3.5. CONSTANT PRESSURE OR DIESEL CYCLE

This cycle was introduced by Dr. R. Diesel in 1897. It differs from Otto cycle, in, that heat is supplied at constant pressure instead of at constant volume. Fig. 3.15 (*a* and *b*) shows the *p-V* and *T-s* diagrams of this cycle respectively.

This cycle comprises of the following ***operations*** :

(*i*) 1-2......*Adiabatic compression.*

(*ii*) 2-3......*Addition of heat at constant pressure.*

(*iii*) 3-4......*Adiabatic expansion.*

(*iv*) 4-1......*Rejection of heat at constant volume.*

Point 1 represents that the cylinder is full of air. Let p_1, V_1 and T_1 be the corresponding pressure, volume and absolute temperature. The piston then compresses the air adiabatically (*i.e.* pV^γ = constant) till the values become p_2, V_2 and T_2 respectively (at the end of the stroke) at point 2. Heat is then added from a hot body at a constant pressure. During this addition of heat let

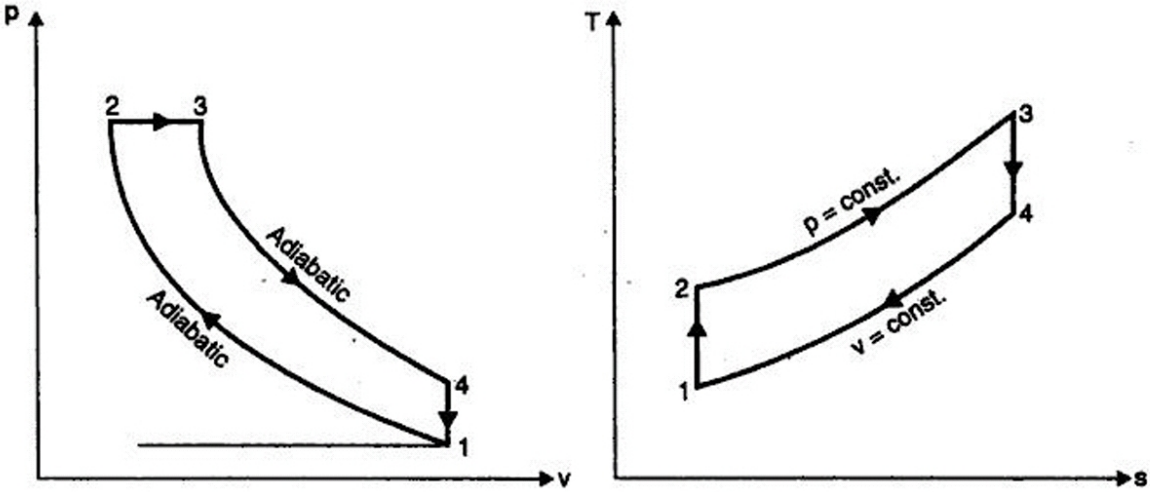

Fig. 3.15

volume increases from V_2 to V_3 and temperature T_2 to T_3, corresponding to point 3. This point (3) is called the *point of cut off*. The air then expands adiabatically to the conditions p_4, V_4 and T_4 respectively corresponding to point 4. Finally, the air rejects the heat to the cold body at constant volume till the point 1 where it returns to its original state.

Consider 1 kg of air.

Heat supplied at constant pressure $= c_p(T_3 - T_2)$

Heat rejected at constant volume $= c_v(T_4 - T_1)$

Work done $=$ Heat supplied – Heat rejected

$$= c_p(T_3 - T_2) - c_v(T_4 - T_1)$$

$$\therefore \quad \eta_{diesel} = \frac{\text{Work done}}{\text{Heat supplied}}$$

$$= \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)}$$

$$= 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \quad \ldots(i) \quad \left[\because \frac{c_p}{c_v} = \gamma\right]$$

Let compression ratio, $r = \dfrac{v_1}{v_2}$ and cut off ratio, $\rho = \dfrac{v_3}{v_2}$ *i.e.* $\dfrac{\text{Volume at cut-off}}{\text{Clearance volume}}$

Now, during *adiabatic compression 1-2,*

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} \quad \text{or} \quad T_2 = T_1 \,.\, (r)^{\gamma - 1}$$

During *constant pressure process 2-3,*

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} = \rho \quad \text{or} \quad T_3 = \rho \,.\, T_2 = \rho \,.\, T_1 \,.\, (r)^{\gamma - 1}$$

During *adiabatic expansion 3-4,*

$$\frac{T_3}{T_4} = \left(\frac{v_4}{v_3}\right)^{\gamma - 1}$$

$$= \left(\frac{r}{\rho}\right)^{\gamma-1} \qquad \left(\because \frac{v_4}{v_3} = \frac{v_1}{v_3} = \frac{v_1}{v_2} \times \frac{v_2}{v_3} = \frac{r}{\rho}\right)$$

$$\therefore \qquad T_4 = \frac{T_3}{\left(\frac{r}{\rho}\right)^{\gamma-1}} = \frac{\rho \,.\, T_1 (r)^{\gamma-1}}{\left(\frac{r}{\rho}\right)^{\gamma-1}} = T_1 \,.\, \rho^{\gamma}$$

By inserting values of T_2, T_3 and T_4 in equation (*i*), we get

$$\eta_{\text{diesel}} = 1 - \frac{(T_1 \,.\, \rho^{\gamma} - T_1)}{\gamma(\rho \,.\, T_1 \,.\, (r)^{\gamma-1} - T_1 \,.\, (r)^{\gamma-1})} = 1 - \frac{(\rho^{\gamma} - 1)}{\gamma (r)^{\gamma-1} (\rho - 1)}$$

or

$$\eta_{\text{diesel}} = 1 - \frac{1}{\gamma (r)^{\gamma-1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1}\right] \qquad ...(3.7)$$

It may be observed that equation (3.7) for efficiency of diesel cycle is different from that of the Otto cycle only in bracketed factor. This factor is always greater than unity, because $\rho > 1$. Hence *for a given compression ratio, the Otto cycle is more efficient.*

The *net work* for diesel cycle can be expressed in terms of pv as follows :

$$W = p_2(v_3 - v_2) + \frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}$$

$$= p_2 (\rho v_2 - v_2) + \frac{p_3 \rho v_2 - p_4 r v_2}{\gamma - 1} - \frac{p_2 v_2 - p_1 r v_1}{\gamma - 1}$$

$$\left[\begin{array}{l} \because \frac{v_3}{v_2} = \rho \quad \therefore \ v_3 = \rho v_2 \ \text{and} \ \frac{v_1}{v_2} = r \quad \therefore \ v_1 = r v_2 \\ \text{But } v_4 = v_1 \quad \therefore \quad v_4 = r v_2 \end{array}\right]$$

$$= p_2 v_2 (\rho - 1) + \frac{p_3 \rho v_2 - p_4 r v_2}{\gamma - 1} - \frac{p_2 v_2 - p_1 r v_2}{\gamma - 1}$$

$$= \frac{v_2 [p_2 (\rho - 1)(\gamma - 1) + p_3 \rho - p_4 r - (p_2 - p_1 r)]}{\gamma - 1}$$

$$= \frac{v_2 \left[p_2 (\rho - 1)(\gamma - 1) + p_3 \left(\rho - \frac{p_4 r}{p_3}\right) - p_2 \left(1 - \frac{p_1 r}{p_2}\right)\right]}{\gamma - 1}$$

$$= \frac{p_2 v_2 [(\rho - 1)(\gamma - 1) + \rho - \rho^{\gamma} \,.\, r^{1-\gamma} - (1 - r^{1-\gamma})}{\gamma - 1}$$

$$\left[\because \frac{p_4}{p_3} = \left(\frac{v_3}{v_4}\right)^{\gamma} = \left(\frac{\rho}{r}\right)^{\gamma} = \rho^{\gamma} r^{-\gamma}\right]$$

$$= \frac{p_1 v_1 r^{\gamma-1} [(\rho - 1)(\gamma - 1) + \rho - \rho^{\gamma} r^{1-\gamma} - (1 - r^{1-\gamma})]}{\gamma - 1}$$

$$\left[\because \frac{p_2}{p_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} \ \text{or} \ p_2 = p_1 \,.\, r^{\gamma} \ \text{and} \ \frac{v_1}{v_2} = r \ \text{or} \ v_2 = v_1 r^{-1}\right]$$

$$= \frac{p_1 v_1 r^{\gamma-1} [\gamma(\rho - 1) - r^{1-\gamma} (\rho^{\gamma} - 1)]}{(\gamma - 1)} \qquad ...(3.8)$$

Mean effective pressure p_m is given by :

$$p_m = \frac{p_1 v_1 r^{\gamma-1}[\gamma(\rho-1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma-1)v_1\left(\frac{r-1}{r}\right)}$$

or

$$p_m = \frac{p_1 r^\gamma [\gamma(\rho-1) - r^{1-\gamma}(\rho^\gamma - 1)]}{(\gamma-1)(r-1)} \quad ...(3.9)$$

Example 3.17. *A diesel engine has a compression ratio of 15 and heat addition at constant pressure takes place at 6% of stroke. Find the air standard efficiency of the engine.*

Take γ for air as 1.4.

Solution. Refer Fig. 3.16.

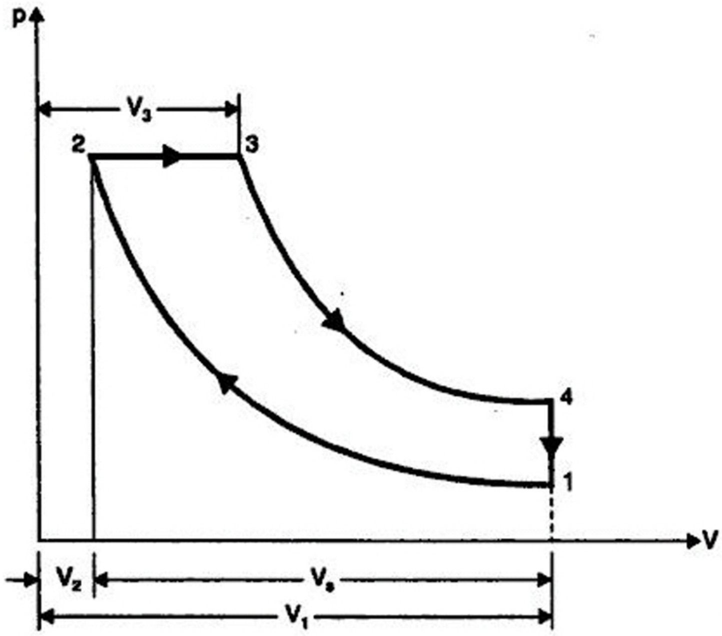

Fig. 3.16

Compression ratio, $r\left(=\frac{V_1}{V_2}\right) = 15$

γ for air = 1.4

Air standard efficiency of diesel cycle is given by

$$\eta_{diesel} = 1 - \frac{1}{\gamma(r)^{\gamma-1}}\left[\frac{\rho^\gamma - 1}{\rho - 1}\right] \quad ...(i)$$

where ρ = cut-off ratio = $\frac{V_3}{V_2}$

But $\quad V_3 - V_2 = \frac{6}{100} V_s \quad (V_s = \text{stroke volume})$

$$= 0.06\,(V_1 - V_2) = 0.06\,(15\,V_2 - V_2)$$

$$= 0.84\,V_2 \text{ or } V_3 = 1.84\,V_2$$

$$\therefore \qquad \rho = \frac{V_3}{V_2} = \frac{1.84\,V_2}{V_2} = 1.84$$

Putting the value in eqn. (*i*), we get

$$\eta_{diesel} = 1 - \frac{1}{1.4\,(15)^{1.4-1}}\left[\frac{(1.84)^{1.4}-1}{1.84-1}\right]$$

$$= 1 - 0.2417 \times 1.605 = \mathbf{0.612 \text{ or } 61.2\%.} \quad \textbf{(Ans.)}$$

Example 3.18. *The stroke and cylinder diameter of a compression ignition engine are 250 mm and 150 mm respectively. If the clearance volume is 0.0004 m³ and fuel injection takes place at constant pressure for 5 per cent of the stroke determine the efficiency of the engine. Assume the engine working on the diesel cycle.*

Solution. Refer Fig. 3.16.

Length of stroke, $L = 250 \text{ mm} = 0.25 \text{ m}$

Diameter of cylinder, $D = 150 \text{ mm} = 0.15 \text{ m}$

Clearance volume, $V_2 = 0.0004 \text{ m}^3$

Swept volume, $V_s = \pi/4\, D^2 L = \pi/4 \times 0.15^2 \times 0.25 = 0.004418 \text{ m}^3$

Total cylinder volume = Swept volume + Clearance volume

$$= 0.004418 + 0.0004 = 0.004818 \text{ m}^3$$

Volume at point of cut-off, $V_3 = V_2 + \frac{5}{100} V_s$

$$= 0.0004 + \frac{5}{100} \times 0.004418 = 0.000621 \text{ m}^3$$

$\therefore$ Cut-off ratio, $\rho = \frac{V_3}{V_2} = \frac{0.000621}{0.0004} = 1.55$

Compression ratio, $r = \frac{V_1}{V_2} = \frac{V_s + V_2}{V_2} = \frac{0.004418 + 0.0004}{0.0004} = 12.04$

Hence, $\eta_{diesel} = 1 - \frac{1}{\gamma (r)^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\rho-1}\right] = 1 - \frac{1}{1.4 \times (12.04)^{1.4-1}}\left[\frac{(1.55)^{1.4}-1}{1.55-1}\right]$

$$= 1 - 0.264 \times 1.54 = \mathbf{0.593 \text{ or } 59.3\%.} \quad \textbf{(Ans.)}$$