

UNIT-I

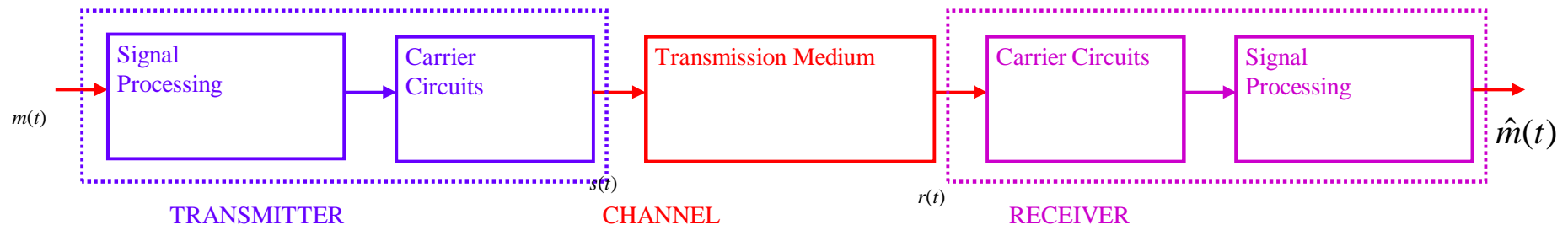
Amplitude Modulation System

Introduction to communications

• Elements of a communication system (cont)

➤ Basic components

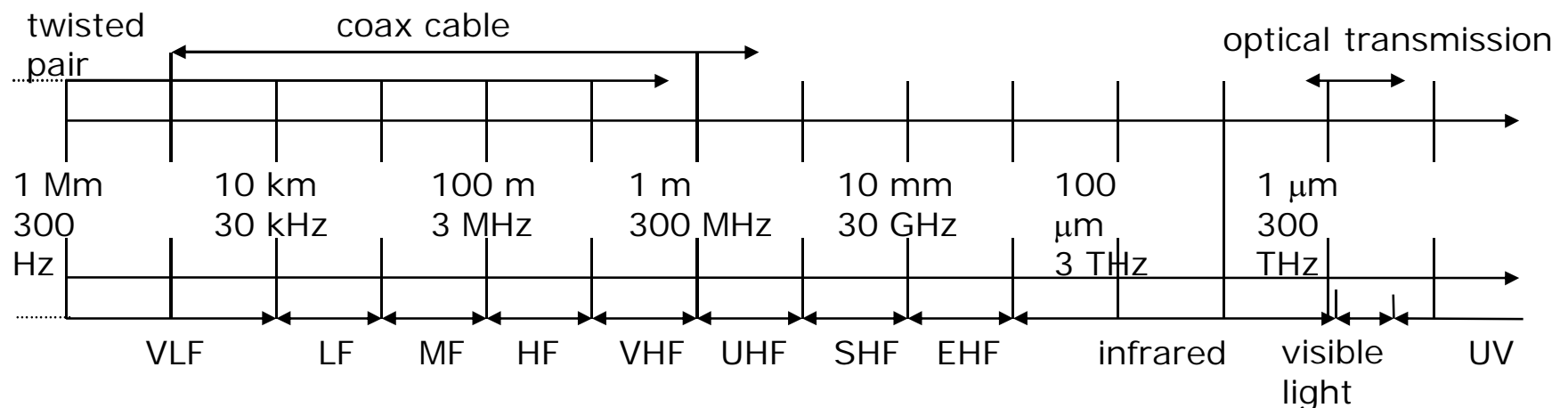
- Transmitter
 - Convert Source (information) to signals
 - Send converted signals to the channel (by antenna if applicable)
- Channel
 - Wireless: atmosphere (free space)
 - Wired: coaxial cables, twisted wires, optical fibre
- Receiver
 - Reconvert received signals to original information
 - Output the original information



Introduction to communications

- Elements of a communication system (cont)

- Frequencies for communication



- VLF = Very Low Frequency
- LF = Low Frequency
- MF = Medium Frequency
- HF = High Frequency
- VHF = Very High Frequency
- UHF = Ultra High Frequency
- SHF = Super High
- EHF = Extra High Frequency
- UV = Ultraviolet Light

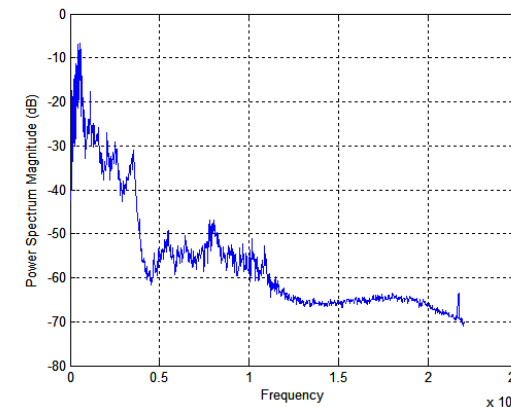
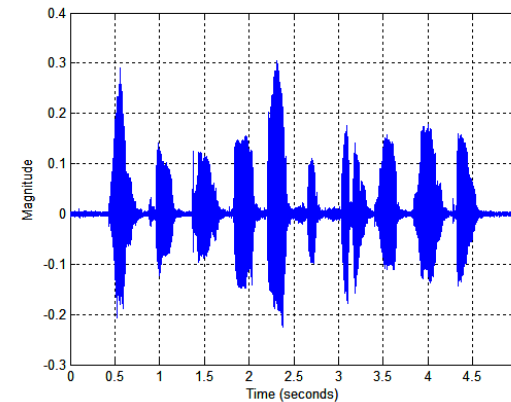
- Frequency and wave length:

$$\lambda = c/f$$

wave length λ , speed of light $c \cong 3 \times 10^8 \text{m/s}$, frequency f

Baseband vs Passband Transmission

- Baseband signals:
 - Voice (0-4kHz)
 - TV (0-6 MHz)
- A signal may be sent in its baseband format when a dedicated wired channel is available.
- Otherwise, it must be converted to passband.



Modulation: What and Why?

- The process of shifting the baseband signal to passband range is called *Modulation*.
- The process of shifting the passband signal to baseband frequency range is called *Demodulation*.
- Reasons for modulation:
 - Simultaneous transmission of several signals
 - Practical Design of Antennas
 - Exchange of power and bandwidth

Types of (Carrier) Modulation

- In modulation, one characteristic of a signal (generally a sinusoidal wave) known as the *carrier* is changed based on the information signal that we wish to transmit (*modulating signal*).
- That could be the amplitude, phase, or frequency, which result in Amplitude modulation (**AM**), Phase modulation (**PM**), or Frequency modulation (**FM**). The last two are combined as Angle Modulation

Types of Amplitude Modulation (AM)

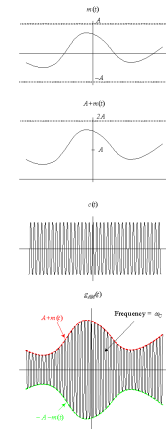
- **Double Sideband with carrier (we will call it AM)**: This is the most widely used type of AM modulation. In fact, all radio channels in the AM band use this type of modulation.
- **Double Sideband Suppressed Carrier (DSBSC)**: This is the same as the AM modulation above but without the carrier.
- **Single Sideband (SSB)**: In this modulation, only half of the signal of the DSBSC is used.
- **Vestigial Sideband (VSB)**: This is a modification of the SSB to ease the generation and reception of the signal.

Definition of AM

- Shift $m(t)$ by some DC value "A" such that $A+m(t) \geq 0$. Or $A \geq m_{\text{peak}}$

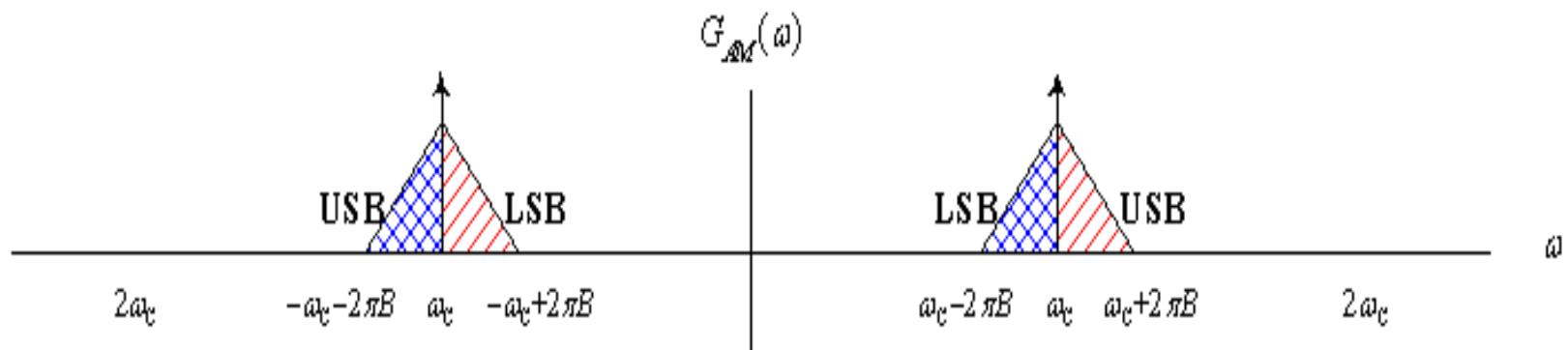
$$\begin{aligned}g_{AM}(t) &= [A + m(t)]\cos(\omega_c t) \\ &= A \cos(\omega_c t) + m(t) \cos(\omega_c t)\end{aligned}$$

- Called DSBWC. Here will refer to it as Full AM, or simply AM
- Modulation index $\mu = m_p / A$.
- $0 \leq \mu \leq 1$



Spectrum of AM

$$g_{AM}(t) \Leftrightarrow \pi A[\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] + \frac{1}{2}[M(\omega - \omega_c) + M(\omega + \omega_c)]$$

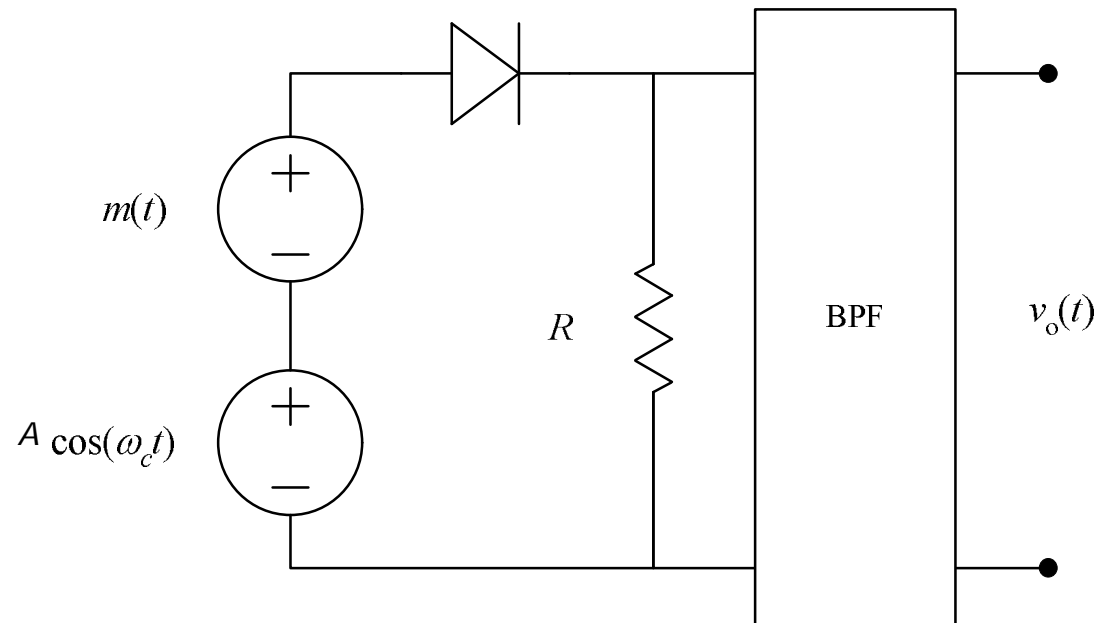


Generation of AM

- AM signals can be generated by any DSBSC modulator, by using $A+m(t)$ as input instead of $m(t)$.
- In fact, the presence of the carrier term can make it even simpler. We can use it for switching instead of generating a local carrier.
- The switching action can be made by a single diode instead of a diode bridge.

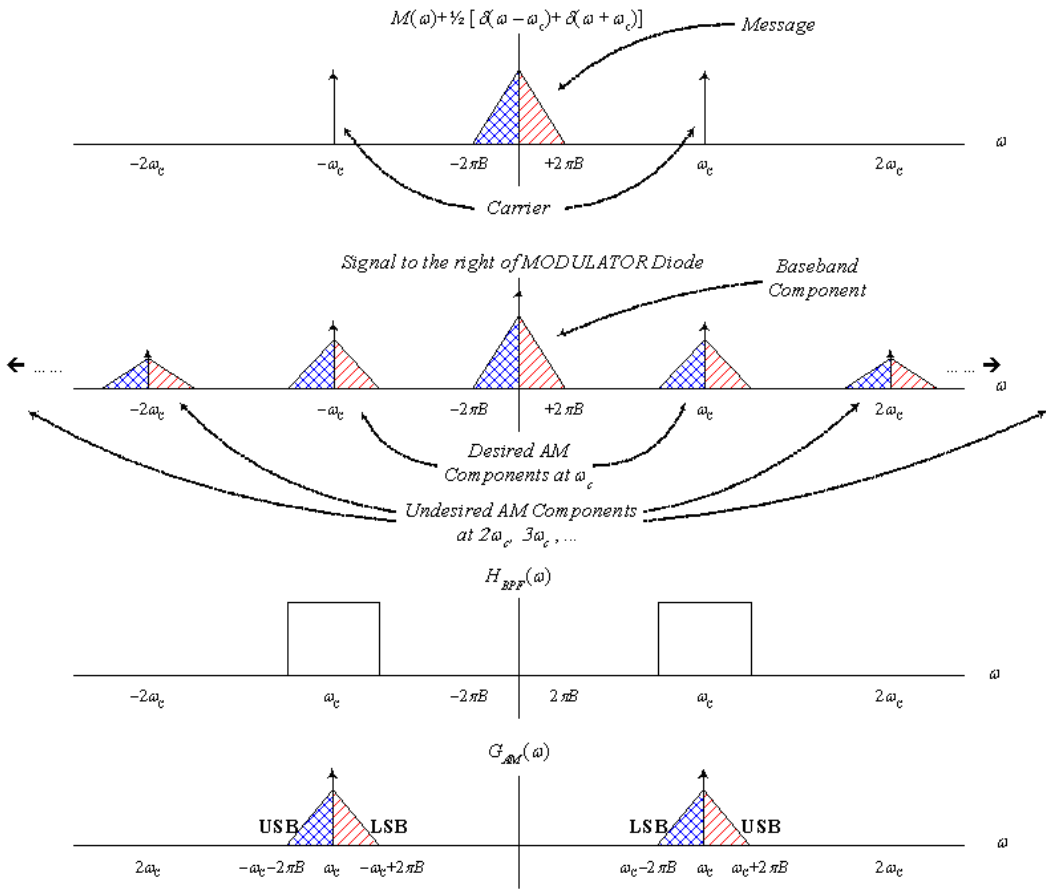
AM Generator

- $A \gg m(t)$
(to ensure switching at every period).



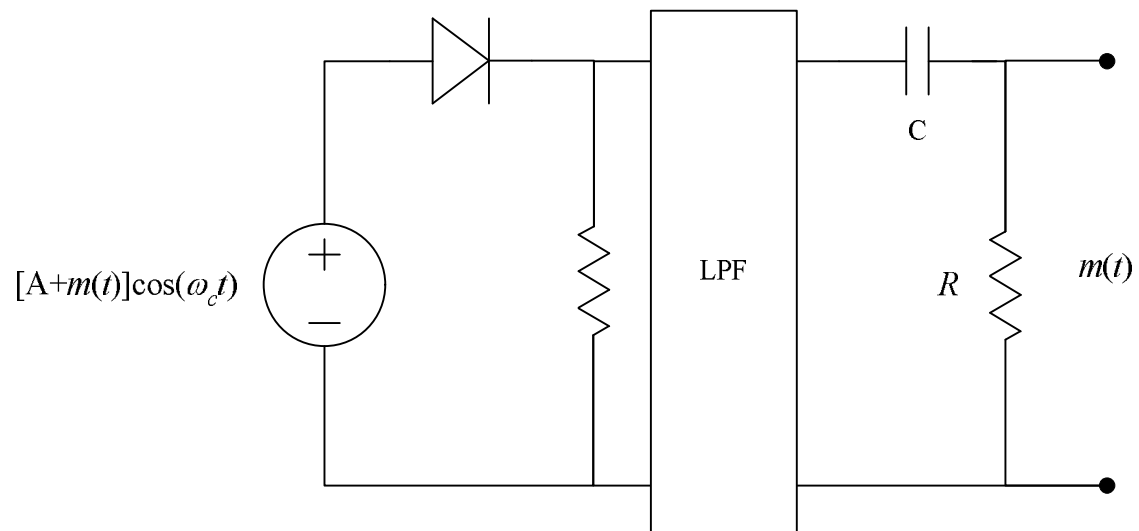
- $V_R = [\cos \omega_c t + m(t)] [1/2 + 2/\pi (\cos \omega_c t - 1/3 \cos 3 \omega_c t + \dots)]$
 $= (1/2) \cos \omega_c t + (2/\pi) m(t) \cos \omega_c t + \text{other terms (suppressed by BPF)}$
- $v_o(t) = (1/2) \cos \omega_c t + (2/\pi) m(t) \cos \omega_c t$

AM Modulation Process (Frequency)



AM Demodulation: Rectifier Detector

- Because of the presence of a carrier term in the received signal, switching can be performed in the same way we did in the modulator.



Rectifier Detector: Time Domain

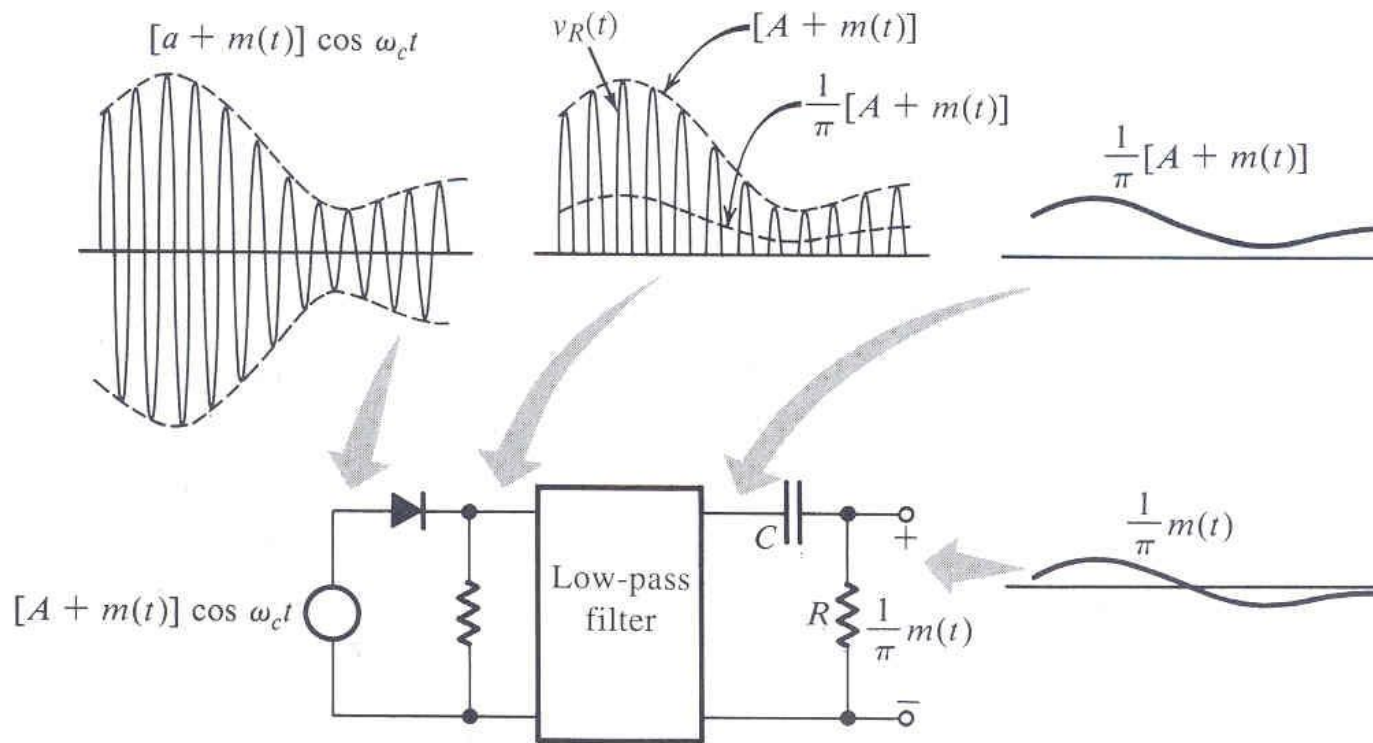
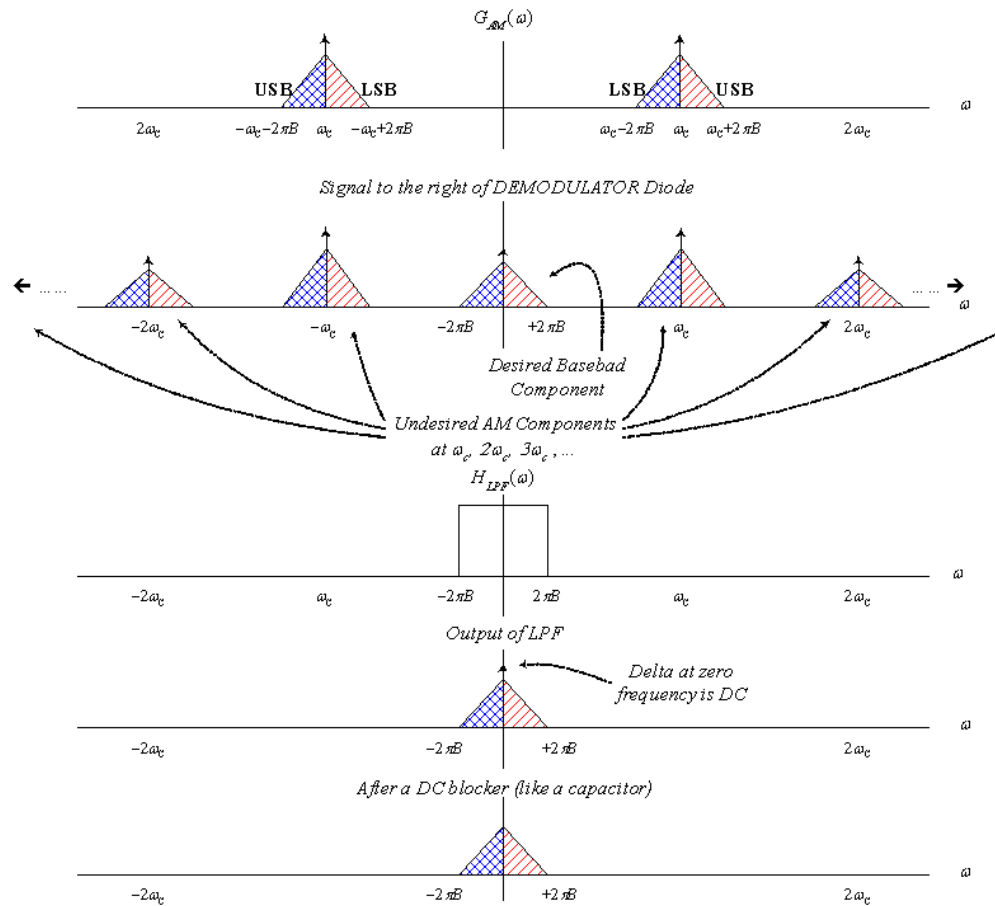
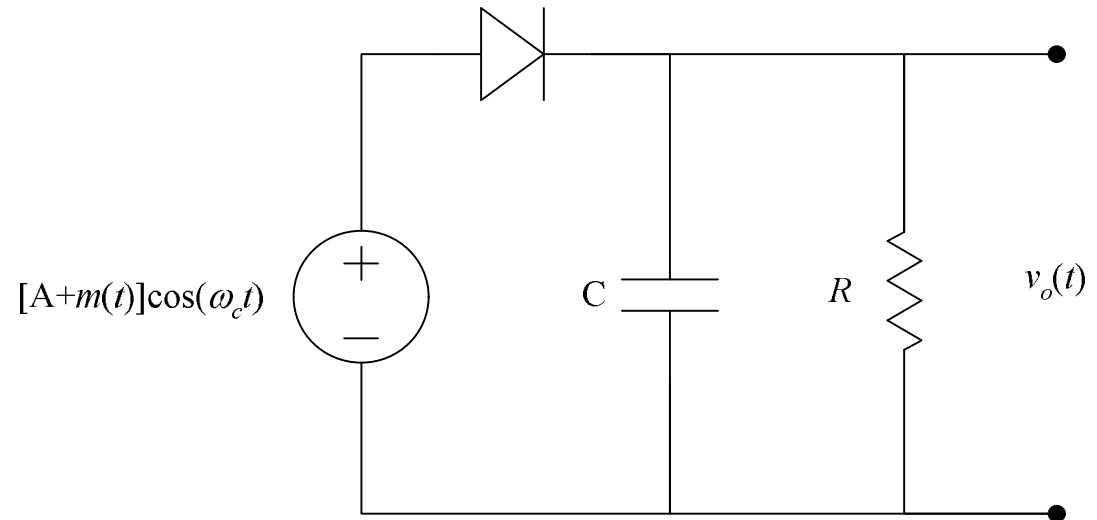


Figure 4.11 Rectifier detector for AM.

Rectifier Detector (Frequency Domain)



Envelope Detector



- When D is forward-biased, the capacitor charges and follows input.
- When D is reverse-biased, the capacitor discharges through R .

Double Sideband Suppressed Carrier (DSBSC)

Handwritten mathematical derivation on a whiteboard:

$$A \cos(\omega t + \theta_c) \text{ message signal } m(t)$$
$$A \propto m(t)$$

$\therefore \theta_c$ is constant

$$\therefore \theta_c = 0$$
$$m(t) \times A \cos \omega_c t$$

if $m(t) \leftrightarrow M(\omega)$

then

$$m(t) \cos \omega_c t \leftrightarrow \frac{1}{2} [M(\omega + \omega_c) + M(\omega - \omega_c)]$$

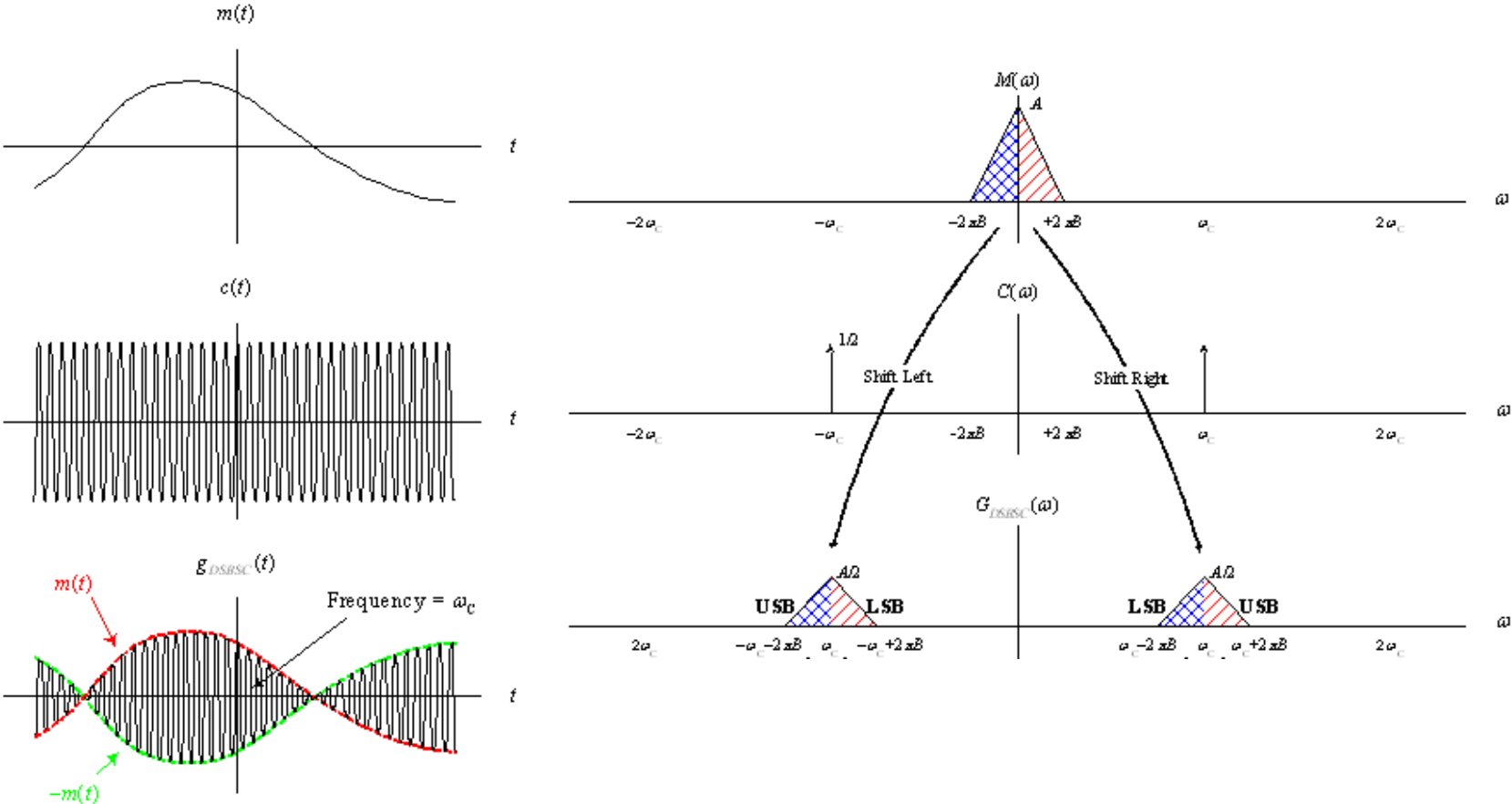
sifting of $m(t)$ to the carrier frequency

DSBSC carrier is filtered or suppressed or receiver. That's why it is called DSBSC

Problem with DSBSC

- 1) Geometrical Carrier or Receiver
- 2) Phase Detection Problem
- 3) Frequency Shifting Properties

Time and Frequency Representation of DSBSC Modulation Process



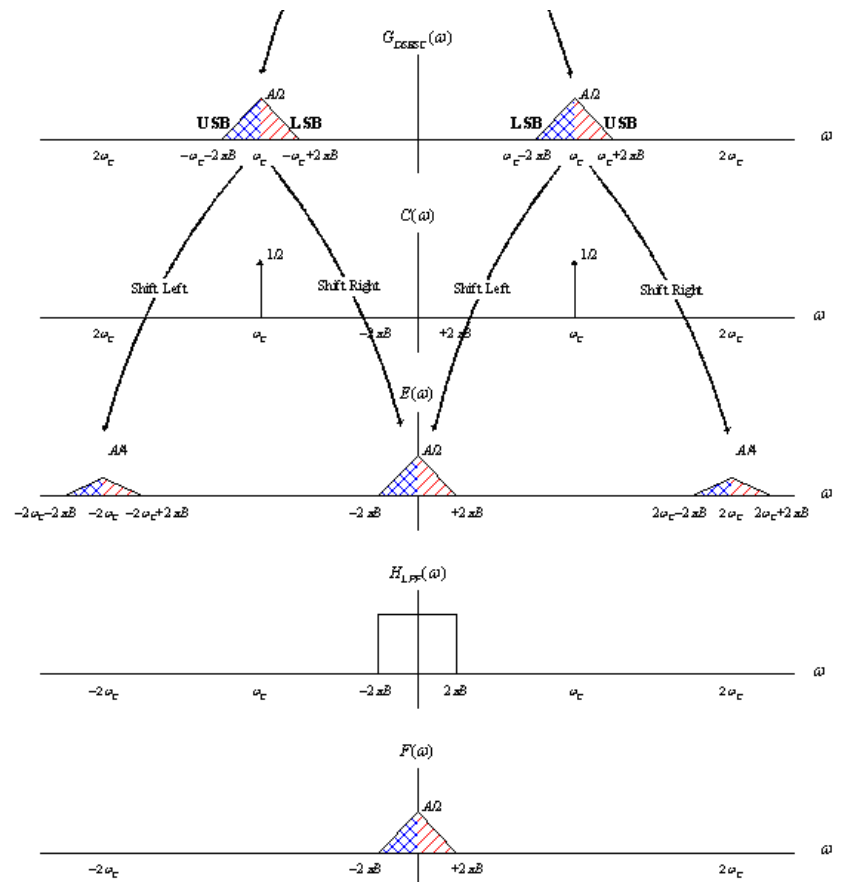
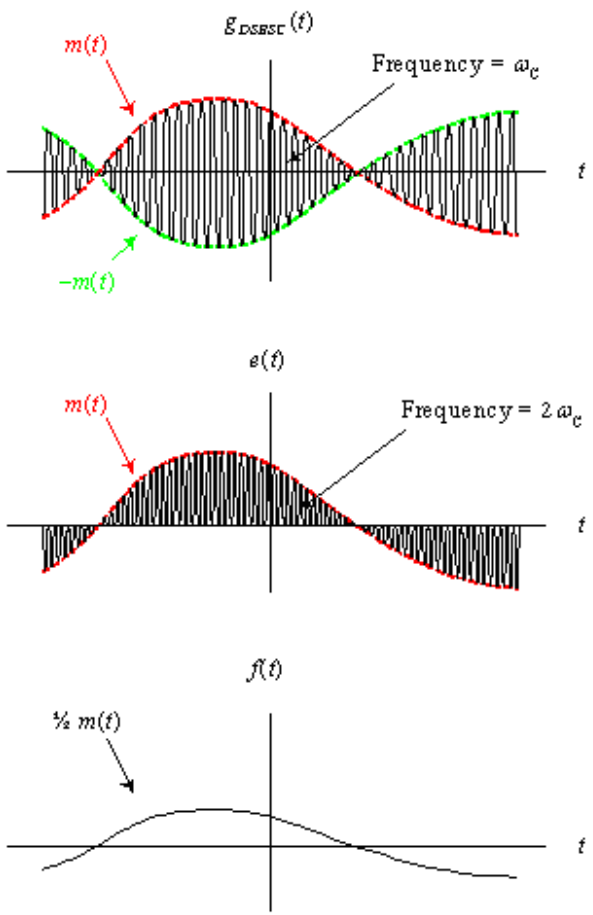
DSBSC Demodulation

Double Sideband Suppressed Carrier (DSBSC)

For a broadcast system it is more economical to have one expensive high power transmitter and expensive receiver, for such application a large carrier signal is transmitted along with the suppressed carrier modulated signal $m(t) \cos(\omega_c t)$, thus no need to generate a local carrier. This is called AM in which the transmitted signal is.

$$\cos \omega_c t [A + m(t)]$$

Time and Frequency Representation of DSBSC Demodulation Process



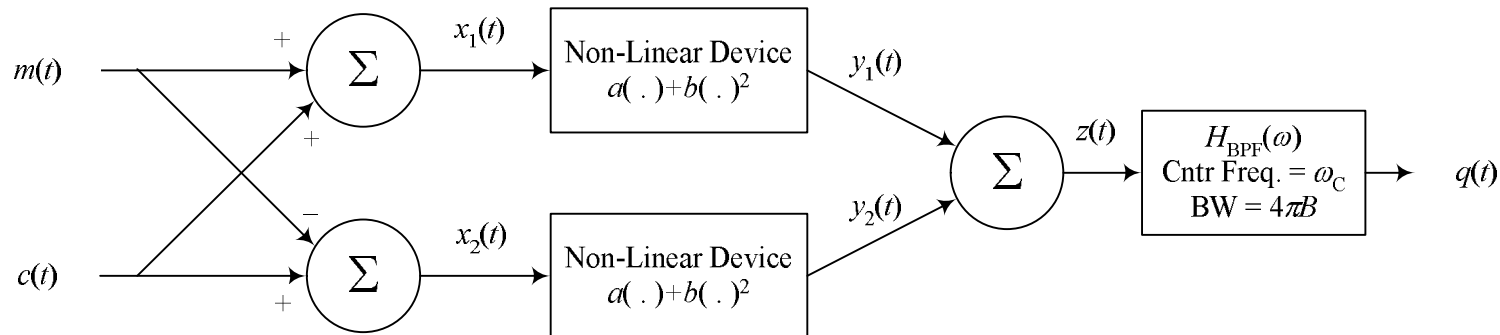
Modulator Circuits

- Basically we are after multiplying a signal with a carrier.
- There are three realizations of this operation:
 - Multiplier Circuits
 - Non-Linear Circuits
 - Switching Circuits

Non-Linear Devices (NLD)

- A NLD is a device whose input-output relation is non-linear. One such example is the diode ($i_D = e^{V_D/V_T}$).
- The output of a NLD can be expressed as a power series of the input, that is
$$y(t) = ax(t) + bx^2(t) + cx^3(t) + \dots$$
- When $x(t) \ll 1$, the higher powers can be neglected, and the output can be approximated by the first two terms.
- When the input $x(t)$ is the sum of two signal, $m(t)+c(t)$, $x^2(t)$ will have the product term $m(t)c(t)$

Non-Linear Modulators



DSBSC modulation using non-linear device

$$x_1(t) = c(t) + m(t) = \cos(\omega_c t) + m(t)$$

$$x_2(t) = c(t) - m(t) = \cos(\omega_c t) - m(t)$$

$$y_1(t) = a[\cos(\omega_c t) + m(t)] + b[\cos(\omega_c t) + m(t)]^2$$

$$= a \cos(\omega_c t) + am(t) + bm^2(t) + 2bm(t) \cdot \cos(\omega_c t) + b \cos^2(\omega_c t)$$

$$= \underbrace{am(t)}_{\text{Undesired}} + \underbrace{bm^2(t)}_{\text{Undesired}} + \underbrace{2bm(t) \cdot \cos(\omega_c t)}_{\text{Desired}} + \underbrace{a \cos(\omega_c t)}_{\text{Undesired}} + \underbrace{\frac{b}{2}}_{\text{Undesired}} + \underbrace{\frac{b}{2} \cos(2\omega_c t)}_{\text{Undesired}}$$

$$z(t) = y_1(t) - y_2(t)$$

$$= \underbrace{2am(t)}_{\text{Undesired}} + \underbrace{4bm(t) \cdot \cos(\omega_c t)}_{\text{Desired}}$$

$$y_2(t) = a[\cos(\omega_c t) - m(t)] + b[\cos(\omega_c t) - m(t)]^2$$

$$= a \cos(\omega_c t) - am(t) + bm^2(t) - 2bm(t) \cdot \cos(\omega_c t) + b \cos^2(\omega_c t)$$

$$= \underbrace{-am(t)}_{\text{Undesired}} + \underbrace{bm^2(t)}_{\text{Undesired}} - \underbrace{2bm(t) \cdot \cos(\omega_c t)}_{\text{Desired}} + \underbrace{a \cos(\omega_c t)}_{\text{Undesired}} + \underbrace{\frac{b}{2}}_{\text{Undesired}} + \underbrace{\frac{b}{2} \cos(2\omega_c t)}_{\text{Undesired}}$$

Switching Modulators

- Any periodic function can be expressed as a series of cosines (Fourier Series).
- The information signal, $m(t)$, can therefore be, equivalently, multiplied by any periodic function, and followed by BPF.
- Let this periodic function be a train of pulses.
- Multiplication by a train of pulses can be realized by simple *switching*.

Switching Modulator Illustration

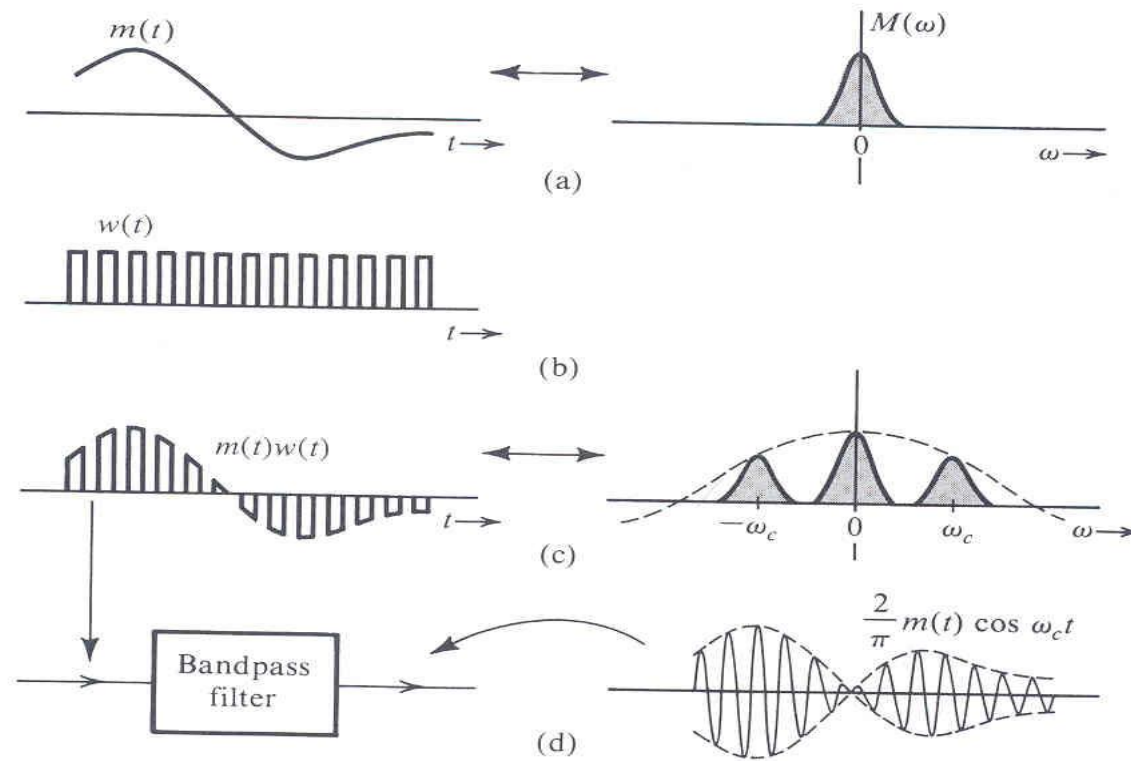


Figure 4.4 Switching modulator for DSB-SC.

Switching Modulator: Diode Bridge

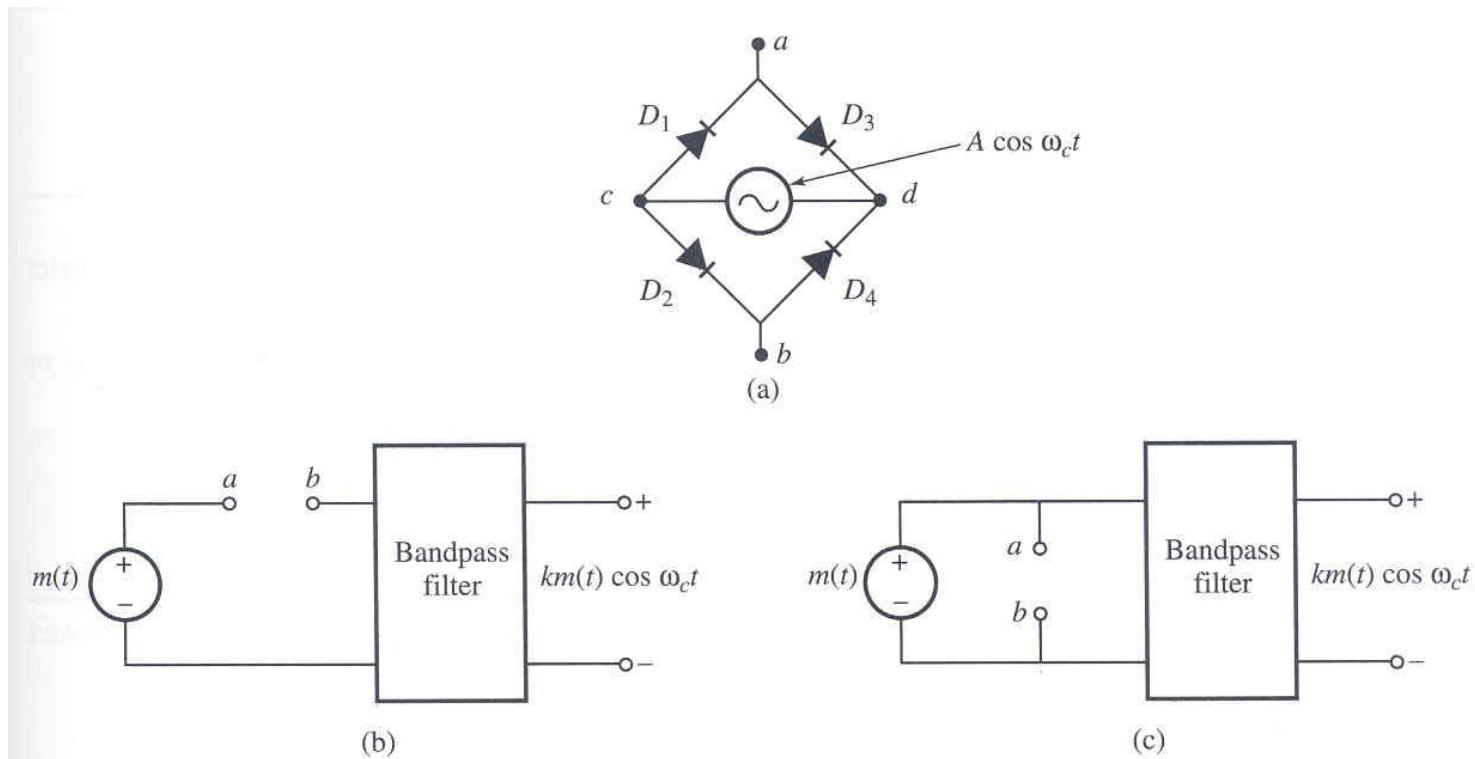


Figure 4.5 (a) Diode-bridge electronic switch. (b) Series-bridge diode modulator. (c) Shunt-bridge diode modulator.

Switching Modulator: Ring

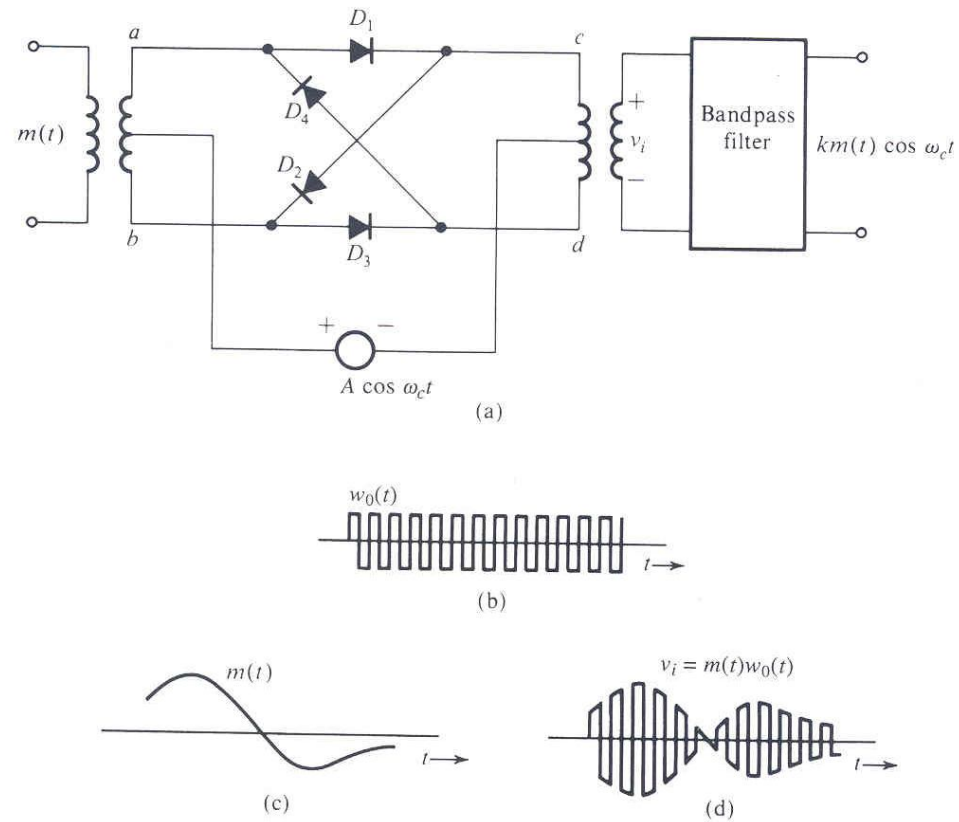
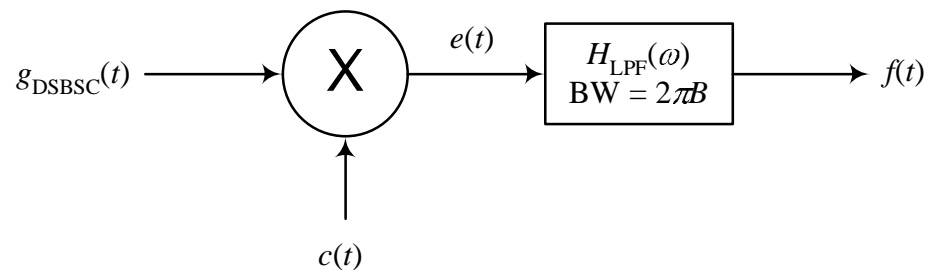


Figure 4.6 Ring modulator.

Demodulation of DSBSC

- The modulator circuits can be used for demodulation, but replacing the BPF by a LPF of bandwidth B Hz.
- The receiver must generate a carrier frequency in phase and frequency synchronization with the incoming carrier.
- This type of demodulation is therefore called *coherent* demodulation (or detection).



DSBSC Demodulator (receiver)

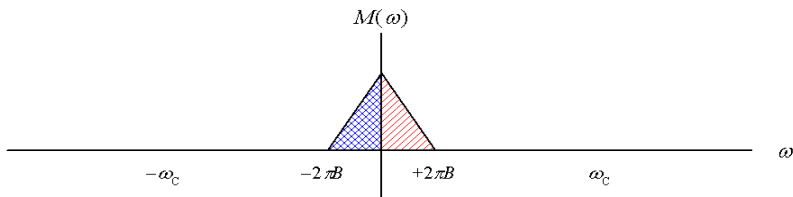
From DSBSC to DSBWC (AM)

- Carrier recovery circuits, which are required for the operation of coherent demodulation, are sophisticated and could be quite costly.
- If we can let $m(t)$ be the envelope of the modulated signal, then a much simpler circuit, the *envelope detector*, can be used for demodulation (non-coherent demodulation).
- How can we make $m(t)$ be the envelope of the modulated signal?

Single-Side Band (SSB) Modulation

- DSBSC (as well as AM) occupies double the bandwidth of the baseband signal, although the two sides carry the same information.
- Why not send only one side, the upper or the lower?
- **Modulation:** similar to DSBSC. Only change the settings of the BPF (center frequency, bandwidth).
- **Demodulation:** similar to DSBSC (coherent)

SSB Representation



How would we represent the SSB signal in the time domain?

$$g_{USB}(t) = ?$$

$$g_{LSB}(t) = ?$$

Time-Domain Representation of SSB (1/2)

$$M(\omega) = M_+(\omega) + M_-(\omega)$$

Let $m_+(t) \leftrightarrow M_+(\omega)$ and $m_-(t) \leftrightarrow M_-(\omega)$

Then: $m(t) = m_+(t) + m_-(t)$ [linearity]

Because $M_+(\omega)$, $M_-(\omega)$ are not even

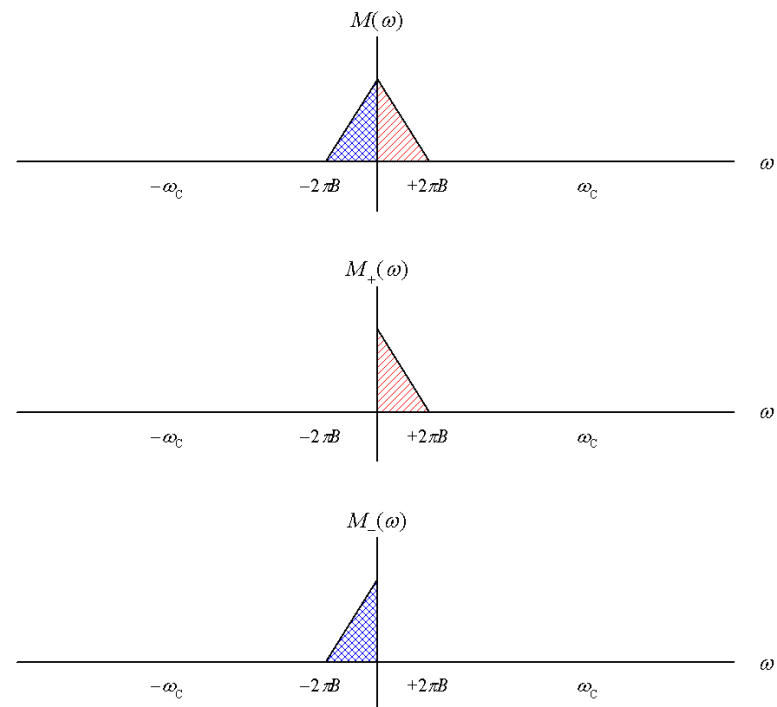
→ $m_+(t)$, $m_-(t)$ are complex.

Since their sum is real they must be conjugates.

$$m_+(t) = \frac{1}{2} [m(t) + j m_h(t)]$$

$$m_-(t) = \frac{1}{2} [m(t) - j m_h(t)]$$

What is $m_h(t)$?



Time-Domain Representation of SSB (2/2)

$$M(\omega) = M_+(\omega) + M_-(\omega)$$

$$M_+(\omega) = M(\omega)u(\omega); M_-(\omega) = M(\omega)u(-\omega)$$

$$\text{sgn}(\omega) = 2u(\omega) - 1 \rightarrow u(\omega) = \frac{1}{2} + \frac{1}{2} \text{sgn}(\omega); u(-\omega) = \frac{1}{2} - \frac{1}{2} \text{sgn}(\omega)$$

$$M_+(\omega) = \frac{1}{2} [M(\omega) + M(\omega)\text{sgn}(\omega)]$$

$$M_-(\omega) = \frac{1}{2} [M(\omega) - M(\omega)\text{sgn}(\omega)]$$

Comparing to:

$$m_+(t) = \frac{1}{2} [m(t) + j m_h(t)] \leftrightarrow \frac{1}{2} [M(\omega) + j M_h(\omega)]$$

$$m_-(t) = \frac{1}{2} [m(t) - j m_h(t)] \leftrightarrow \frac{1}{2} [M(\omega) - j M_h(\omega)]$$

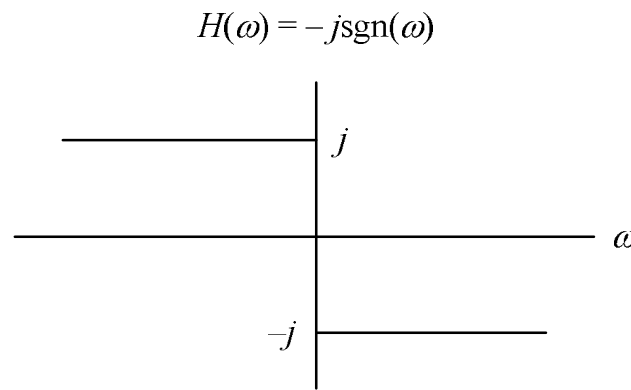
We find

$$M_h(\omega) = -j M(\omega) \cdot \text{sgn}(\omega) \quad \text{where } m_h(t) \leftrightarrow M_h(\omega)$$

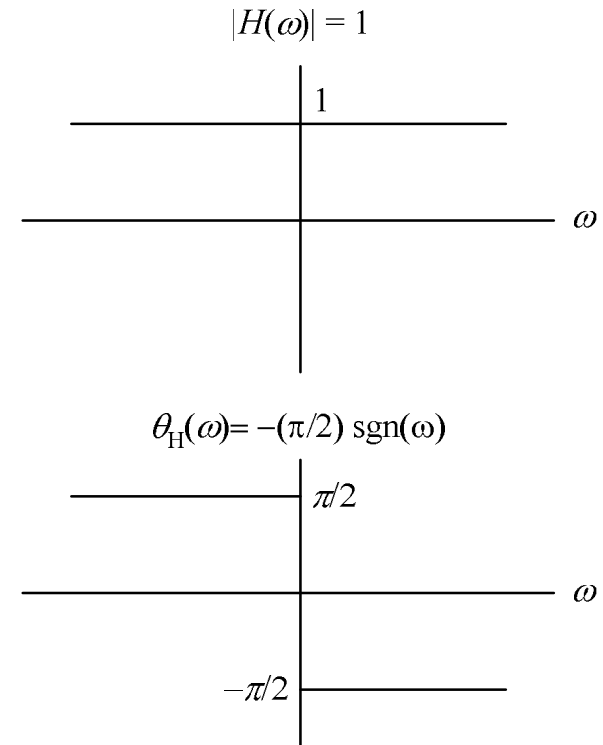
Hilbert Transform

- $m_h(t)$ is known as the Hilbert Transform (HT) of $m(t)$.
- The transfer function of this transform is given by:

$$H(\omega) = -j \operatorname{sgn}(\omega)$$



≡



- It is basically a $\pi/2$ phase shifter



Hilbert Transform of $\cos(\omega_c t)$

$$\cos(\omega_c t) \leftrightarrow \pi [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)]$$

$$\begin{aligned} \text{HT}[\cos(\omega_c t)] &\leftrightarrow -j \operatorname{sgn}(\omega) \pi [\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] \\ &= j \operatorname{sgn}(\omega) \pi [-\delta(\omega - \omega_c) - \delta(\omega + \omega_c)] \\ &= j \pi [-\delta(\omega - \omega_c) + \delta(\omega + \omega_c)] \\ &= j \pi [\delta(\omega + \omega_c) - \delta(\omega - \omega_c)] \leftrightarrow \sin(\omega_c t) \end{aligned}$$

Which is expected since:

$$\cos(\omega_c t - \pi/2) = \sin(\omega_c t)$$

Time-Domain Operation for Hilbert Transformation

For Hilbert Transformation $H(\omega) = -j \operatorname{sgn}(\omega)$.

What is $h(t)$?

$\operatorname{sgn}(t) \leftrightarrow 2/(j\omega)$ [From FT table]

$2/(jt) \leftrightarrow 2\pi \operatorname{sgn}(-\omega)$ [symmetry]

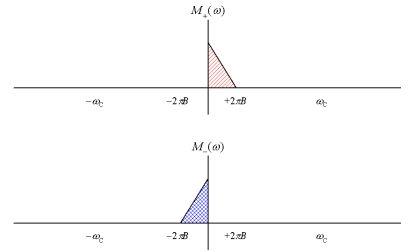
$1/(\pi t) \leftrightarrow -j \operatorname{sgn}(\omega)$

Since $M_h(\omega) = -j M(\omega) \cdot \operatorname{sgn}(\omega) = H(\omega) \cdot M(\omega)$

Then

$$\begin{aligned} m_h(t) &= \frac{1}{\pi t} * m(t) \\ &= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{m(\alpha)}{t - \alpha} d\alpha \end{aligned}$$

Finally ...

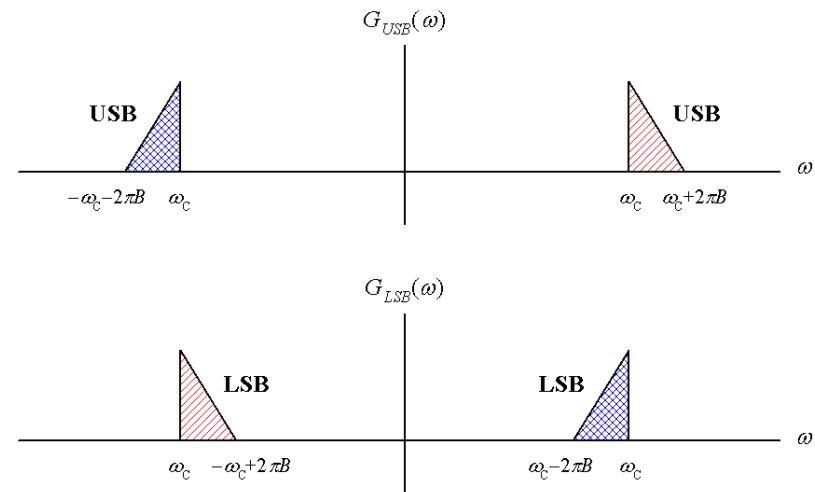


$$g_{USB}(t) = m_+(t)e^{j\omega_c t} + m_-(t)e^{-j\omega_c t}$$

$$g_{LSB}(t) = m_+(t)e^{-j\omega_c t} + m_-(t)e^{j\omega_c t}$$

$$\begin{aligned} g_{USB}(t) &= \frac{1}{2}m(t)e^{j\omega_c t} + \frac{1}{2}jm_h(t)e^{j\omega_c t} \\ &\quad + \frac{1}{2}m(t)e^{-j\omega_c t} - \frac{1}{2}jm_h(t)e^{-j\omega_c t} \\ &= m(t)\cos(\omega_c t) - m_h(t)\sin(\omega_c t) \end{aligned}$$

$$\begin{aligned} g_{LSB}(t) &= \frac{1}{2}m(t)e^{j\omega_c t} - \frac{1}{2}jm_h(t)e^{j\omega_c t} \\ &\quad + \frac{1}{2}m(t)e^{-j\omega_c t} + \frac{1}{2}jm_h(t)e^{-j\omega_c t} \\ &= m(t)\cos(\omega_c t) + m_h(t)\sin(\omega_c t) \end{aligned}$$



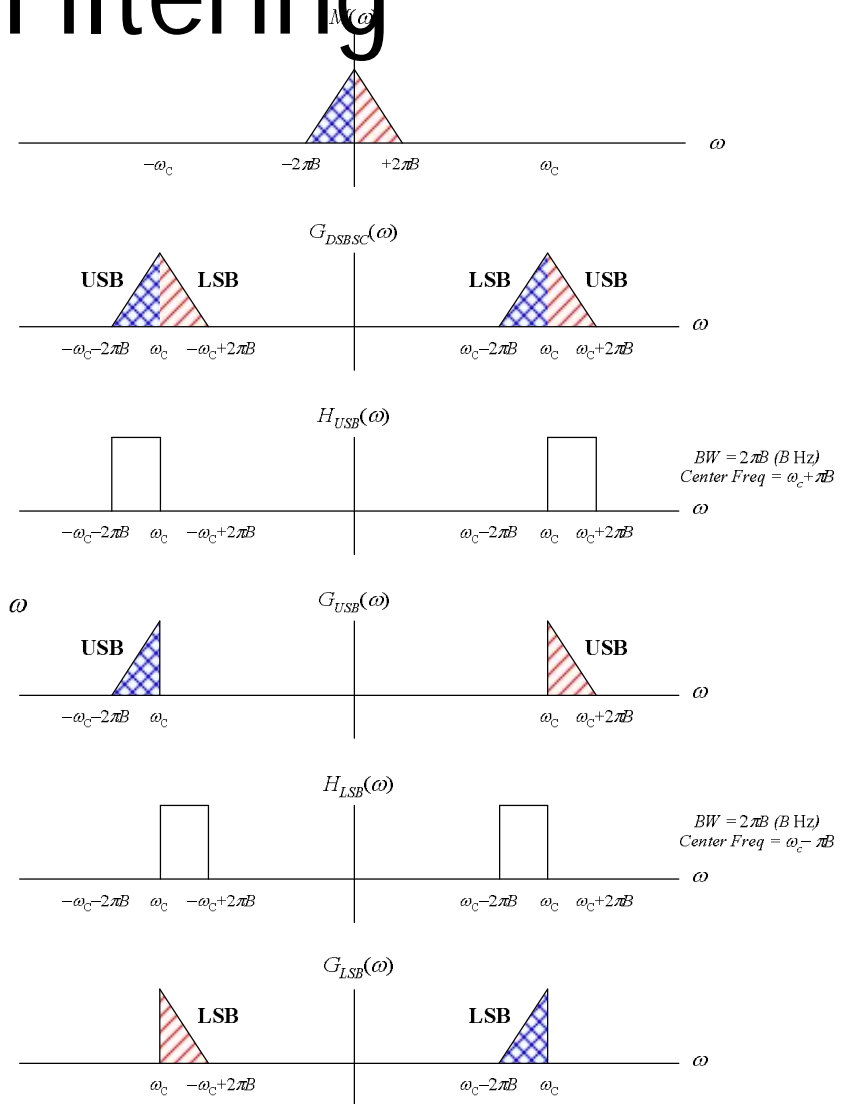
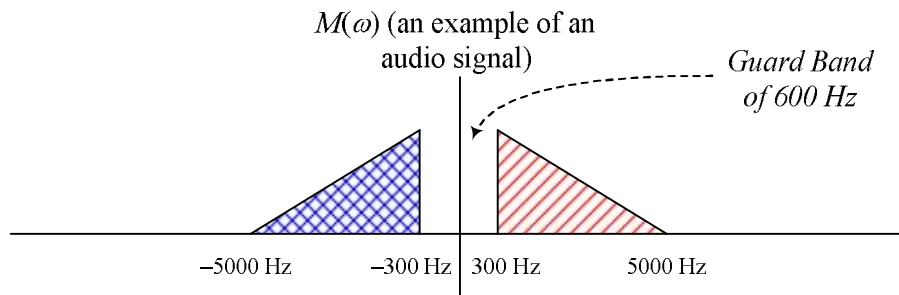
$$G_{USB}(\omega) = M_+(\omega - \omega_c) + M_-(\omega + \omega_c)$$

$$G_{LSB}(\omega) = M_+(\omega + \omega_c) + M_-(\omega - \omega_c)$$

Generation of SSB

- Selective Filtering Method
Realization based on spectrum analysis
- Phase-Shift Method
Realization based on time-domain expression of the modulated signal

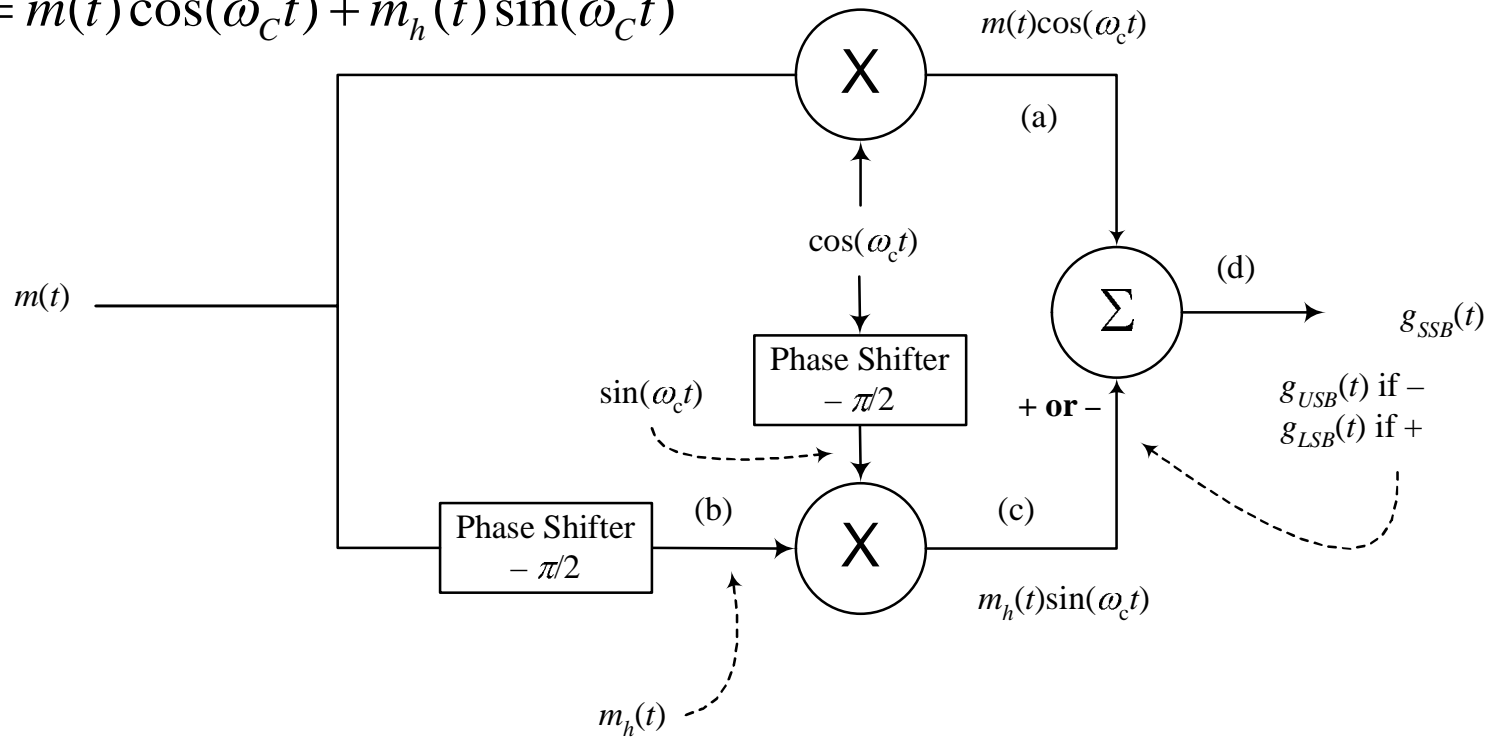
Selective Filtering



Phase Shifting

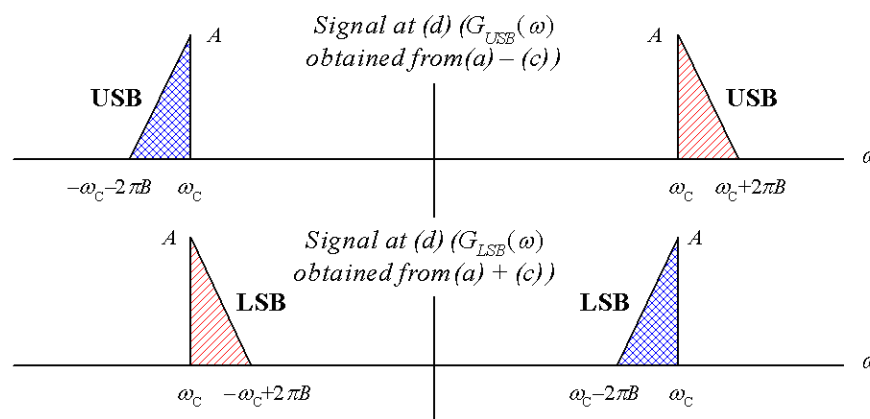
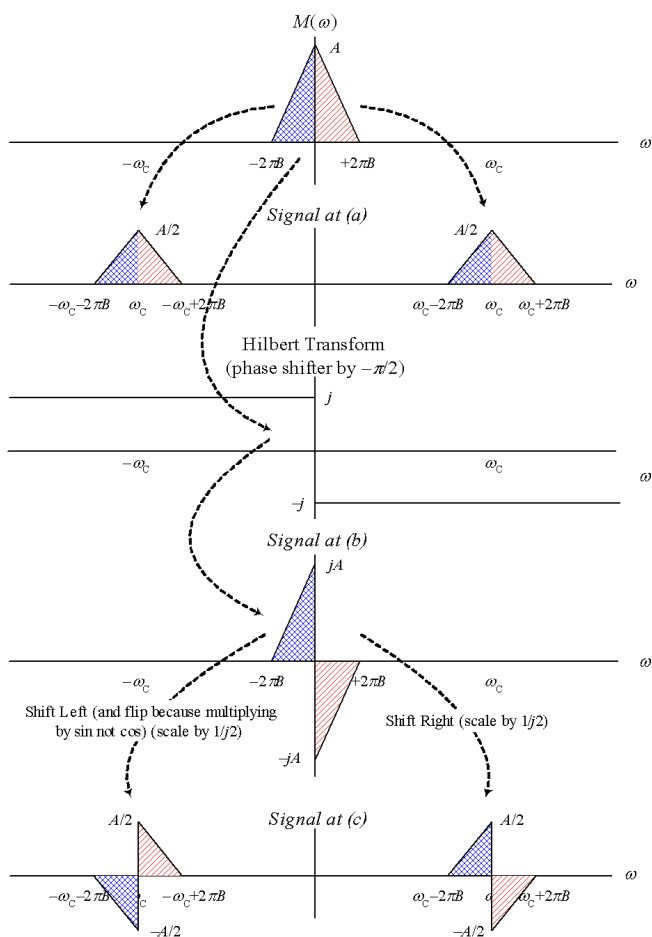
$$g_{USB}(t) = m(t) \cos(\omega_c t) - m_h(t) \sin(\omega_c t)$$

$$g_{LSB}(t) = m(t) \cos(\omega_c t) + m_h(t) \sin(\omega_c t)$$



SSB Modulator

Phase-shifting Method: Frequency-Domain Illustration

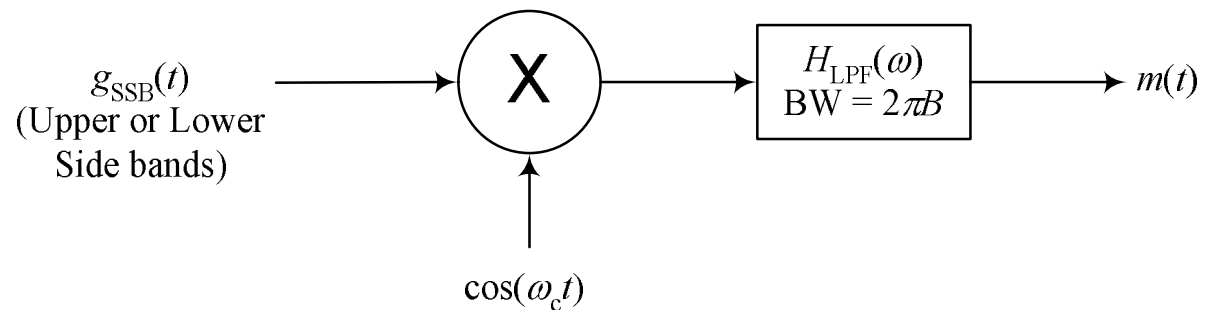


SSB Demodulation (Coherent)

$$g_{SSB}(t) = m(t) \cos(\omega_c t) \pm m_h(t) \sin(\omega_c t)$$

$$g_{SSB}(t) \cos(\omega_c t) = \frac{1}{2} m(t) [1 + \cos(2\omega_c t)] \pm \frac{1}{2} m_h(t) \sin(2\omega_c t)$$

$$\text{LPF Output} = \frac{1}{2} m(t)$$

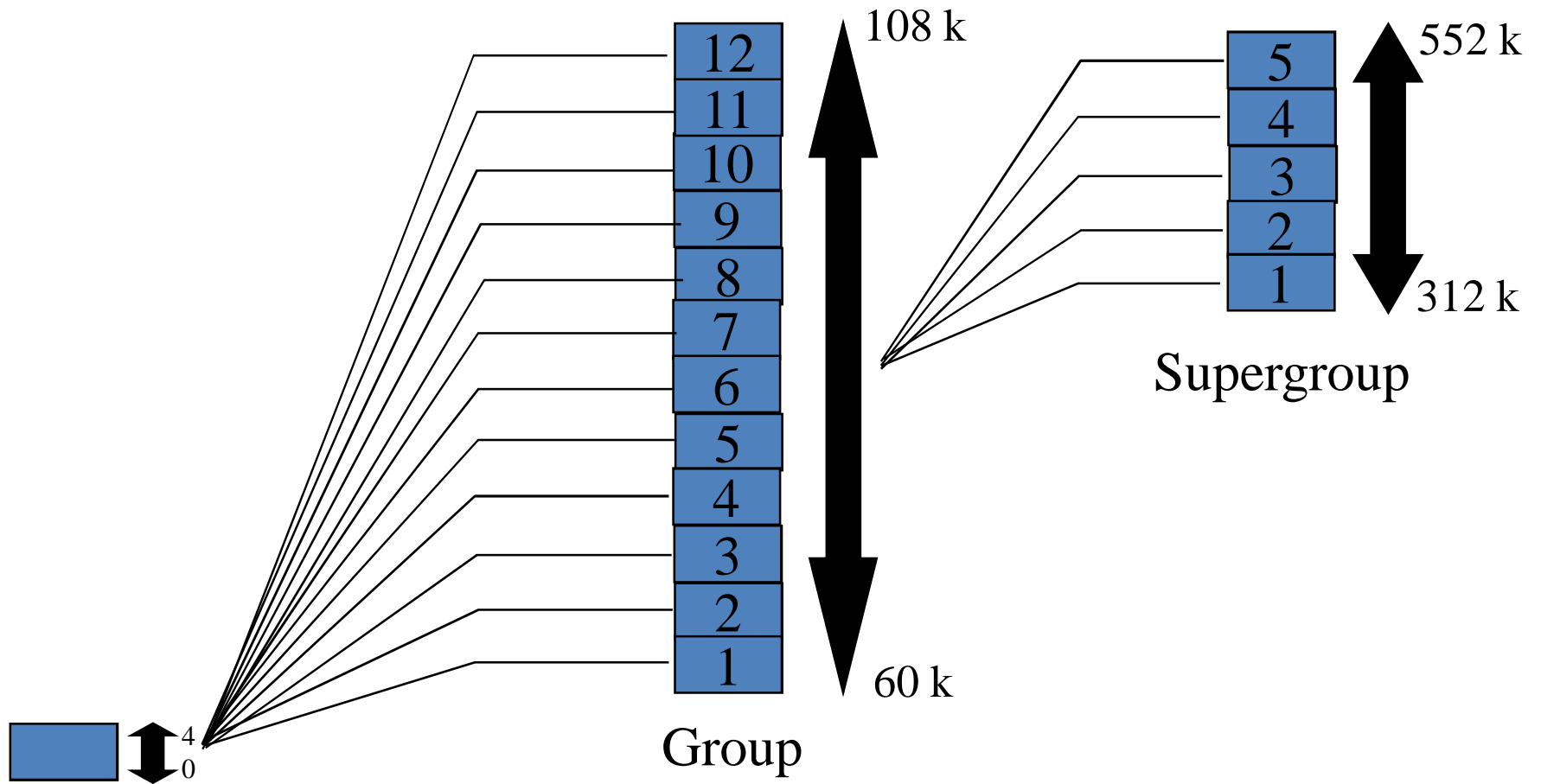


SSB Demodulator (receiver)

FDM in Telephony

- FDM is done in stages
 - Reduce number of carrier frequencies
 - More practical realization of filters
- Group: $12 \text{ voice channels} \times 4 \text{ kHz} = 48 \text{ kHz}$
occupy the band 60-108 kHz
- Supergroup: $5 \text{ groups} \times 48 \text{ kHz} = 240 \text{ kHz}$
occupy the band 312-552
- Mastergroup: $10 \text{ S-G} \times 240 \text{ kHz} = 2400 \text{ kHz}$
occupy the band 564-3084 kHz

FDM Hierarchy



Vestigial Side Band Modulation (VSB)

- What if we want to generate SSB using selective filtering but there is no guard band between the two sides?
 - ➔ We will filter-in a *vestige* of the other band.
- Can we still recover our message, without distortion, after demodulation?
Yes. If we use a proper LPF.

Filtering Condition of VSB

$$g_{DSBSC}(t) = 2m(t) \cos(\omega_c t)$$

$$G_{DSBSC}(\omega) = M(\omega - \omega_c) + M(\omega + \omega_c)$$

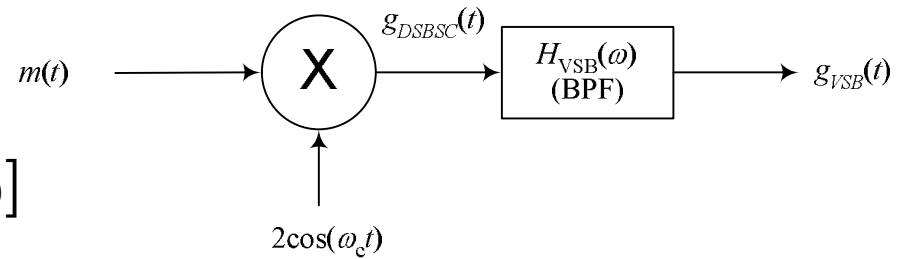
$$G_{VSB}(\omega) = H_{VSB}(\omega) [M(\omega - \omega_c) + M(\omega + \omega_c)]$$

$$X(\omega) = H_{VSB}(\omega - \omega_c) \left[\underbrace{M(\omega - 2\omega_c)}_{at + 2\omega_c} + \underbrace{M(\omega)}_{Baseband} \right]$$

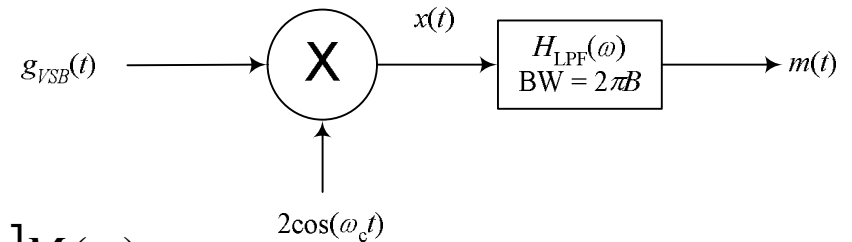
$$+ H_{VSB}(\omega + \omega_c) \left[\underbrace{M(\omega)}_{baseband} + \underbrace{M(\omega + 2\omega_c)}_{at - 2\omega_c} \right]$$

$$Z(\omega) = H_{LPF}(\omega) [H_{VSB}(\omega - \omega_c) + H_{VSB}(\omega + \omega_c)] M(\omega)$$

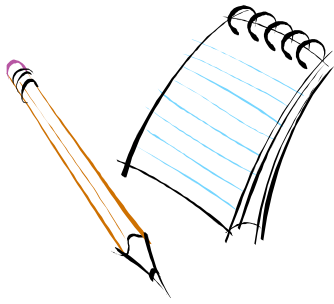
$$H_{LPF}(\omega) = \frac{1}{H_{VSB}(\omega - \omega_c) + H_{VSB}(\omega + \omega_c)} \quad ; \quad |\omega| \leq 2\pi B$$



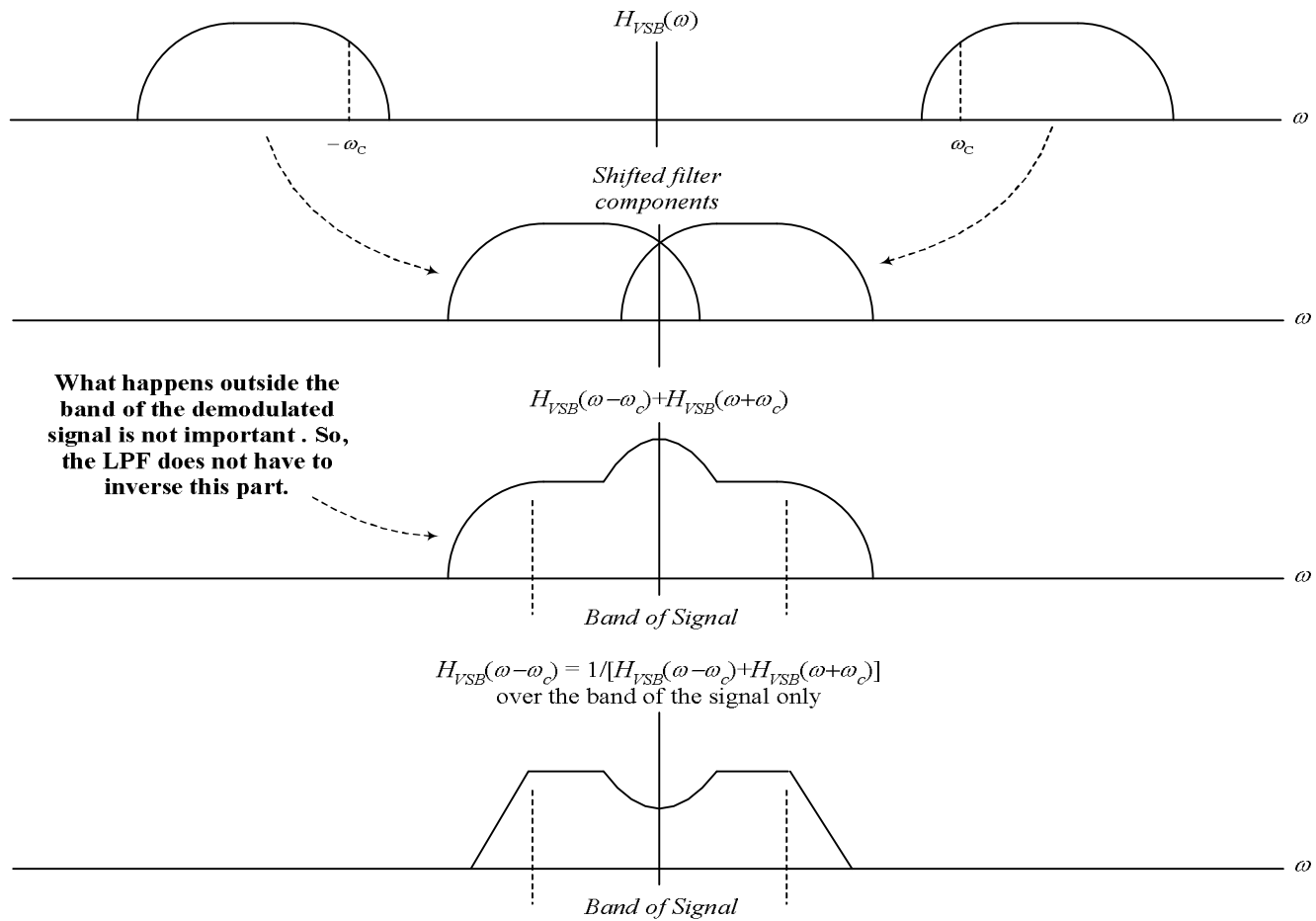
VSB Modulator (transmitter)



VSB Demodulator (receiver)



VSB Filtering



What happens outside the band of the demodulated signal is not important. So, the LPF does not have to invert this part.

VSB Filter: Special Case

- Condition For distortionless demodulation:

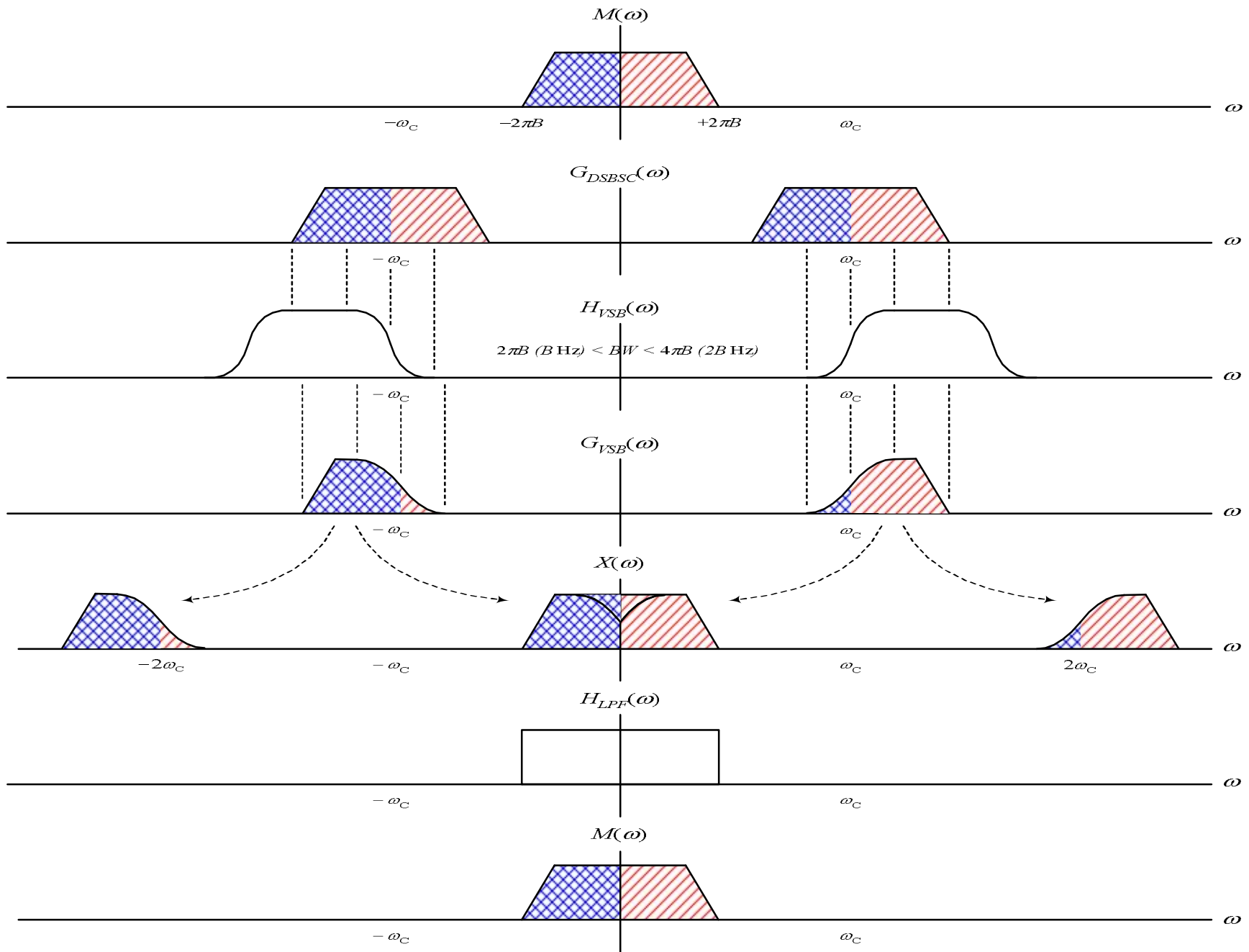
$$H_{LPF}(\omega) = \frac{1}{H_{VSB}(\omega - \omega_c) + H_{VSB}(\omega + \omega_c)} ; |\omega| \leq 2\pi B$$

- If we impose the condition on the filter at the modulator:

$$H_{VSB}(\omega - \omega_c) + H_{VSB}(\omega + \omega_c) = 1 ; |\omega| \leq 2\pi B$$

Then $H_{LPF} = 1$ for $|\omega| \leq 2\pi B$ (Ideal LPF)

- $H_{VSB}(\omega)$ will then have odd symmetry around ω_c over the transition period.

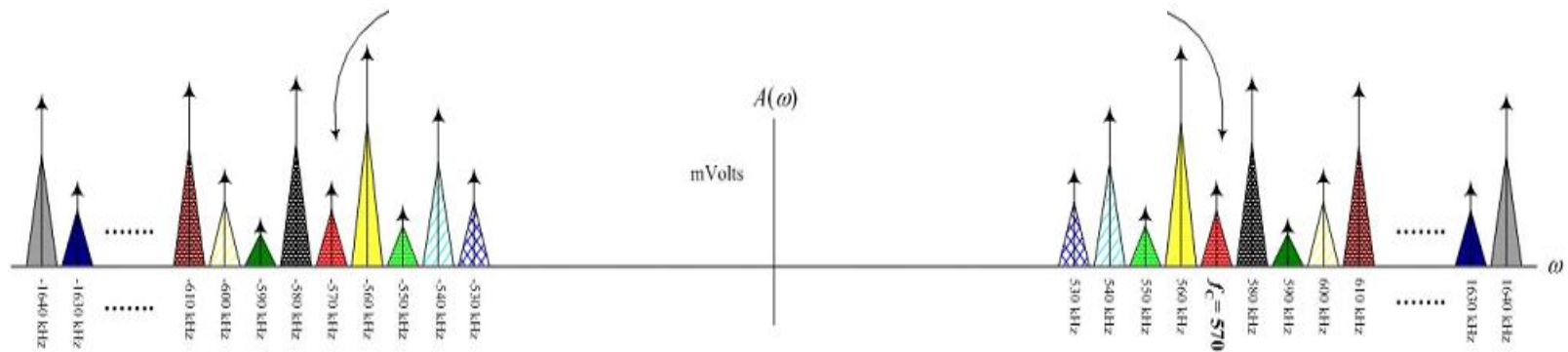


AM Broadcasting

- Allocated the band 530 kHz – 1600 kHz (with minor variations)
- 10 kHz per channel. (9 kHz in some countries)
- More than 100 stations can be licensed in the same geographical area.
- Uses AM modulation (DSB + C)

AM station Reception

- In theory, any station can be extracted from the stream of spectra by tuning the receiver BPF to its center frequency. Then demodulated.



- Impracticalities:
 - Requires a BPF with very high Q-factor ($Q = f_c / B$).
 - Particularly difficult if the filter is to be tunable.