1.5 COMPRESSIBILITY AND BULK MODULUS

Compressibility is the reciprocal of the bulk modulus of elasticity, K which is defined as the ratio of compressive stress to volumetric strain.

Consider a cylinder fitted with a piston as shown in Fig. 1.9.

Let $\forall$ = Volume of a gas enclosed in the cylinder

p = Pressure of gas when volume is $\forall$

Let the pressure is increased to $p + dp$, the volume of gas decreases from $\forall$ to $\forall - d\forall$.

CYLINDER

Fig. 1.9

Then increase in pressure $= dp \ \text{kgf/m}^2$

Decrease in volume $= d\forall$

$\therefore$ Volumetric strain $= -\dfrac{d\forall}{\forall}$

– ve sign means the volume decreases with increase of pressure.

$\therefore$ Bulk modulus $K = \dfrac{\text{Increase of pressure}}{\text{Volumetric strain}}$

$$= \dfrac{dp}{\dfrac{-d\forall}{\forall}} = \dfrac{-dp}{d\forall} \forall \qquad \qquad ...(1.10)$$

Compressibility is given by $= \dfrac{1}{K}$ $\qquad\qquad ...(1.11)$

Relationship between Bulk Modulus (K) and Pressure (p) for a Gas

The relationship between bulk modulus of elasticity (K) and pressure for a gas for two different processes of compression are as :

(*i*) **For Isothermal Process.** Equation (1.6) gives the relationship between pressure (p) and density (ρ) of a gas as
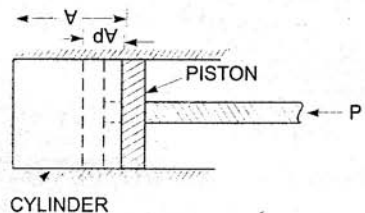
$$\dfrac{p}{\rho} = \text{Constant}$$

or $$p\forall = \text{Constant} \qquad \left\{ \because \forall = \frac{1}{\rho} \right\}$$

Differentiating this equation, we get (p and $\forall$ both are variables)

$$pd\forall + \forall dp = 0 \quad \text{or} \quad pd\forall = -\forall dp \quad \text{or} \quad p = \frac{-\forall dp}{d\forall}$$

Substituting this value in equation (1.10), we get

$$K = p \qquad\qquad\qquad ...(1.12)$$

(*ii*) **For Adiabatic Process.** Using equation (1.7) for adiabatic process

$$\frac{p}{\rho^k} = \text{Constant or } p\,\forall^k = \text{Constant}$$

Differentiating, we get $pd(\forall^k) + \forall^k(dp) = 0$

or $\qquad p \times k \times \forall^{k-1}\, d\forall + \forall^k\, dp = 0$

or $\qquad pkd\forall + \forall dp = 0 \qquad\qquad$ [Cancelling $\forall^{k-1}$ to both sides]

or $$pkd\forall = -\forall dp \quad \text{or} \quad pk = -\frac{\forall dp}{d\forall}$$

Hence from equation (1.10), we have

$$K = pk \qquad\qquad\qquad ...(1.13)$$

where K = Bulk modulus and k = Ratio of specific heats.

Problem 1.23 *Determine the bulk modulus of elasticity of a liquid, if the pressure of the liquid is increased from 70 N/cm² to 130 N/cm². The volume of the liquid decreases by 0.15 per cent.*

Solution. Given :

Initial pressure $\qquad\qquad = 70$ N/cm²

Final pressure $\qquad\qquad = 130$ N/cm²

$\therefore \quad dp$ = Increase in pressure $= 130 - 70 = 60$ N/cm²

Decrease in volume $\qquad = 0.15\%$

$\therefore \qquad\qquad\qquad -\dfrac{d\forall}{\forall} = +\dfrac{0.15}{100}$

Bulk modulus, K is given by equation (1.10) as

$$K = \frac{dp}{-\dfrac{d\forall}{\forall}} = \frac{60 \text{ N/cm}^2}{\dfrac{.15}{100}} = \frac{60 \times 100}{.15} = 4 \times 10^4 \text{ N/cm}^2. \text{ Ans.}$$

Problem 1.24 *What is the bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of 0.0125 m³ at 80 N/cm² pressure to a volume of 0.0124 m³ at 150 N/cm² pressure ?*

Solution. Given :

Initial volume, $\qquad\qquad \forall = 0.0125$ m³

Final volume $\qquad\qquad = 0.0124$ m³

$\therefore \quad$ Decrease in volume, $\quad d\forall = .0125 - .0124 = .0001$ m³

$$\therefore \qquad -\frac{d\forall}{\forall} = \frac{.0001}{.0125}$$

Initial pressure $= 80 \text{ N/cm}^2$

Final pressure $= 150 \text{ N/cm}^2$

$\therefore$ Increase in pressure, $dp = (150 - 80) = 70 \text{ N/cm}^2$

Bulk modulus is given by equation (1.10) as

$$K = \frac{dp}{-\dfrac{d\forall}{\forall}} = \frac{70}{\dfrac{.0001}{.0125}} = 70 \times 125 \text{ N/cm}^2$$

$$= 8.75 \times 10^3 \text{ N/cm}^2. \text{ Ans.}$$

immiscible

1.6 SURFACE TENSION AND CAPILLARITY

Surface tension is defined as the tensile force acting on the surface of a liquid in contact with a gas or on the surface between two immiscible liquids such that the contact surface behaves like a membrane under tension. The magnitude of this force per unit length of the free surface will have the same value as the surface energy per unit area. It is denoted by Greek letter σ (called sigma). In MKS units, it is expressed as kgf/m while in SI units as N/m.

The phenomenon of surface tension is explained by Fig. 1.10. Consider three molecules A, B, C of a liquid in a mass of liquid. The molecule A is attracted in all directions equally by the surrounding molecules of the liquid. Thus the resultant force acting on the molecule A is zero. But the molecule B, which is situated near the free surface, is acted upon by upward and downward forces which are unbalanced. Thus a net resultant force on molecule B is acting in the downward direction. The molecule C, situated on the free surface of liquid, does experience a resultant downward force. All the molecules on the free surface experience a downward force. Thus the free surface of the liquid acts like a very thin film under tension of the surface of the liquid act as though it is an elastic membrane under tension.
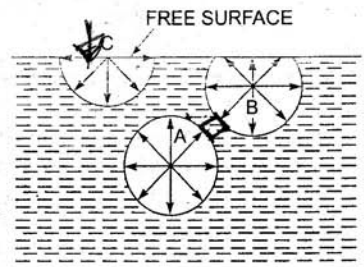

FREE SURFACE

Fig. 1.10 *Surface tension.*

1.6.1 Surface Tension on Liquid Droplet. Consider a small spherical droplet of a liquid of radius 'r'. On the entire surface of the droplet, the tensile force due to surface tension will be acting.

Let $\sigma =$ Surface tension of the liquid

$p =$ Pressure intensity inside the droplet (in excess of the outside pressure intensity)

$d =$ Dia. of droplet.

Let the droplet is cut into two halves. The forces acting on one half (say left half) will be

(i) tensile force due to surface tension acting around the circumference of the cut portion as shown in Fig. 1.11 (b) and this is equal to
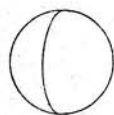
$$= \sigma \times \text{Circumference}$$

$$= \sigma \times \pi d$$

1

(ii) pressure force on the area $\frac{\pi}{4} d^2$ and $= p \times \frac{\pi}{4} d^2$ as shown in Fig. 1.11 (c). These two forces will be equal and opposite under equilibrium conditions, i.e.,
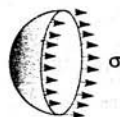
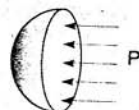$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d$$

or

$$\boxed{p = \frac{\sigma \times \pi d}{\frac{\pi}{4} \times d^2} = \frac{4\sigma}{d}} \quad ...(1.14)$$

Equation (1.14) shows that with the decrease of diameter of the droplet, pressure intensity inside the droplet increases.

(a) DROPLET   (b) SURFACE TENSION

(c) PRESSURE FORCES

Fig. 1.11 Forces on droplet.

1.6.2 Surface Tension on a Hollow Bubble.

A hollow bubble like a soap bubble in air has two surfaces in contact with air, one inside and other outside. Thus two surfaces are subjected to surface tension. In such case, we have

$$p \times \frac{\pi}{4} d^2 = 2 \times (\sigma \times \pi d)$$

∴

$$p = \frac{2\sigma \pi d}{\frac{\pi}{4} d^2} = \frac{8\sigma}{d} \quad ...(1.15)$$

1.6.3 Surface Tension on a Liquid Jet.

Consider a liquid jet of diameter 'd' and length 'L' as shown in Fig. 1.12.

Let p = Pressure intensity inside the liquid jet above the outside pressure
  σ = Surface tension of the liquid.

Consider the equilibrium of the semi jet, we have

Force due to pressure   = p × area of semi jet
                             = p × L × d
Force due to surface tension = σ × 2L.

Equating the forces, we have

$$p \times L \times d = \sigma \times 2L$$

∴

$$p = \frac{\sigma \times 2L}{L \times d} \quad ...(1.16)$$
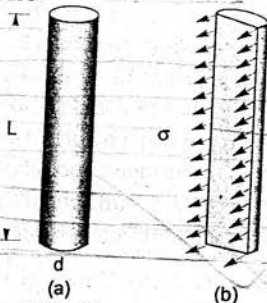

(a)   (b)

Fig. 1.12 Forces on liquid jet.

Problem 1.25 *The surface tension of water in contact with air at 20°C is 0.0725 N/m. The pressure inside a droplet of water is to be 0.02 N/cm² greater than the outside pressure. Calculate the diameter of the droplet of water.*

Solution. Given :

Surface tension,   σ = 0.0725 N/m

Pressure intensity, p in excess of outside pressure is

$$p = 0.02 \text{ N/cm}^2 = 0.02 \times 10^4 \ \frac{\text{N}}{\text{m}^2}$$

Let   d = dia. of the droplet

Using equation (1.14), we get $p = \dfrac{4\sigma}{d}$ or $0.02 \times 10^4 = \dfrac{4 \times 0.0725}{d}$

$\therefore$ $d = \dfrac{4 \times 0.0725}{0.02 \times (10)^4} = .00145 \text{ m} = .00145 \times 1000 =$ **1.45 mm. Ans.**

Problem 1.26 *Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/m² above atmospheric pressure.*

Solution. Given :

Dia. of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$

Pressure in excess of outside, $p = 2.5 \text{ N/m}^2$

For a soap bubble, using equation (1.15), we get

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m} = \textbf{0.0125 N/m. Ans.}$$

Problem 1.27 *The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.*

Solution. Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3} \text{ m}$

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

The pressure inside the droplet, in excess of outside pressure is given by equation (1.14)

or $p = \dfrac{4\sigma}{d} = \dfrac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \dfrac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$

$\therefore$ Pressure inside the droplet $= p +$ Pressure outside the droplet

$= 0.725 + 10.32 = $ **11.045 N/cm². Ans.**

1.6.4 Capillarity. [Capillarity is defined as a phenomenon of rise or fall of a liquid surface in a small tube relative to the adjacent general level of liquid when the tube is held vertically in the liquid. The rise of liquid surface is known as capillary rise while the fall of the liquid surface is known as capillary depression. It is expressed in terms of cm or mm of liquid. Its value depends upon the specific weight of the liquid, diameter of the tube and surface tension of the liquid.]

Expression for Capillary Rise. Consider a glass tube of small diameter '*d*' opened at both ends and is inserted in a liquid, say water. The liquid will rise in the tube above the level of the liquid.

Let *h* = height of the liquid in the tube. Under a state of equilibrium, the weight of liquid of height *h* is balanced by the force at the surface of the liquid in the tube. But the force at the surface of the liquid in the tube is due to surface tension.

Fig. 1.13 *Capillary rise.*

Let σ = Surface tension of liquid

θ = Angle of contact between liquid and glass tube.

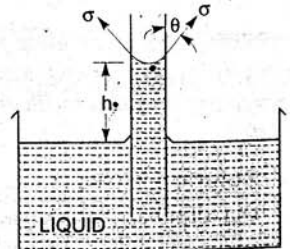The weight of liquid of height *h* in the tube = (Area of tube $\times h$) $\times \rho \times $

$$= \frac{\pi}{4} d^2 \times h \times \rho \times g \qquad \qquad ...(1.17)$$

where ρ = Density of liquid

Vertical component of the surface tensile force

$$= (\sigma \times \text{Circumference}) \times \cos \theta$$
$$= \sigma \times \pi d \times \cos \theta \qquad \qquad ...(1.18)$$

For equlibrium, equating (1.17) and (1.18), we get

$$\frac{\pi}{4} d^2 \times h \times \rho \times g = \sigma \times \pi d \times \cos \theta$$

or

$$h = \frac{\sigma \times \pi d \times \cos \theta}{\frac{\pi}{4} d^2 \times \rho \times g} = \frac{4 \sigma \cos \theta}{\rho \times g \times d} \qquad \qquad ...(1.19)$$

The value of θ between water and clean glass tube is approximately equal to zero and hence $\cos \theta$ is equal to unity. Then rise of water is given by

$$h = \frac{4\sigma}{\rho \times g \times d} \qquad \qquad ... (1.20)$$

Expression for Capillary Fall. If the glass tube is dipped in mercury, the level of mercury in the tube will be lower than the general level of the outside liquid as shown in Fig. 1.14.

Let h = Height of depression in tube.

Then in equilibrium, two forces are acting on the mercury inside the tube. First one is due to surface tension acting in the downward direction and is equal to $\sigma \times \pi d \times \cos \theta$.

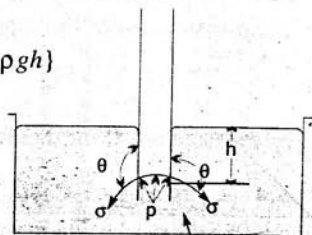Second force is due to hydrostatic force acting upward and is equal to intensity of pressure at a depth 'h' × Area

$$= p \times \frac{\pi}{4} d^2 = \rho g \times h \times \frac{\pi}{4} d^2 \; \{\because p = \rho g h\}$$

Equating the two, we get

$$\sigma \times \pi d \times \cos \theta = \rho g h \times \frac{\pi}{4} d^2$$

$$\therefore \qquad h = \frac{4 \sigma \cos \theta}{\rho g d} \qquad \qquad ...(1.21)$$

Value of θ for mercury and glass tube is 128°.

MERCURY
Fig. 1.14.

Problem 1.28 *Calculate the capillary rise in a glass tube of 2.5 mm diameter when immersed vertically in (a) water and (b) mercury. Take surface tensions $\sigma = 0.0725$ N/m for water and $\sigma = 0.52$ N/m for mercury in contact with air. The specific gravity for mercury is given as 13.6 and angle of contact = 130°.*

Solution. Given :

Dia. of tube, $\qquad d = 2.5$ mm $= 2.5 \times 10^{-3}$ m

Surface tenstion, σ for water $= 0.0725$ N/m

σ for mercury $\qquad = 0.52$ N/m

Sp. gr. of mercury $\qquad = 13.6$