Example *Two triangulation stations A and B are 60 kilometres apart and have elevations 240 m and 280 m respectively. Find the minimum height of signal required at B so that the line of sight may not pass near the ground than 2 metres. The intervening ground may be assumed to have a uniform elevation of 200 metres.*

Solution.

Minimum elevation of line of sight = 200 + 2 = 202 m

Let us take this elevation as the datum

∴ Height of A above this datum = $h_1 = 240 - 202 = 38$ m

The tangent distance D_1 corresponding to h_1 is given by

$$D_1 = 3.8553\sqrt{h_1} = 3.8553\sqrt{38} = 23.766 \text{ km.}$$

∴ Distance of B from the point of tangency

$$= D_2 = D - D_1 = 60 - 23.766 = 36.234 \text{ km.}$$

The elevation h_2 (of B above the datum) corresponding to the distance D_2 is given by

$$h_2 = 0.06728\, D_2^2 = 0.06728\,(36.234)^2 = 88.33 \text{ m}$$

∴ Elevation of line of sight at B = 202 + 88.33 = 290.33 m

Ground level at B = 280 m

∴ Minimum height of signal above ground at B = 290.33 − 280 m = **10.33** m.

Example *The altitude of two proposed stations A and B 130 km apart are respectively 220 m and 1160 m. The altitudes of two points C and D on the profiles between them are respectively 308 m and 632 m, the distances being AC=50 km and AD=90 km. Determine whether A and B are intervisible, and if necessary, find the minimum height of a scaffolding at B, assuming A as the ground station.*

Solution.

Let *acedb* be the visible horizon (level line) and a horizontal sight Ab_1 through *A* meet the horizon tangentially in *e*. *Ao*, *Co*, *Do* and *Bo* are the vertical lines through *A*, *C*, *D* and *B* respectively, *O* being the centre of the earth.

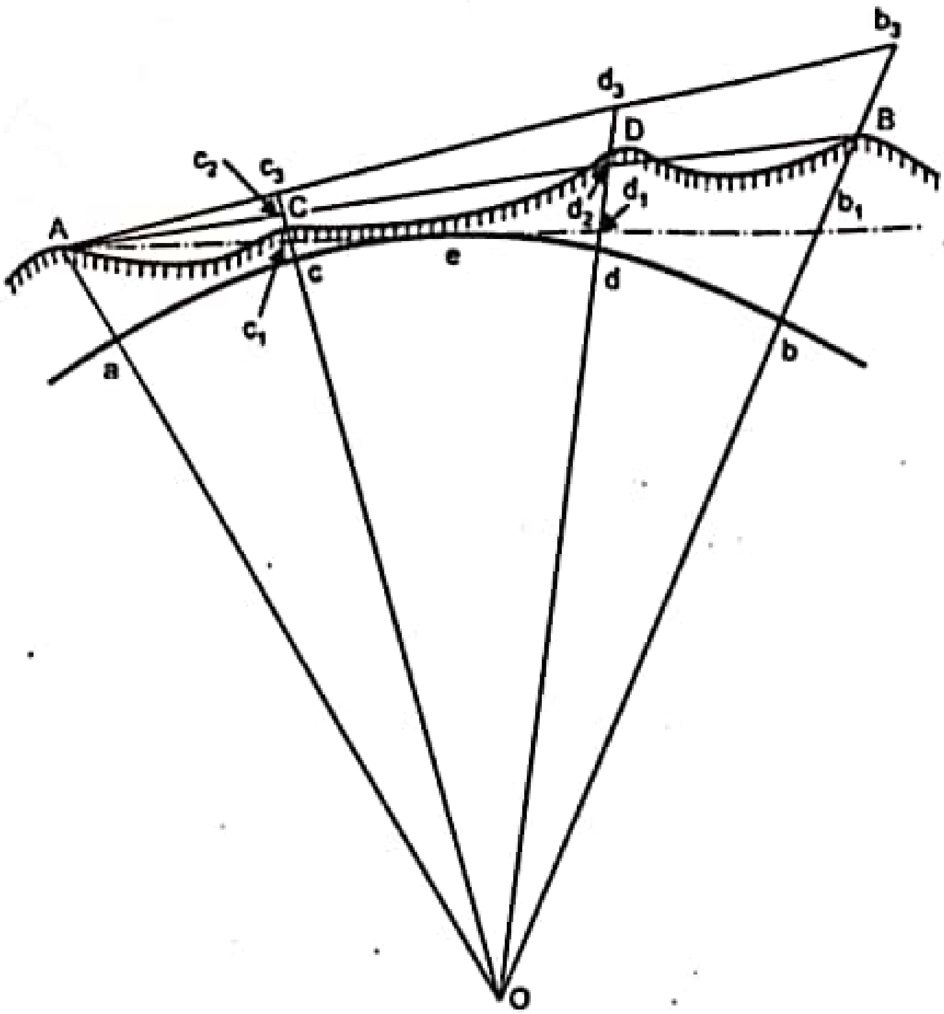

FIG. 1

The distance *Ae* to the visible horizon from station *A* of an altitude 220 metres is given by

$$D = Ae = 3.8553\sqrt{h} = 3.8553\sqrt{220} = 57.18 \text{ km.}$$

Let *a*, *c*, *d* and *b* be the points in which the vertical lines through *A*, *C*, *D* and *B* cuts the level line.

Now $AC = 50$ km ; $AD = 90$ km ; $AB = 130$ km

$\therefore$ $ce = Ae - AC = 57.18 - 50 = 7.18$ km

$ed = AD - Ae = 90 - 57.18 = 32.82$ km

$eb = AB - Ae = 130 - 57.18 = 72.82$ km.

Let c_1, d_1 and b_1 be the points in which a horizontal line through A cut the vertical lines through C, D and B respectively. The corresponding heights cc_1, dd_1 and bb_1 are given by

$$cc_1 = 0.06728\ (ce)^2 = 0.06728\ (7.18)^2 = 3.49 \text{ m}$$

$$dd_1 = 0.06728\ (ed)^2 = 0.06728\ (32.82)^2 = 72.47 \text{ m}$$

and
$$bb_1 = 0.06728\ (eb)^2 = 0.06728\ (72.82)^2 = 356.77 \text{ m}$$

Now
$$Bb = \text{Elev. of } B = 1160 \text{ m}$$

$\therefore$
$$Bb_1 = Bb - bb_1 = 1160 - 356.77 = 803.23 \text{ m}$$

Let AB be the line of sight.

Now from triangles Ac_1c_2, Ad_1d_2 and Ab_1B

$$c_1c_2 = Bb_1 \frac{Ac_1}{Ab_1} = 803.23 \times \frac{50}{130} = 308.93 \text{ m}$$

and
$$d_1d_2 = Bb_1 \frac{Ad_1}{Ab} = 803.23 \times \frac{90}{130} = 556.08 \text{ m}$$

Elevation of line of sight at C = elevation of $c_2 = cc_1 + c_1c_2 = 3.49 + 308.93 = 312.42$ m

Elevation of line of sight at D = elevation of $d_2 = dd_1 + d_1d_2 = 72.47 + 556.08 = 628.55$ m

Elevation of $C = 308$ m and that of $D = 632$ m

Thus, the line of sight clears the peak C, but fails to clear the peak D by $632 - 628.55 = 3.45$ m $= d_2D$.

Let Ad_3 be the new line of sight, such that

$$Dd_3 = 3 \text{ metres (minimum)}$$

Hence
$$d_2d_3 = d_3D + d_2D = 3 + 3.45 = 6.45 \text{ m}$$

Hence
$$Bb_3 = d_2d_3 \frac{AB}{Ad_2} = 6.45 \times \frac{130}{90} = 9.32 \text{ m} \approx 9.5 \text{ m (say)}.$$

Hence minimum height of scaffold at $B = 9.5$ m.