UNIT – IV

Force in Magnetic fields and Magnetic Potential

- Magnetic force Moving charges in a Magnetic field – Lorentz force equation
- Force on a current element in a magnetic field
- Force on a straight and a long current carrying conductor in a magnetic field
- Force between two straight long and parallel current carrying conductors
- Magnetic dipole and dipole moment
- a differential current loop as a magnetic dipole
- Torque on a current loop placed in a magnetic field.
- Scalar Magnetic potential and its limitations
- Vector magnetic potential and its properties
- Vector magnetic potential due to simple configurations
- Vector Poisson's equations.
- Self and Mutual inductance – Neumann's formulae
- Determination of self-inductance of a solenoid and toroid
- Mutual inductance between a straight long wire and a square loop wire in the same plane
- Energy stored and density in a magnetic field.
- Introduction to permanent magnets, their characteristics and applications.

Magnetic forces:

There are three ways in which the force due to magnetic fields can be experienced. The force can be

(a) Force on a charged particle:

We have $F_e = QE$

This shows that if Q is positive, F_e and E are in same direction. It is found that the magnetic force F_m experienced by a charge Q moving with a velocity u in magnetic field B is

$F_m = Qu \times B$

For a moving change Q in the presence of both electric and magnetic fields, the total force on the charge is given by

$F = F_e + F_m$

or

$F = Q(E + u \times B)$

This is known as Lorentz force equation.

(b) Force on a current element:

To determine the force on a current element Idl of a current carrying conductor due to the magnetic field B, we take the equation

$J = P_e\, u$

We have $Idl = \frac{dQ}{dt}.dl = dQ = \frac{dl}{dt} = dQ.u$

Hence

$Idl = dQ.u$

This shows that an elemental charge dQ moving with velocity u (thereby producing convection current element dQu) is equivalent to a conduction current element Idl. Thus the force on current element is give by

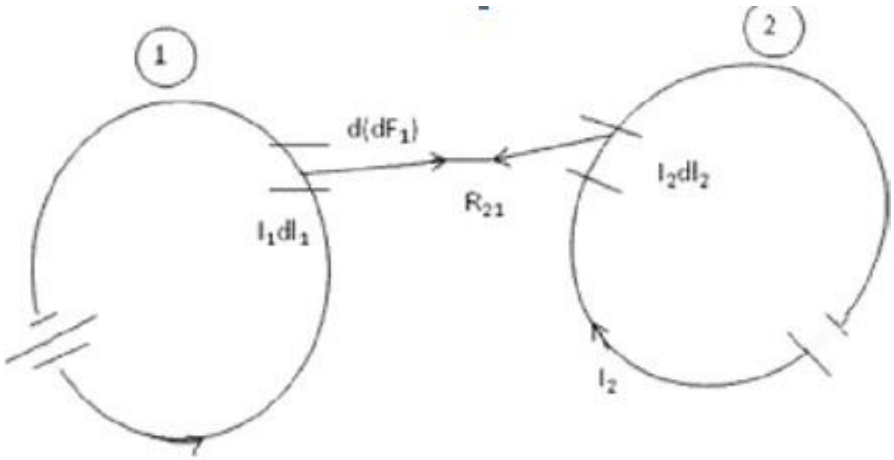
$dF = Idl \times B$

If the current I is through a closed path L or circuit, the force on the circuit is given by

$$F = \oint_L Idl \times B$$

(c) Force between two current elements:

Consider the force between two elements I_1dl_1 and I_2dl_2. According to biotsavarts law, both current elements produce magnetic fields. Force $d(dF_1)$ on element I_1dl_1 due to field dB_2 produced by element $I_2\, dl_2$ as shown in figure below:



$d(dF_1) = I_1 Dl_1 \times dB_2$

But from biot Savarts law

$$dB_2 = \frac{\mu_0 I_2 dl_2 \times a_{R21}}{4\pi R_{21}^2}$$

Hence

$$d(dF_1) = \frac{\mu_0 I_1 dl_1 \times (I_2 dl_2 \times a_{R21})}{4\pi R_{21}^2}$$

This equation is the law of force between two current elements.

We have $F1 = \frac{\mu_0 I_1 I_2 \times a_{R21}}{4\pi} \oint_{L_1}\oint_{L_2} \frac{dI_1 \times (dI_2 \times aR_{21})}{R_{21}^2}$

Scalar Magnetic Potential and its limitations:

In studying electric field problems, we introduced the concept of electric potential that simplified the computation of electric fields for certain types of problems. In the same manner let us relate the magnetic field intensity to a scalar magnetic potential and write:

$\vec{H} = -\nabla V_m$ ....(18)

From Ampere's law , we know that

$\nabla \times \vec{H} = \vec{J}$ ....(19)

Therefore, $\nabla \times (-\nabla V_m) = \vec{J}$ ....(20)

But using vector identity, $\nabla \times (\nabla V) = 0$ we find that $\vec{H} = -\nabla V_m$ is valid only where $\vec{J} = 0$. Thus the scalar magnetic potential is defined only in the region where $\vec{J} = 0$. Moreover, Vm in general is not a single valued function of position.

This point can be illustrated as follows. Let us consider the cross section of a coaxial line as shown in fig 7.

In the region $a<\rho<b$, $\vec{J}=0$ and $\vec{H}=\frac{I}{2\pi\rho}\hat{a}_\phi$

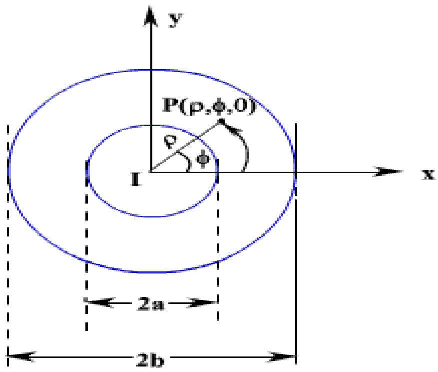


Fig. 7: Cross Section of a Coaxial Line

If Vm is the magnetic potential then,

$$-\nabla V_m = -\frac{1}{\rho}\frac{\partial V_m}{\partial \phi}$$
$$= \frac{I}{2\pi\rho}$$

If we set Vm = 0 at $\phi = 0$ then c=0 and $V_m = -\frac{I}{2\pi}\phi$

$$\therefore \text{At } \phi = \phi_0 \quad V_m = -\frac{I}{2\pi}\phi_0$$

We observe that as we make a complete lap around the current carrying conductor , we reach ϕ_0 again but Vm this time becomes

$$V_m = -\frac{I}{2\pi}(\phi_0 + 2\pi)$$

We observe that value of Vm keeps changing as we complete additional laps to pass through the same point. We introduced Vm analogous to electostatic potential V. But for static electric fields, $\nabla\times\vec{E}=0$ and $\oint\vec{E}\cdot d\vec{l}=0$ $\nabla\times\vec{H}=0$, whereas for steady magnetic field $\nabla\times\vec{H}=0$ wherever $\vec{J}=0$ but $\oint H\cdot dl = I$ even if $\vec{J}=0$ along the path of integration.

Vector magnetic potential due to simple configurations:

We now introduce the vector magnetic potential which can be used in regions where current density may be zero or nonzero and the same can be easily extended to time varying cases. The use of vector magnetic potential provides elegant ways of solving EM field problems.

Since $\nabla.\vec{B} = 0$ and we have the vector identity that for any vector $\vec{A}$, $\nabla.\left(\nabla\times\vec{A}\right) = 0$, we can write $\vec{B} = \nabla\times\vec{A}$.

Here, the vector field $\vec{A}$ is called the vector magnetic potential. Its SI unit is Wb/m. Thus if can find $\vec{A}$ of a given current distribution, $\vec{B}$ can be found from $\vec{A}$ through a curl operation. We have introduced the vector function $\vec{B}$ and $\vec{A}$ related its curl to $\vec{B}$. A vector function is defined fully in terms of its curl as well as divergence. The choice of $\nabla.\vec{A}$ is made as follows.

$$\nabla\times\nabla\times\vec{A} = \mu\nabla\times\vec{H} = \mu\vec{J} \quad ....(23)$$

By using vector identity, $\nabla\times\nabla\times\vec{A} = \nabla(\nabla.\vec{A}) - \nabla^2\vec{A}$ ....(24)

$$\nabla(\nabla.\vec{A}) - \nabla^2\vec{A} = \mu\vec{J} \quad ....(25)$$

Great deal of simplification can be achieved if we choose $\nabla.\vec{A} = 0$.

Putting $\nabla.\vec{A} = 0$, we get $\nabla^2\vec{A} = -\mu\vec{J}$ which is vector poisson equation.

In Cartesian coordinates, the above equation can be written in terms of the components as

$$\nabla^2 A_x = -\mu J_x \quad ....(26a)$$

$$\nabla^2 A_y = -\mu J_y \quad ....(26b)$$

$$\nabla^2 A_z = -\mu J_z \quad ....(26c)$$

The form of all the above equation is same as that of

$$\nabla^2 V = -\frac{\rho}{\varepsilon} \quad ....(27)$$

for which the solution is

$$V = \frac{1}{4\pi\varepsilon}\int_{V'}\frac{\rho}{R}dv', \qquad R = \left|\vec{r} - \vec{r}\,'\right| \quad ....(28)$$

In case of time varying fields we shall see that $\nabla.\vec{A} = \mu\varepsilon\frac{\partial V}{\partial t}$, which is known as Lorentz condition, V being the electric potential. Here we are dealing with static magnetic field, so $\nabla.\vec{A} = 0$.

By comparison, we can write the solution for Ax as

$$A_x = \frac{\mu}{4\pi}\int_{V'}\frac{J_x}{R}dv' \quad \text{..................................(30)}$$

Computing similar solutions for other two components of the vector potential, the vector potential can be written as

$$\vec{A} = \frac{\mu}{4\pi}\int_{V'}\frac{\vec{J}}{R}dv' \quad \text{......................................(31)}$$

This equation enables us to find the vector potential at a given point because of a volume current density $\vec{J}$. Similarly for line or surface current density we can write

$$\vec{A} = \frac{\mu}{4\pi}\int_{L'}\frac{I}{R}d\vec{l}' \quad \text{..(32)}$$

respectively. $$\vec{A} = \frac{\mu}{4\pi}\int_{S'}\frac{\vec{K}}{R}ds' \quad \text{..............................(33)}$$

The magnetic flux ψ through a given area S is given by

$$\psi = \int_S \vec{B}.d\vec{s} \quad \text{...(34)}$$

Substituting $\vec{B} = \nabla\times\vec{A}$

$$\psi = \int_S \nabla\times\vec{A}.d\vec{s} = \oint_C \vec{A}.d\vec{l} \quad \text{...(35)}$$

Vector potential thus have the physical significance that its integral around any closed path is equal to the magnetic flux passing through that path.

Self and Mutual inductance – Neumann's formulae:

Resistance, capacitance and inductance are the three familiar parameters from circuit theory. We have already discussed about the parameters resistance and capacitance in the earlier chapters. In this section, we discuss about the parameter inductance. Before we start our discussion, let us first introduce the concept of flux linkage. If in a coil with N closely wound turns around where a current I produces a flux ϕ and this flux links or encircles each of the N turns, the flux linkage is defined as Λ. In a linear medium $\Lambda = N\phi$, where the flux is proportional to the current, we define the self inductance L as the ratio of the total flux linkage to the current which they link.

i.e., $$L = \frac{\Lambda}{I} = \frac{N\phi}{I}$$ ....(36)

To further illustrate the concept of inductance, let us consider two closed loops C1 and C2 as shown in the figure 8, S1 and S2 are respectively the areas of C1 and C2 .

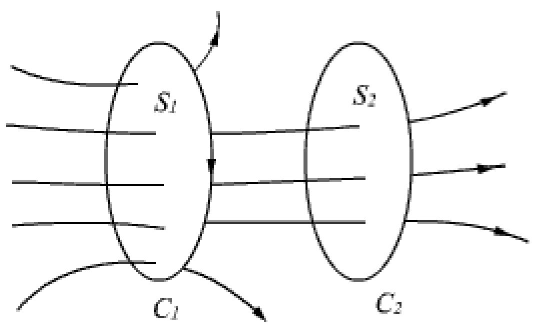


Fig:8

If a current I1 flows in C1 , the magnetic flux B1 will be created part of which will be linked to C2 as shown in Figure 8:

$$\phi_{12} = \int_{S_2} \vec{B}_1 . d\vec{S}_2$$ ....(37)

In a linear medium, ϕ_{12} is proportional to I 1. Therefore, we can write

$$\phi_{12} = L_{12} I_1$$ ....(38)

where L12 is the mutual inductance. For a more general case, if C2 has N2 turns then

$$\Lambda_{12} = N_2 \phi_{12}$$ ....(39)

and $\Lambda_{12} = L_{12} I_1$

or $L_{12} = \frac{\Lambda_{12}}{I_1}$ ....(40)

., the mutual inductance can be defined as the ratio of the total flux linkage of the second circuit to the current flowing in the first circuit.

As we have already stated, the magnetic flux produced in C1 gets linked to itself and if C1 has N1 turns then $\Lambda_{11} = N_1\phi_{11}$, where ϕ_{11} is the flux linkage per turn.

Therefore, self inductance

$$L_{11}\left(\text{or } L \text{ as defined earlier}\right) = \frac{\Lambda_{11}}{I_1} \qquad ....(41)$$

As some of the flux produced by I1 links only to C1 & not C2.

$$\Lambda_{11} = N_1\phi_{11} > N_2\phi_{12} = \Lambda_{12} \qquad ....(42)$$

Further in general, in a linear medium, $L_{12} = \frac{d\Lambda_{12}}{dI_1}$ and $L_{11} = \frac{d\Lambda_{11}}{dI_1}$

Inductance:

Inductance is the ability of the material to hold energy in form of magnetic field.

L, I are inductance of material and current flowing in the material.

$$E = \frac{1}{2}LI^2$$

$$\text{Inductance, L} = \frac{\text{Total flux linking current I}}{\text{current (I)}}$$

'B' is induced by I

$$\therefore \phi = \int_s \mathbf{B}.d\bar{s}$$

Total Flux depends on no of turns

Flux linking for n turns is 'Nϕ'.

$$\therefore L = \frac{\lambda}{I}$$

λ=Nϕ(depending on condition i.e total Flux linking the current)

Inductance of a solenoid:

In the application of ampere's law to solenoid we found that

$$B = \frac{\mu NI}{l} Tesla$$

$$\therefore \phi = B.A = \frac{\mu NIA}{l}$$

With in a loop of N turns, the flux is linking the current N times.

$\therefore$ Total flux linking I = Nϕ

$$= \frac{\mu N^2 IA}{l}$$

$$L = \frac{\lambda}{I} = \frac{\mu N^2 A}{l}$$

Some times inductors are given for unit length as well

$$\therefore \frac{l}{l} = \mu\left(\frac{N}{l}\right)^2 .A$$

Energy stored and density in a magnetic field.

Energy stored in Magnetic Field:

So far we have discussed the inductance in static forms. In earlier chapter we discussed the fact that work is required to be expended to assemble a group of charges and this work is stated as electric energy. In the same manner energy needs to be expended in sending currents through coils and it is stored as magnetic energy. Let us consider a scenario where we consider a coil in which the current is increased from 0 to a value I. As mentioned earlier, the self inductance of a coil in general can be written as

$$L = \frac{d\Lambda}{di} = N\frac{d\phi}{di} \text{ ....(43a)}$$

or $Ldi = Nd\phi$ ....(43b)

If we consider a time varying scenario,

$$L\frac{di}{dt} = N\frac{d\phi}{dt} \text{ ....(44)}$$

We will later see that $N\frac{d\phi}{dt}$ is an induced voltage.

$\therefore v = L\frac{di}{dt}$ is the voltage drop that appears across the coil and thus voltage opposes the change of current.

Therefore in order to maintain the increase of current, the electric source must do an work against this induced voltage.

$$dW = vi\,dt$$

$$= Li\,di \quad ....(45)$$

$$W = \int_0^I Li\,di = \frac{1}{2}LI^2 \quad \text{(Joule)}..................................(46)$$

which is the energy stored in the magnetic circuit.

We can also express the energy stored in the coil in term of field quantities.

For linear magnetic circuit

$$W = \frac{1}{2}\frac{N\phi}{I}I^2 = \frac{1}{2}N\phi I \quad ....(47)$$

Now,

$$\phi = \int_s \vec{B}.d\vec{S} = BA \quad ....(48)$$

where A is the area of cross section of the coil. If l is the length of the coil

$$NI = Hl$$

$$\therefore W = \frac{1}{2}HBAl \quad ....(49)$$

Al is the volume of the coil. Therefore the magnetic energy density i.e., magnetic energy/unit volume is given by

$$W_m = \frac{W}{Al} = \frac{1}{2}BH \quad ....(50)$$

In vector form

$$W_m = \frac{1}{2}\vec{B}.\vec{H} \quad \text{J/mt3} \ ....(51)$$

is the energy density in the magnetic field.