STEADY STATE ERROR
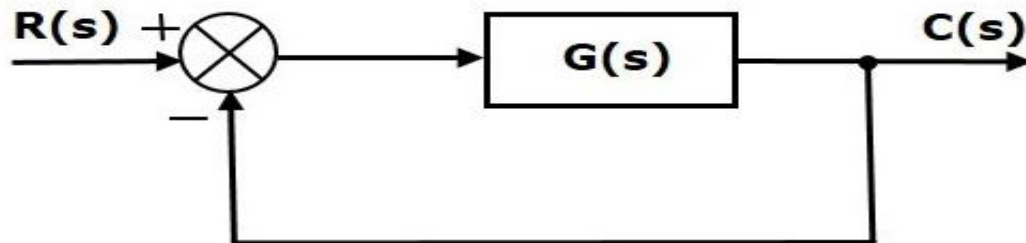
Consider the following block diagram of closed loop control system, which is having unity negative feedback.

Where,

- R(s) is the Laplace transform of the reference Input signal $r(t)$
- C(s) is the Laplace transform of the output signal $c(t)$

We know the transfer function of the unity negative feedback closed loop control system as

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)}$$

$$\Rightarrow C(s) = \frac{R(s)G(s)}{1 + G(s)}$$

The output of the summing point is -

$$E(s) = R(s) - C(s)$$

Substitute $C(s)$ value in the above equation.

$$E(s) = R(s) - \frac{R(s)G(s)}{1 + G(s)}$$

$$\Rightarrow E(s) = \frac{R(s) + R(s)G(s) - R(s)G(s)}{1 + G(s)}$$

$$\Rightarrow E(s) = \frac{R(s)}{1 + G(s)}$$

Substitute $E(s)$ value in the steady state error formula

$$e_{ss} = \lim_{s \to 0} \frac{sR(s)}{1 + G(s)}$$

The following table shows the steady state errors and the error constants for standard input signals like unit step, unit ramp & unit parabolic signals.

Input signal	Steady state error e_{ss}	Error constant
unit step signal	$\frac{1}{1+k_p}$	$K_p = \lim_{s \to 0} G(s)$
unit ramp signal	$\frac{1}{K_v}$	$K_v = \lim_{s \to 0} sG(s)$
unit parabolic signal	$\frac{1}{K_a}$	$K_a = \lim_{s \to 0} s^2 G(s)$

Where, Kp, Kv and Ka are position error constant, velocity error constant and acceleration error constant respectively.

Note − If any of the above input signals has the amplitude other than unity, then multiply corresponding steady state error with that amplitude.

Note − We can't define the steady state error for the unit impulse signal because, it exists only at origin. So, we can't compare the impulse response with the unit impulse input as **t** denotes infinity

Example

Let us find the steady state error for an input signal $r(t) = \left(5 + 2t + \frac{t^2}{2}\right)u(t)$ of unity negative feedback control system with $G(s) = \frac{5(s+4)}{s^2(s+1)(s+20)}$

The given input signal is a combination of three signals step, ramp and parabolic. The following table shows the error constants and steady state error values for these three signals.

Input signal	Error constant	Steady state error
$r_1(t) = 5u(t)$	$K_p = \lim_{s \to 0} G(s) = \infty$	$e_{ss1} = \frac{5}{1+k_p} = 0$
$r_2(t) = 2tu(t)$	$K_v = \lim_{s \to 0} sG(s)$ $= \infty$	$e_{ss2} = \frac{2}{K_v} = 0$
$r_3(t) = \frac{t^2}{2}u(t)$	$K_a = \lim_{s \to 0} s^2 G(s)$ $= 1$	$e_{ss3} = \frac{1}{k_a} = 1$

We will get the overall steady state error, by adding the above three steady state errors.

$$e_{ss} = e_{ss1} + e_{ss2} + e_{ss3}$$
$$\Rightarrow e_{ss} = 0+0+1=1 \Rightarrow e_{ss} = 0+0+1=1$$

Therefore, we got the steady state error e_{ss} as **1** for this example.