 = 145.368$

R.L. of Q $= 145.368 - 2$

$=$ **143.368 m.**

Example *In order to ascertain the elevation of the top (Q) of the signal on a hill, observations were made from two instrument stations P and R at a horizontal distance 100 metres apart, the station P and R being in the line with Q. The angles of elevation of Q at P and R were 28° 42′ and 18° 6′ respectively. The staff reading upon the bench mark of elevation 287.28 were respectively 2.870 and 3.750 when the instrument was at P and at R, the telescope being horizontal. Determine the elevation of the foot of the signal if the height of the signal above its base is 3 metres.*

Solution. (Fig. 5.6)

Elevation of instrument axis at P = R.L of B.M. + Staff reading

$= 287.28 + 2.870 = 290.15$ m

Elevation of instrument axis at R = R.L. of B.M. + staff reading

$= 287.28 + 3.750 = 291.03$ m

Difference in level of the instrument axes at the two stations $= s = 291.03 - 290.15 = 0.88$ m

$\alpha_1 = 28° 42'$ and $\alpha_2 = 18° 6'$

$$D = \frac{b}{\tan\alpha_1 - \tan\alpha_2} = \frac{100}{\tan 28°\,42' - \tan 18°\,6'} = 152.1 \text{ m.}$$

$\therefore$ $h_1 = D \tan\alpha_1 = 152.1 \tan 28°\,42' = 83.272$ m

$\therefore$ R.L. of foot of signal = R.L. of inst. axis at $P + h_1$ – ht. of signal

$= 290.15 + 83.272 - 3 = \mathbf{370.422}$ m.

Check : $(b + D) = 100 + 152.1 = 252.1$ m

$h_2 = (b + D) \tan\alpha_2 = 252.1 \times \tan 18°\,6' = 82.399$ m

R.L. of foot of signal = R.L. of inst. axis at $R + h_2$ + ht. of signal

$= 291.03 + 82.399 - 3 = \mathbf{370.429}$ m.

Example *The top (Q) of a chimney was sighted from two stations P and R at very different levels, the stations P and R being in the line with the top of the chimney. The angle of elevation from P to the top of the chimney was 38° 21′ and that from R to the top of the chimney was 21° 18′. The angle of elevation from R to a vane 2 m above the foot of the staff held at P was 15° 11′. The heights of the instrument at P and R were 1.87 m and 1.64 m respectively. The horizontal distance between P and R was 127 m and the reduced level of R was 112.78 m. Find the R.L. of the top of the chimney and the horizontal distance from P to the chimney.*

Solution.

(*i*) When the observations were taken from R to P

$h = b \tan\alpha = 127 \tan 15°\,11' = 34.47$ m

R.L. of P = R.L. of R + height of instrument at $R + h - r$

$= 112.78 + 1.64 + 34.47 - 2 = 146.89$ m

R.L. of instrument axis at P = R.L. of P + ht. of instrument at P

$= 146.89 + 1.87 = 148.76$ m

Difference in elevation between the instrument axes $= s$

$= 148.76 - (112.78 + 1.64) = 34.34$ m

$\therefore$ $D = \dfrac{(b \tan\alpha_2 - s)}{\tan\alpha_1 - \tan\alpha_2} = \dfrac{127 \tan 21°\,18' - 34.34}{\tan 38°\,21' - \tan 21°\,18'} = \dfrac{49.25 - 34.34}{0.78812 - 0.38988} = 37.8$ m

$h_1 = D \tan\alpha_1 = 37.8 \tan 38°\,21' = 29.91$

$\therefore$ R.L. of Q = R.L. of instrument axis at $P + h_1$

$= 148.76 + 29.91 = \mathbf{178.67}$ m.

Check : R.L. of Q = R.L. of instruments axis at $R + h_2$

$= (112.78 + 1.64) + (b + D) \tan\alpha_2 = 114.42 + (127 + 37.8) \tan 21°\,18'$

$= 114.42 + 64.25 = \mathbf{178.67}$ m.