

Objective

- Understand the transformer nameplate
- Describe the basic construction features of a transformer.
- Explain the relationship between voltage, current, impedance, and power in a transformer.
- Define transformer exciting current.
- Develop transformer equivalent circuits from open-circuit and short-circuit test data.
- Analyze transformer operation.
- Calculate transformer voltage regulation and efficiency.
- Use K-factor-rated transformer to solve nonlinear load problems.
- Explain the four standard three-phase transformer configurations

Introduction

A **transformer** is an electrical device that transfers energy from one circuit to another purely by magnetic coupling.

Relative motion of the parts of the transformer is not required for transfer of energy.

Transformers are often used to convert between high and low voltages and to change impedance.

Transformers alone **cannot** do the following:

- Convert DC to AC or vice versa
- Change the voltage or current of DC
- Change the AC supply frequency.

However, transformers are **components** of the systems that perform all these functions.

Transformer Nameplate Data



Transformer nameplates contain information about the size of the transformer in terms of how much apparent power (rated in kVA) it is designated to deliver to the load on a continuous basis as well primary and secondary voltages and currents.

Example: 75 kVA, 720-240*120V

U-W primary winding is rated U volts and secondary winding is rated V volts

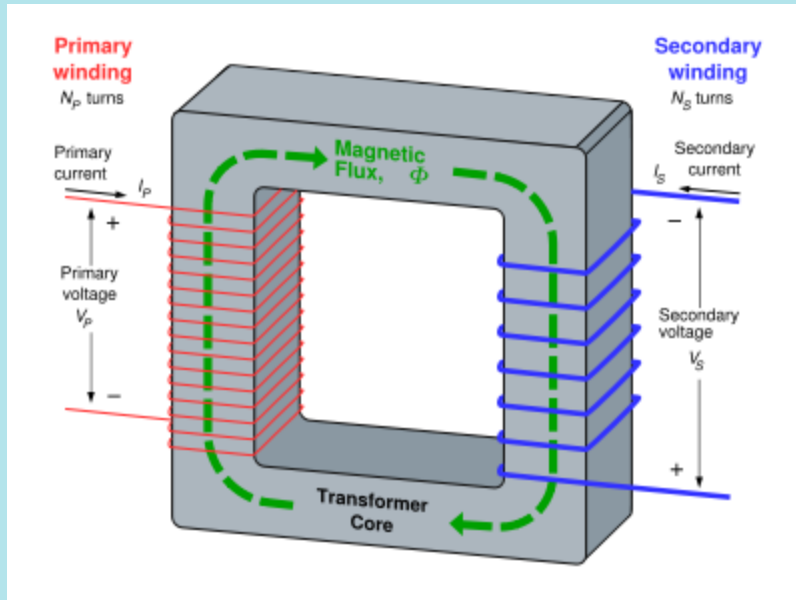
U/W indicates that two voltages are from the same winding and that both voltages are available

U*V two part winding that can be connected in series or parallel to give higher voltage but only one voltage is available at a time.

U Y/W the Y indicates a 3-phase winding connected in a WYE configuration.

Basic principles

An idealized step-down transformer showing resultant flux in the core



The **transformer** may be considered as a simple two-wheel 'gearbox' for electrical voltage and current.

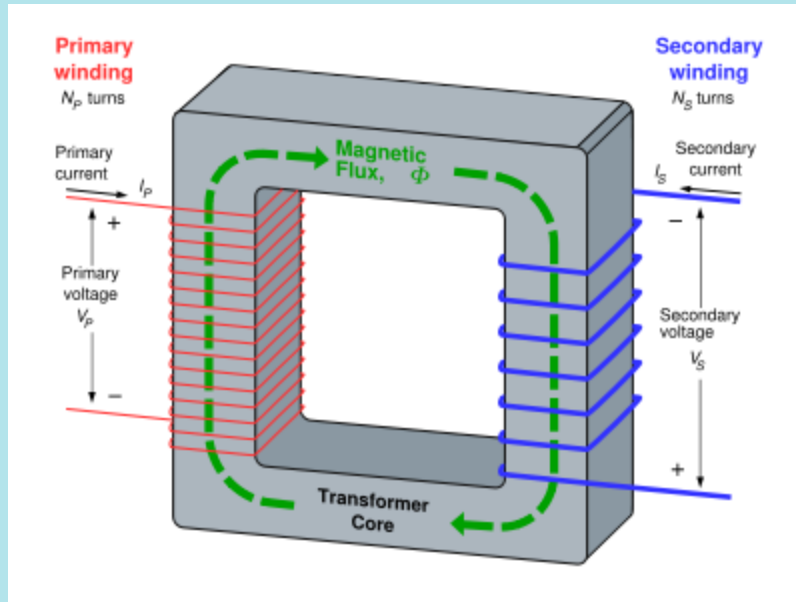
The primary winding is analogous to the input shaft

The secondary winding is analogous to the output shaft.

In this comparison, *current is equivalent to shaft speed and voltage to shaft torque.*

In a gearbox, mechanical power (speed multiplied by torque) is constant (neglecting losses) and is equivalent to electrical power (voltage multiplied by current) which is also constant. 5

An idealized step-down transformer showing resultant flux in the core



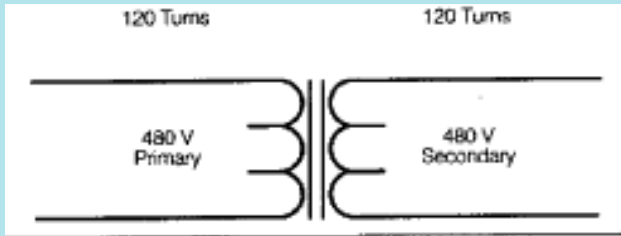
The gear ratio is equivalent to the transformer step-up or step-down ratio.

A **step-up transformer** acts analogously to a reduction gear (in which mechanical power is transferred from a small, rapidly rotating gear to a large, slowly rotating gear): it trades current (speed) for voltage (torque), by transferring power from a primary coil to a secondary coil having more turns.

A **step-down transformer** acts analogously to a multiplier gear (in which mechanical power is transferred from a large gear to a small gear): it trades voltage (torque) for current (speed), by transferring power from a primary coil to a secondary coil having fewer turns.

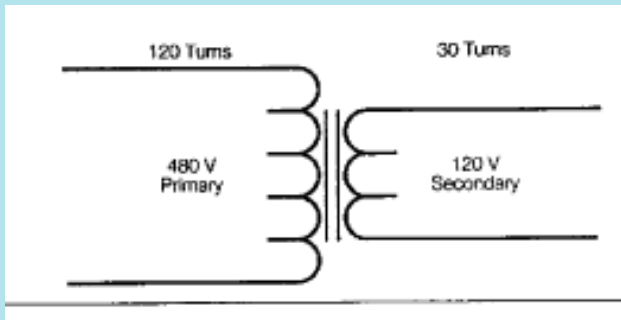
1/1 Transformer

When the primary winding and the secondary winding have the same amount of turns there is no change voltage, the ratio is 1/1 unity.



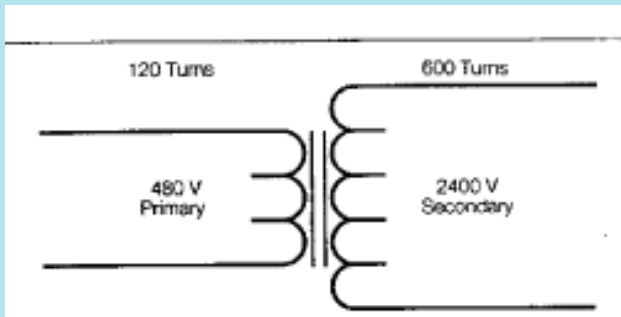
Step-Down Transformer

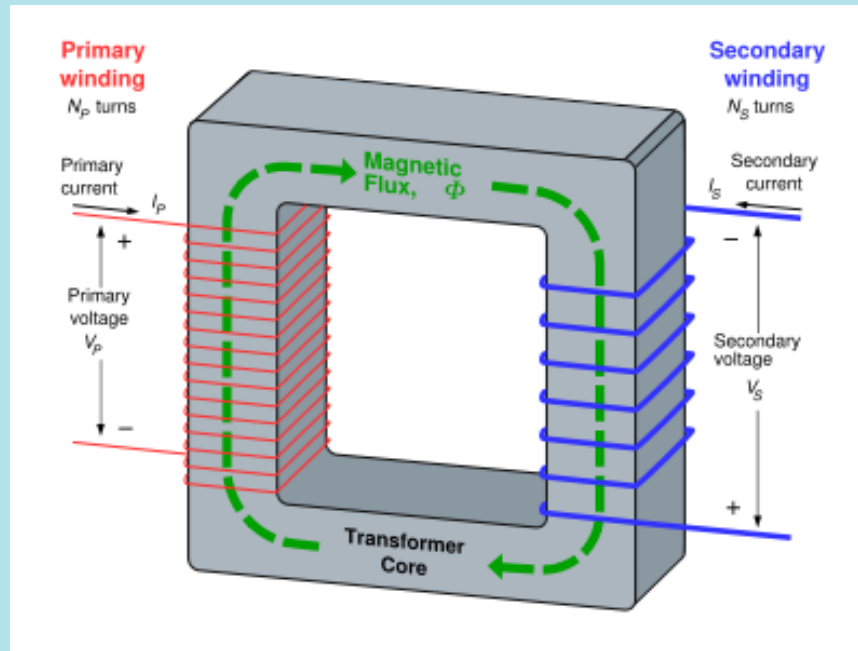
If there are fewer turns in the secondary winding than in the primary winding, the secondary voltage will be lower than the primary.



Step Up Transformers

If there are fewer turns in the primary winding than in the secondary winding, the secondary voltage will be higher than the secondary circuit.





A simple transformer consists of two electrical conductors called the **primary winding** and the **secondary winding**. If a time-varying voltage V_P is applied to the primary winding of N_P turns, a current will flow in it producing a [magnetomotive force](#) (MMF).

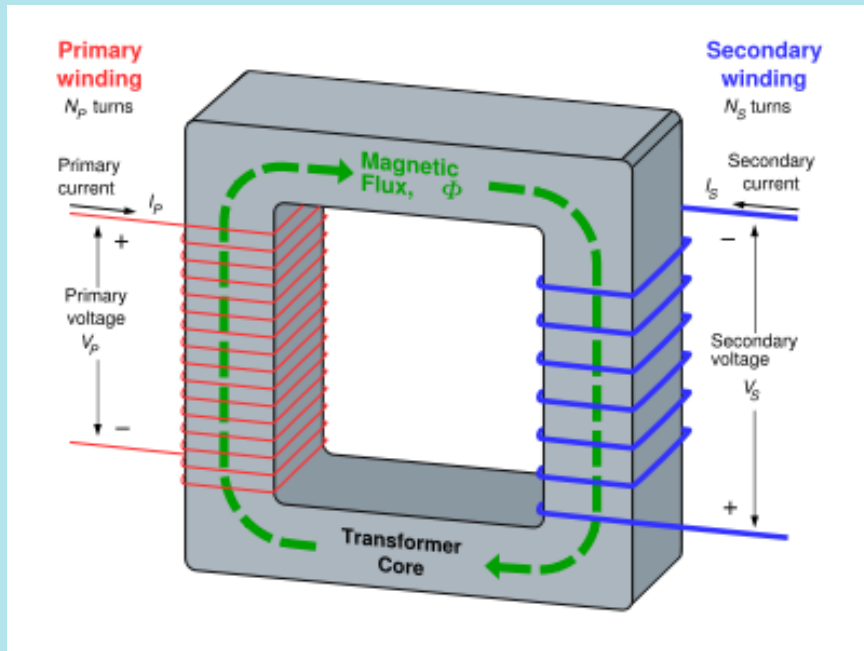
The primary MMF produces a varying [magnetic flux](#) Φ_P in the core.

In accordance with [Faraday's Law](#), the voltage induced across the primary winding is proportional to the rate of change of flux :

$$v_P = N_P \frac{d\Phi_P}{dt}$$

Similarly, the voltage induced across the secondary winding is:

$$v_S = N_S \frac{d\Phi_S}{dt}$$



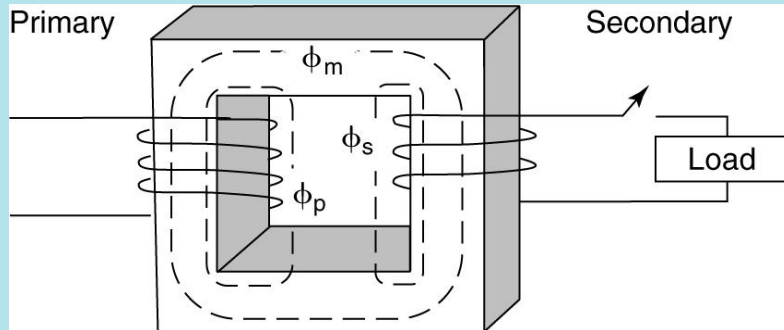
With perfect flux coupling, the flux in the secondary winding will be equal to that in the primary winding, and so we can equate Φ_P and Φ_S .

$$v_P = N_P \frac{d\Phi_P}{dt} \quad / \quad v_S = N_S \frac{d\Phi_S}{dt} \quad \text{————} \quad \frac{v_P}{v_S} = \frac{N_P}{N_S}.$$

Hence, in an ideal transformer, the ratio of the primary and secondary voltages is equal to the ratio of the number of turns in their windings, or alternatively, the voltage per turn is the same for both windings.

This leads to the most common use of the transformer: to convert electrical energy at one voltage to energy at a different voltage by means of windings with different numbers of turns.

The Universal EMF equation



Faraday's law tells:

$$e = N \frac{d\phi}{dt}$$

If we apply sinusoidal voltage to the transformer:

$$e(t) = \sqrt{2}E_{RMS}\sin(\omega t)$$

Flux is given by:

$$\phi(t) = \frac{1}{N} \int_0^t \sqrt{2}E_{RMS}\sin(\omega t) dt$$

$$\begin{aligned}\phi(t) &= \frac{-\sqrt{2}E_{RMS}}{\omega N} \cos(\omega t) = \frac{-\sqrt{2}E_{RMS}}{2\pi f N} \cos(\omega t) = -\phi_{max} \cos(\omega t) \\ E_{RMS} &= \frac{2\pi f N \phi_{max}}{\sqrt{2}} \\ E_{RMS} &= 4.44 f N \phi_{max} = 4.44 f N B_{max} A_c\end{aligned}$$

This equation demonstrates a definite relation between the voltage in a coil, the flux density, and the size of the core. The designer must make trade-offs among the variables when design a transformer.

Voltage and Current

For the ideal transformer, all the flux is confined to the iron core and thus links the primary and secondary.

$$E_{RMS} = 4.44fN\phi_{max} = 4.44fNB_{max}A_c$$



$$E_p = 4.44fN_p\phi_{max}$$

$$E_s = 4.44fN_s\phi_{max}$$



$$\frac{E_p}{E_s} = \frac{N_p}{N_s} = a$$

Turns ratio

For step-down transformer, the primary side has more turns than secondary, therefore $a > 1$;

For step-up transformer, the primary side has fewer turns than secondary, therefore $a < 1$;

Because the losses are zero in the ideal transformer, the apparent power in and out of the transformer must be the same:

$$P_{in} = P_{out} = V_p I_p = V_s I_s$$

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{1}{a}$$

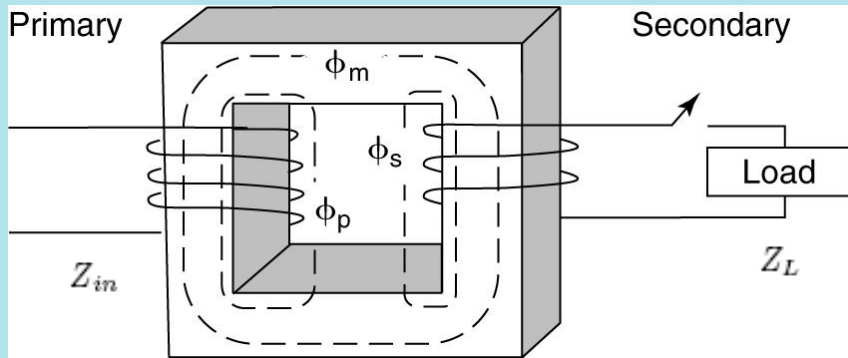
Ratio of the currents is inverse of the voltage ratio or the inverse of the turns ratio.

It makes sense: if we raise the voltage level to a load with a step-up transformer, then the secondary current drawn by the load would have to be less than the primary current, since the apparent power is constant

Example

Impedance

Due to the fact that the transformer changes the voltage and current levels in opposite directions, it also changes the apparent impedance as seen from the two sides of the transformer.



Ohm's law applied at the load:

$$Z_L = \frac{V_s}{I_s}$$

Recollect:

$$\frac{I_p}{I_s} = \frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{1}{a}$$

$$Z_L = \frac{V_p}{a I_p} = \frac{V_p}{a^2 I_p} = \frac{Z_{in}}{a^2}$$

$$Z_{in} = a^2 Z_L$$

The Reflected (referred) impedance
(the impedance looking into
the primary side of the
transformer)

When we move an impedance from the secondary to the primary side of the transformer we multiply by the turns ratio squared. When moving the impedance from the primary to the secondary, we divide it by the turns ratio squared.

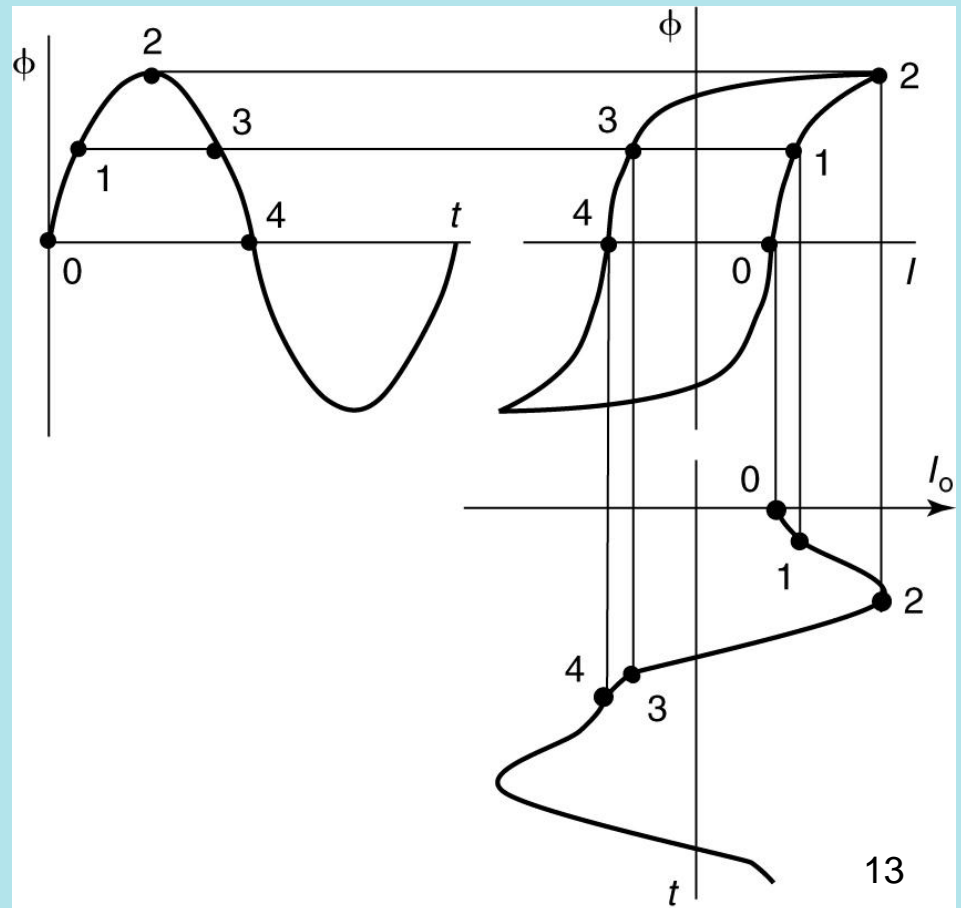
This process is called referring the impedance to the side we move it, and allows us to use transformers to match impedances between a source and a load

Exciting Current

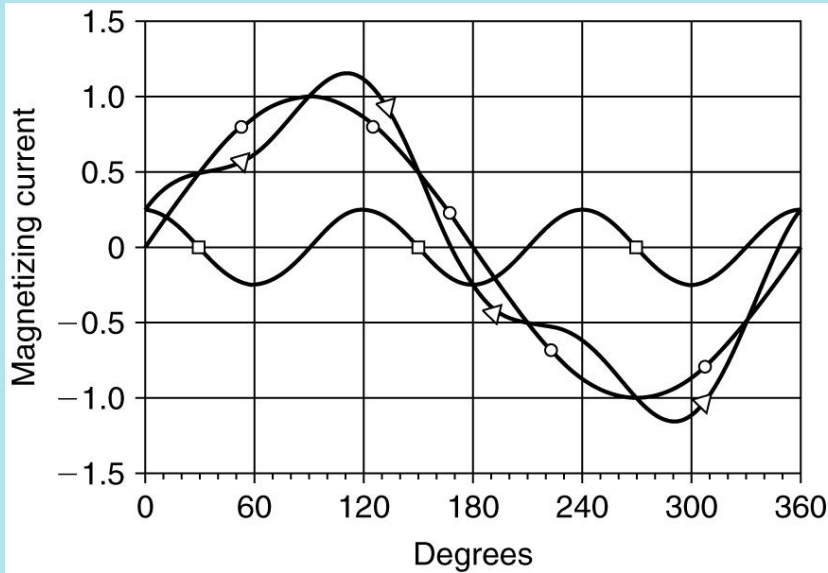
In real life we deal with real transformers which require current in the primary winding to establish the flux in the core. The current that establishes the flux is called the exciting current. Magnitude of the exciting current is usually about 1%-5% of the rated current of the primary.

According to Faraday's law if we apply a sinusoidal voltage to the transformer, then the flux will also be sinusoidal.

Due to the non-linearity of B-H curve, the current will not be sinusoidal even if the flux is sinusoidal and the current will be out of phase with flux.



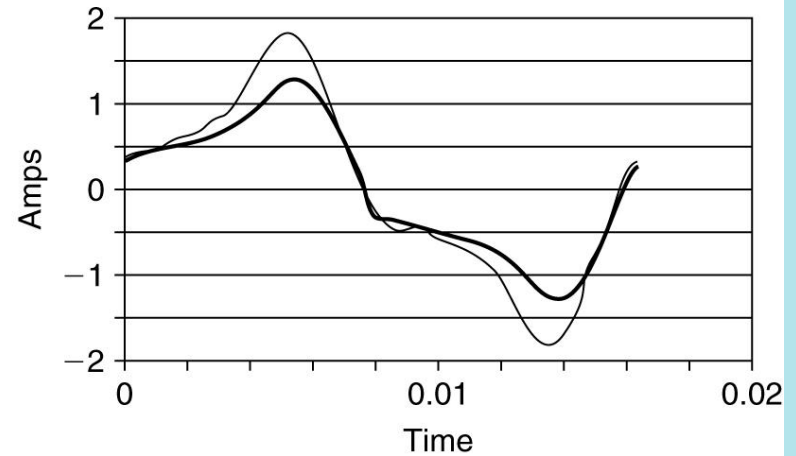
The current is not sinusoidal but it is periodic, thus can be represented by a Fourier series.



○ Fundamental □ Third harmonic △ First and third harmonics

(a)

a. Harmonic content of exciting current.



— 130 volts applied — 120 volts applied

(b)

b. Measured exciting current.

A slightly higher applied voltage causes the transformer to draw a much higher exciting current which would also increase the core losses.

RMS value of the exciting current is calculated :

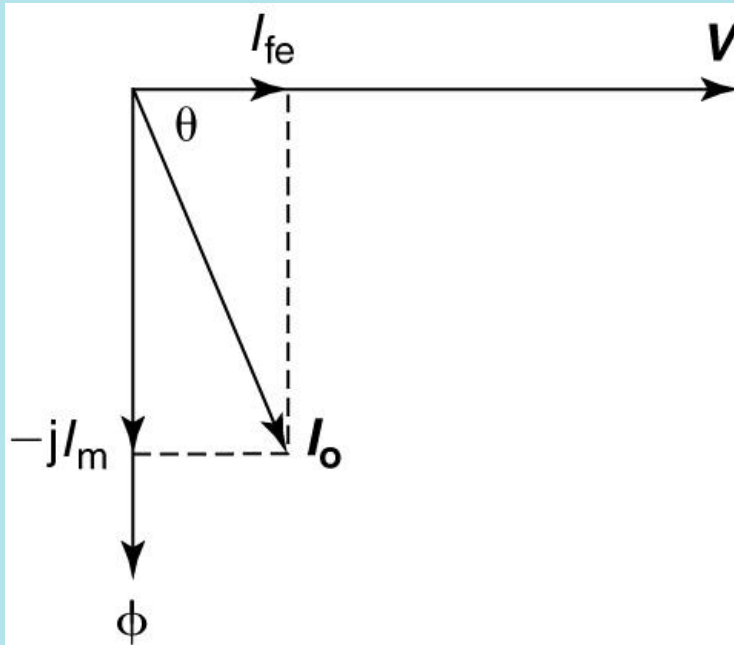
$$I_{O,RMS} = \sqrt{\frac{\int_0^{2\pi} [i_o(wt)]^2 d(wt)}{2\pi}}$$

The exciting current is not in phase with the flux.

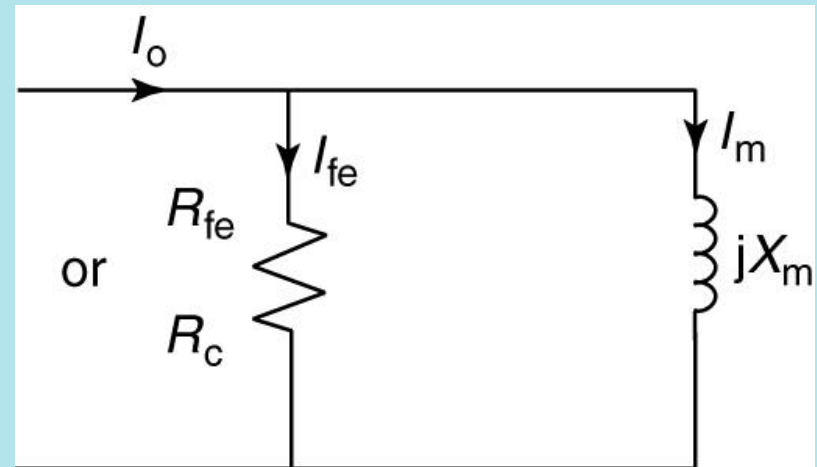
The voltage is 90 degrees ahead of the flux, since voltage is the derivative of the flux.

The exciting current phasor lies between the voltage phasor and the flux phasor, therefore the current can be separated into two components:

- One in phase with the voltage, I_{fe} . Represents real power being consumed and is called: **Core-loss current**
- One in phase with the flux, I_m . Represents reactive power and is called: **Magnetizing current**



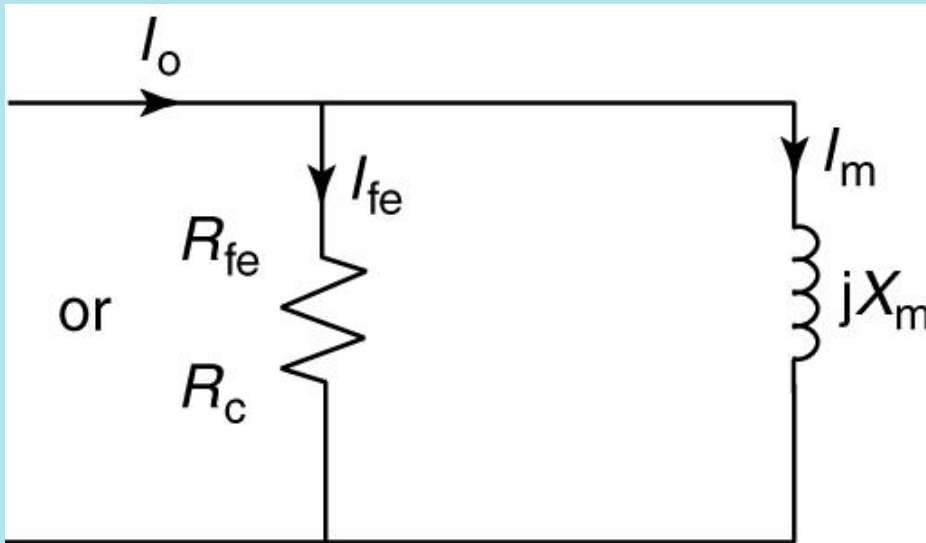
Phasor diagram of exciting current



Equivalent circuit of transformer core

The resistance R_c consumes real power corresponding to the core loss of the transformer.

The inductive reactance X_m draws the current to create the magnetic field in the transformer core.



We can calculate R_{fe} and X_m :

1. With no load on the transformer measure:
 - the RMS current into the transformer
 - the input voltage
 - the real power into transformer
2. The product of the voltage and the current gives the apparent power, S.
3. Since the real power is known, we can find the power factor and the impedance angle. The elements then can be calculated.