Noise in Communication Systems

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1. Introduction

Noise is a general term which is used to describe an unwanted signal which affects a wanted signal. These unwanted signals arise from a variety of sources which may be considered in one of two main categories:-

- *Interference, usually from a human source (man made)*
- *Naturally occurring random noise*

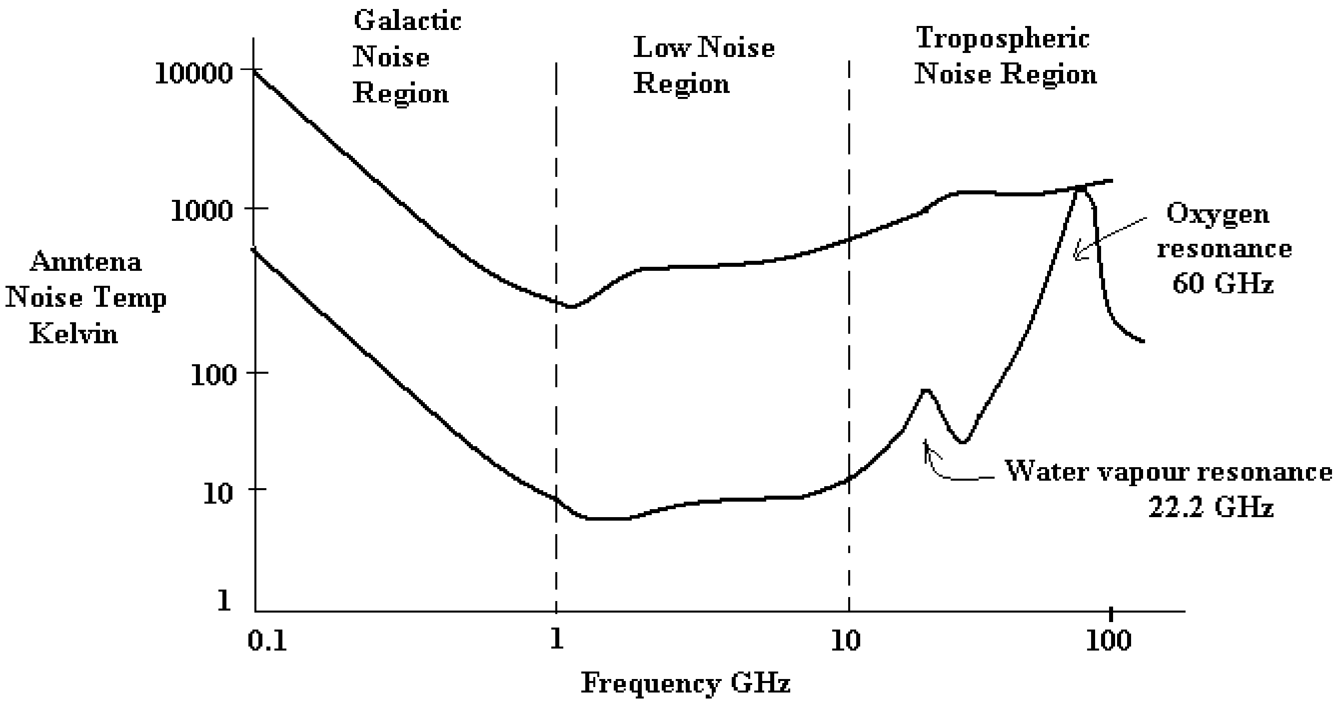
Interference

Interference arises for example, from other communication systems (cross talk), 50 Hz supplies (hum) and harmonics, switched mode power supplies, thyristor circuits, ignition (car spark plugs) motors … etc.

1. Introduction (Cont'd)

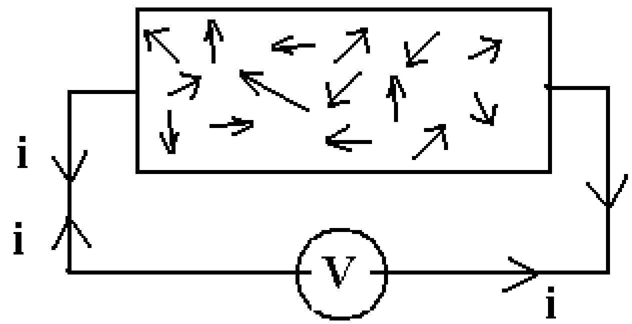
Natural Noise

Naturally occurring external noise sources include atmosphere disturbance (e.g. electric storms, lighting, ionospheric effect etc), so called 'Sky Noise' or Cosmic noise which includes noise from galaxy, solar noise and 'hot spot' due to oxygen and water vapour resonance in the earth's atmosphere.



2. Thermal Noise (Johnson Noise)

This type of noise is generated by all resistances (e.g. a resistor, semiconductor, the resistance of a resonant circuit, i.e. the real part of the impedance, cable etc).



Experimental results (by Johnson) and theoretical studies (by Nyquist) give the mean square noise voltage as

$$\bar{V}^2 = 4kTBR \ (volt^2)$$

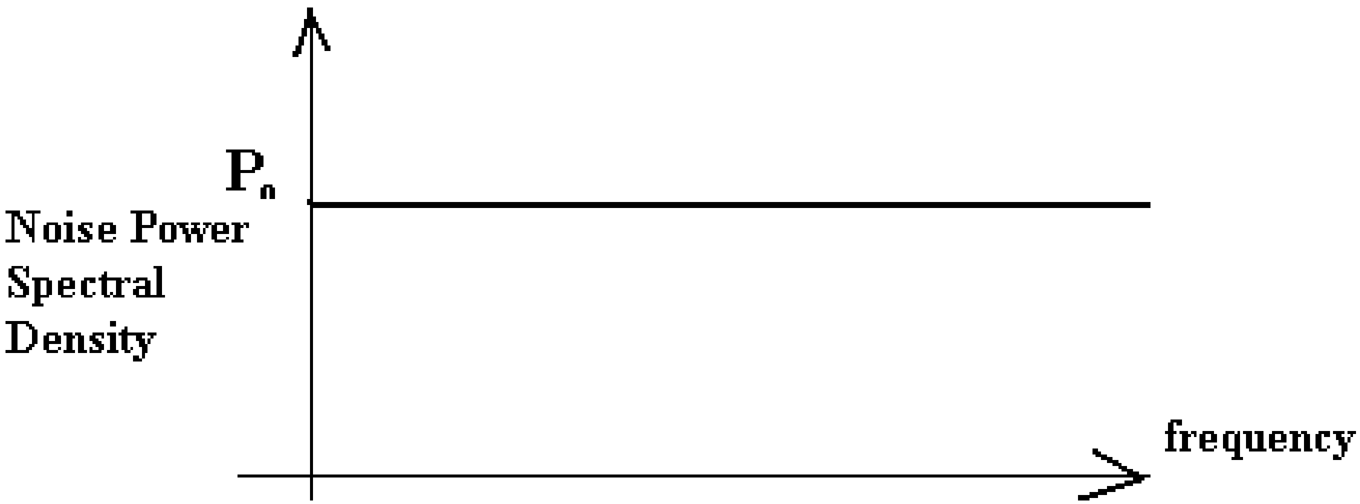
Where k = Boltzmann's constant = 1.38 x 10^{-23} Joules per K
T = absolute temperature
B = bandwidth noise measured in (Hz)
R = resistance (ohms)

2. Thermal Noise (Johnson Noise) (Cont'd)

The law relating noise power, N, to the temperature and bandwidth is

N = k TB watts

Thermal noise is often referred to as 'white noise' because it has a uniform 'spectral density'.



Uniform Noise Power Spectral Density

3. Shot Noise

- Shot noise was originally used to describe noise due to random fluctuations in electron emission from cathodes in vacuum tubes (called shot noise by analogy with lead shot).
- Shot noise also occurs in semiconductors due to the liberation of charge carriers.
- For *pn* junctions the mean square shot noise current is

$$I_n^2 = 2\left(I_{DC} + 2I_o\right)q_e B \qquad (amps)^2$$

Where

is the direct current as the *pn* junction (amps)

is the reverse saturation current (amps)

is the electron charge = 1.6 x 10-19 coulombs

B is the effective noise bandwidth (Hz)

- Shot noise is found to have a uniform spectral density as for thermal noise

4. Low Frequency or Flicker Noise

Active devices, integrated circuit, diodes, transistors etc also exhibits a low frequency noise, which is frequency dependent (i.e. non uniform) known as flicker noise or 'one – over – f' noise.

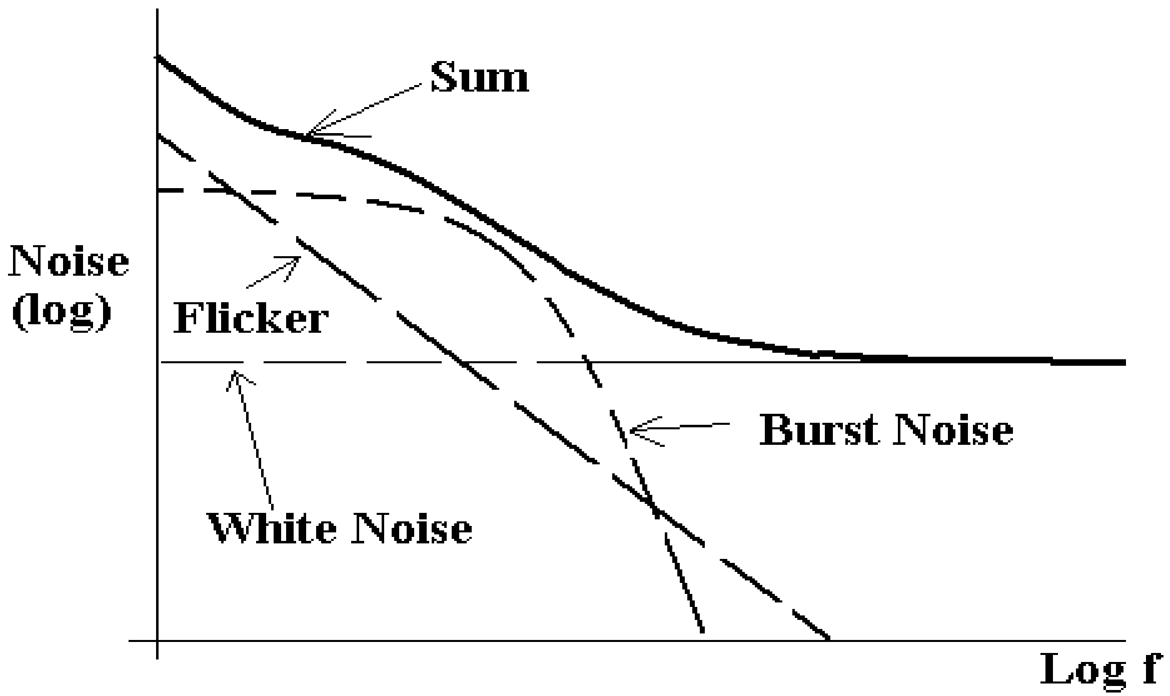
5. Excess Resistor Noise

Thermal noise in resistors does not vary with frequency, as previously noted, by many resistors also generates as additional frequency dependent noise referred to as excess noise.

6. Burst Noise or Popcorn Noise

Some semiconductors also produce burst or popcorn noise with a spectral density which is proportional to $\left(1/f\right)^2$

7. General Comments



For frequencies below a few KHz (low frequency systems), flicker and popcorn noise are the most significant, but these may be ignored at higher frequencies where 'white' noise predominates.

8. Noise Evaluation

The essence of calculations and measurements is to determine the signal power to Noise power ratio, i.e. the (S/N) ratio or (S/N) expression in dB.

$$\left(\frac{S}{N}\right)_{ratio} = \frac{S}{N}$$

$$\left(\frac{S}{N}\right)_{dB} = 10 \log_{10}\left(\frac{S}{N}\right)$$

Also recall that

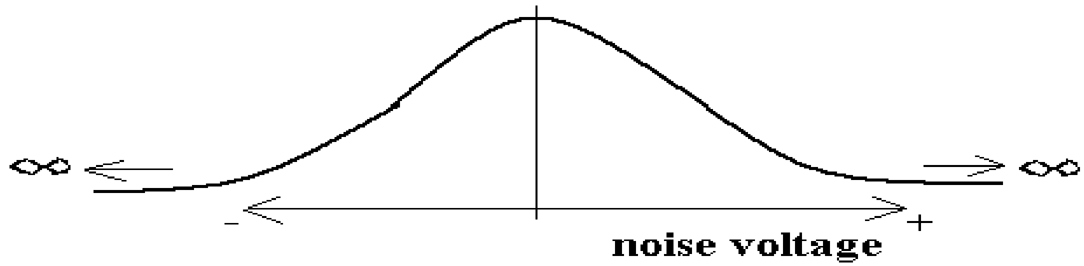
$$S_{dBm} = 10 \log_{10}\left(\frac{S(mW)}{1mW}\right)$$

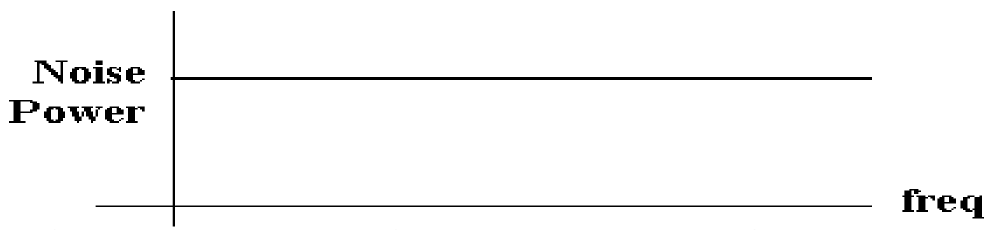
$$and \quad N_{dBm} = 10 \log_{10}\left(\frac{N(mW)}{1mW}\right)$$

$$i.e. \left(\frac{S}{N}\right)_{dB} = 10 \log_{10} S - 10 \log_{10} N$$

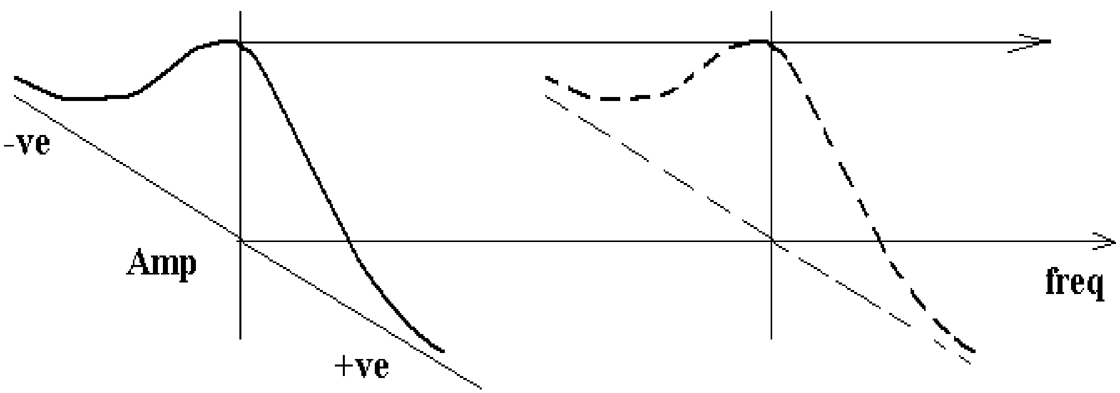
$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$

8. Noise Evaluation (Cont’d)





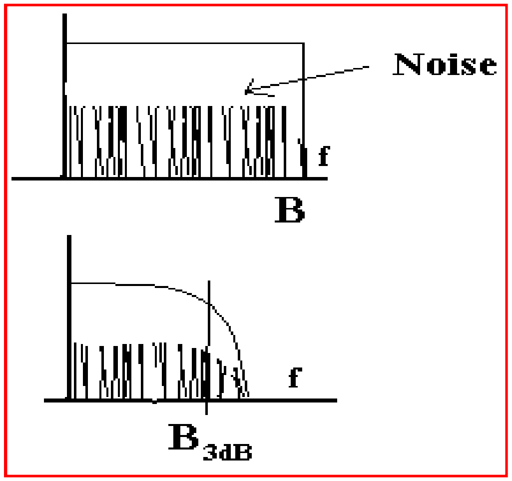
The probability of amplitude of noise at any frequency or in any band of frequencies (e.g. 1 Hz, 10Hz… 100 KHz .etc) is a Gaussian distribution.



8. Noise Evaluation (Cont'd)

Noise may be quantified in terms of noise power spectral density, p_o watts per Hz, from which Noise power N may be expressed as

$$N = p_o B_n \text{ watts}$$



Ideal low pass filter

Bandwidth B Hz = B_n

$N = p_o B_n$ watts

Practical LPF

3 dB bandwidth shown, but noise does not suddenly cease at B_{3dB}

Therefore, $Bn > B_{3dB}$, Bn depends on actual filter.

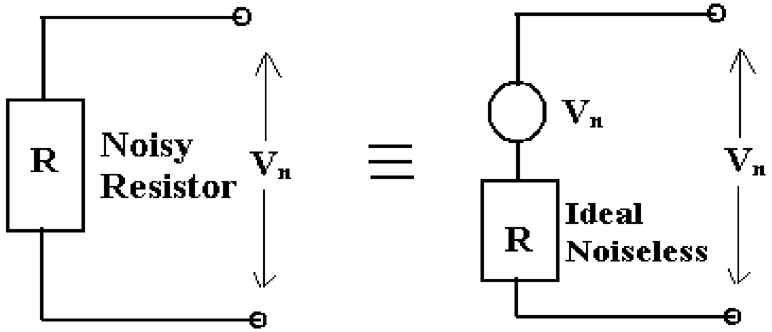
$N = p0\ B_n$

In general the equivalent noise bandwidth is > B_{3dB}.

9. Analysis of Noise In Communication Systems

Thermal Noise (Johnson noise)

This thermal noise may be represented by an equivalent circuit as shown below



$$\overline{V^2} = 4kTBR\ (volt^2)$$

(mean square value , power)

then $V_{RMS} = \sqrt{\overline{V^2}} = 2\sqrt{kTBR} = V_n$

i.e. V_n is the RMS noise voltage.

A) System BW = B Hz

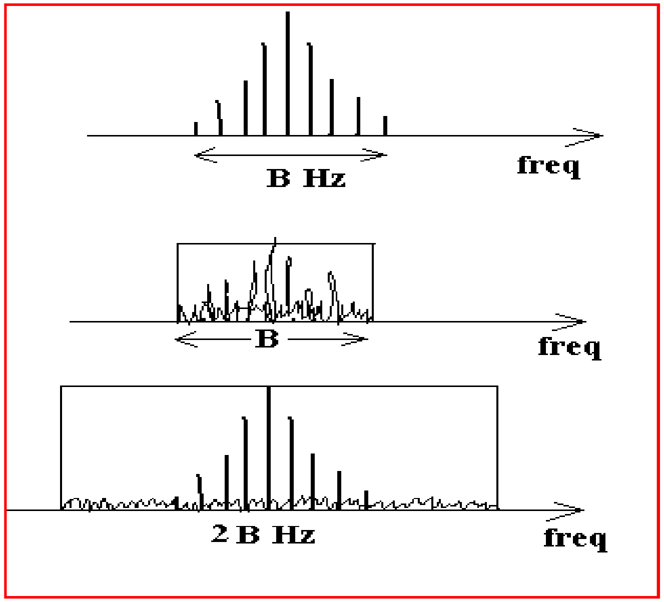
N= Constant B (watts) = KB

B) System BW

N= Constant 2B (watts) = K2B

For A, $\frac{S}{N} = \frac{S}{KB}$

For B, $\frac{S}{N} = \frac{S}{K2B}$



9. Analysis of Noise In Communication Systems (Cont'd)

Resistors in Series

Assume that R_1 at temperature T_1 and R_2 at temperature T_2, then

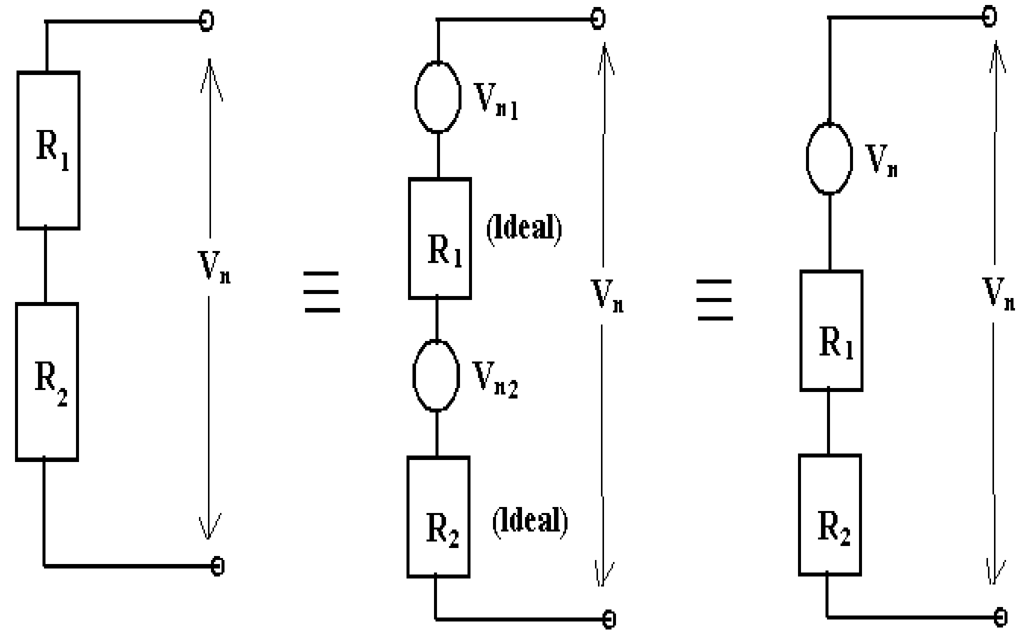
$$\overline{V_n^2} = \overline{V_{n1}^2} + \overline{V_{n2}^2}$$

$$\overline{V_{n1}^2} = 4kT_1BR_1$$

$$\overline{V_{n2}^2} = 4kT_2BR_2$$

$$\therefore \overline{V_n^2} = 4kB(T_1R_1 + T_2R_2)$$

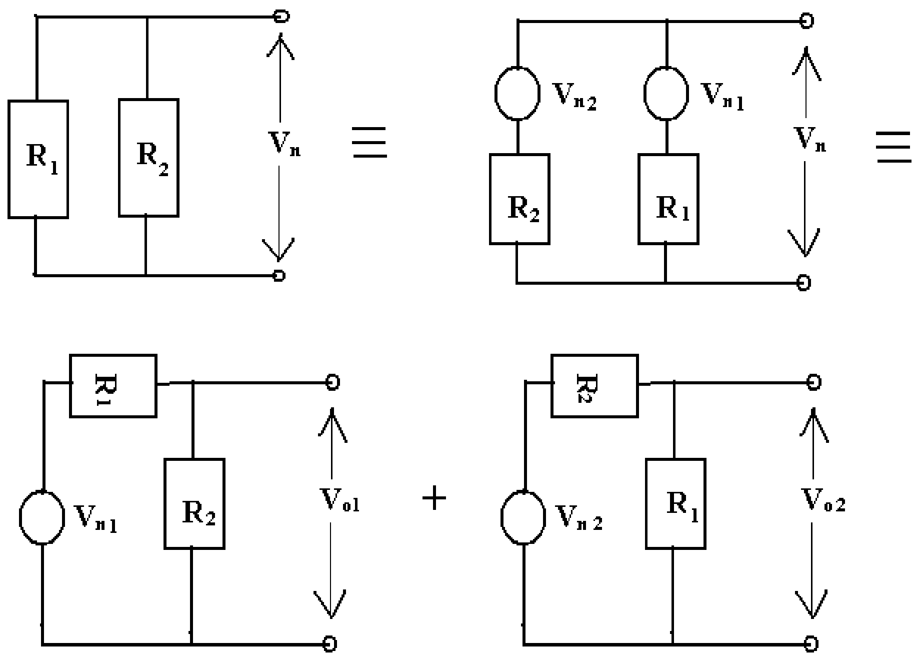
$$\overline{V_n^2} = 4kTB(R_1 + R_2)$$



i.e. The resistor in series at same temperature behave as a single resistor

9. Analysis of Noise In Communication Systems (Cont'd)

Resistance in Parallel



$$V_{o1} = V_{n1}\frac{R_2}{R_1 + R_2} \qquad V_{o2} = V_{n2}\frac{R_1}{R_1 + R_2}$$

$$\overline{V_n^{\,2}} = \overline{V_{o1}}^{\,2} + \overline{V_{o2}}^{\,2}$$

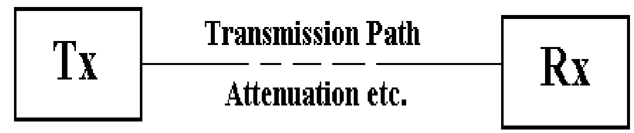
$$\overline{V_n^{\,2}} = \frac{4kB}{(R_1 + R_2)^2}\left[R_2^2\, T_1 R_1 + R_1^2\, T_2 R_2\right] \times \left(\frac{R_1 R_2}{R_1 R_2}\right)$$

$$\overline{V_n^{\,2}} = \frac{4kB\; R_1 R_2 (T_1 R_1 + T_2 R_2)}{(R_1 + R_2)^2}$$

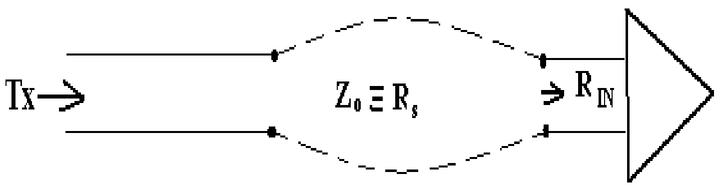
$$\overline{V_n^2} = 4kTB\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

10. Matched Communication Systems

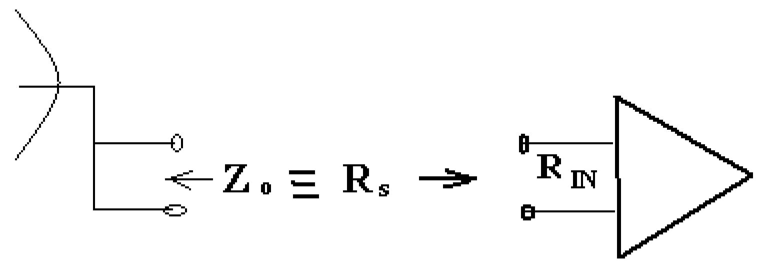
In communication systems we are usually concerned with the noise (i.e. S/N) at the receiver end of the system.



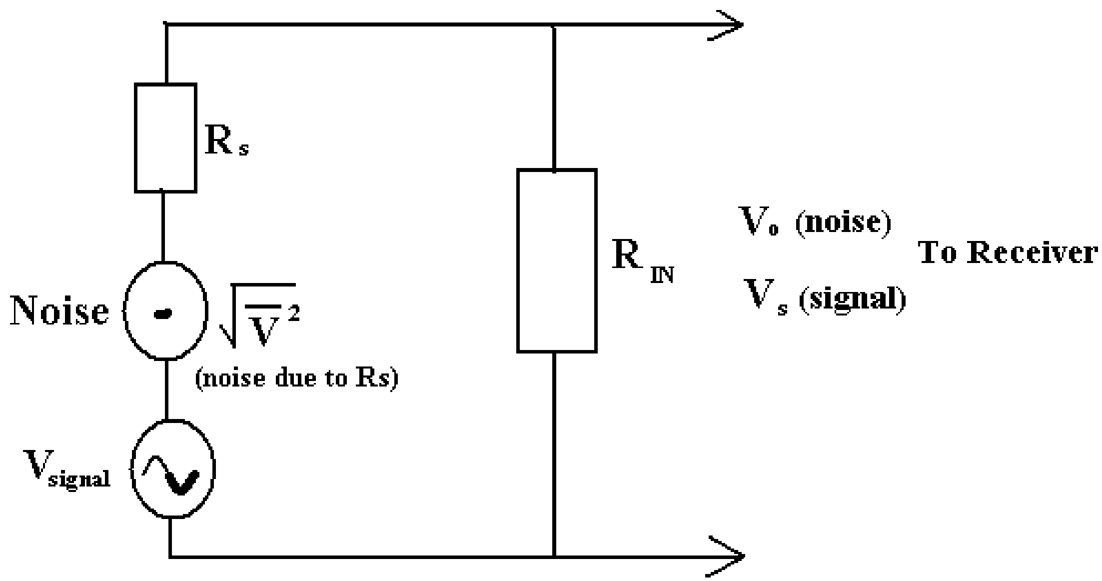
The transmission path may be for example:-



Or



An equivalent circuit, when the line is connected to the receiver is shown below.



10. Matched Communication Systems (Cont'd)

The RMS voltage output, V_o (noise) is

$$V_o(noise) = \sqrt{\overline{v^2}}\left(\frac{R_{IN}}{R_{IN} + R_S}\right)$$

Similarly, the signal voltage output due to V_{signal} at input is

$$Vs(signal) = (V_{signal})\left(\frac{R_{IN}}{R_{IN} + R_s}\right)$$

For maximum power transfer, the input R_{IN} is matched to the source R_S, i.e. R_{IN}

$= R_S = \text{R (say)}$

Then $$V_o(noise) = \sqrt{\overline{v^2}}\left(\frac{R}{2R}\right) = \frac{\sqrt{\overline{v^2}}}{2} \quad (RMS\ \ Value)$$

And signal, $V_{S(signal)} = \dfrac{V_{signal}}{2}$

11. Signal to Noise

The signal to noise ratio is given by

$$\frac{S}{N} = \frac{Signal\ Power}{Noise\ Power}$$

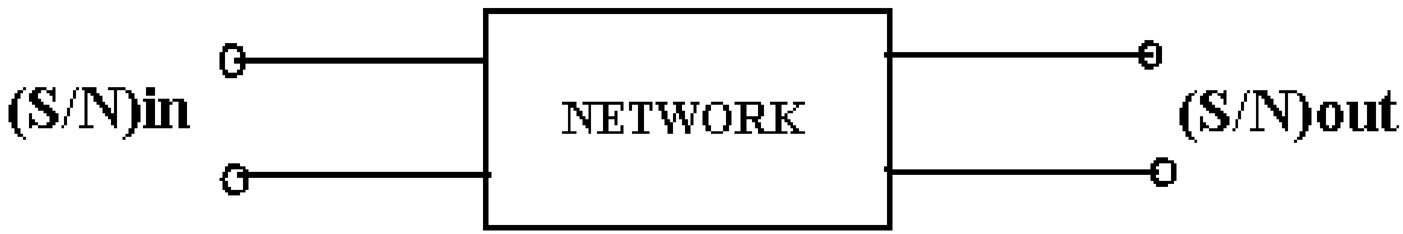
The signal to noise in dB is expressed by

$$\left(\frac{S}{N}\right)_{dB} = 10\log_{10}\left(\frac{S}{N}\right)$$

$$\left(\frac{S}{N}\right)_{dB} = S_{dBm} - N_{dBm}$$ for S and N measured in mW.

12. Noise Factor- Noise Figure

Consider the network shown below,



12. Noise Factor- Noise Figure (Cont’d)

• The amount of noise added by the network is embodied in the Noise Factor F, which is defined by

$$\text{Noise factor } F = \frac{\left(S/N\right)_{IN}}{\left(S/N\right)_{OUT}}$$

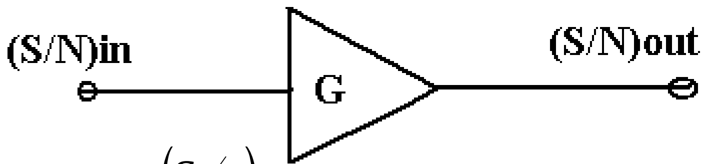
• F equals to 1 for noiseless network and in general F > 1. The noise figure in the noise factor quoted in dB

i.e. Noise Figure F dB = 10 log10 F F $\geq$ 0 dB

• The noise figure / factor is the measure of how much a network degrades the (S/N)IN, the lower the value of F, the better the network.

13. Noise Figure – Noise Factor for Active Elements

For active elements with power gain G>1, we have

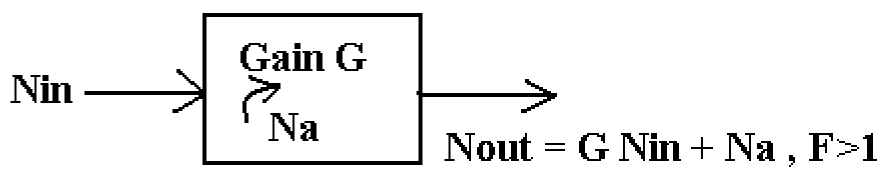


$$F = \frac{\left(S/N\right)_{IN}}{\left(S/N\right)_{OUT}} = \frac{S_{IN}}{N_{IN}}\frac{N_{OUT}}{S_{OUT}} \qquad \text{But} \qquad S_{OUT} = G\, S_{IN}$$

Therefore

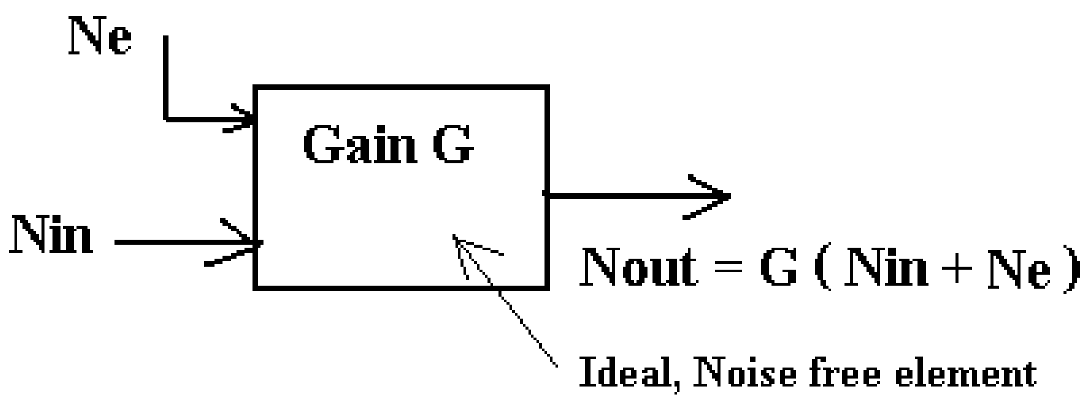
$$F = \frac{S_{IN}}{N_{IN}}\frac{N_{OUT}}{G\, S_{IN}} = \frac{N_{OUT}}{G\, N_{IN}}$$

Since in general F v> 1 , then N_{OUT} is increased by noise due to the active element i.e.



Na represents ‘added’ noise measured at the output. This added noise may be referred to the input as extra noise, i.e. as equivalent diagram is

13. Noise Figure – Noise Factor for Active Elements (Cont'd)



Ne is extra noise due to active elements referred to the input; the element is thus effectively noiseless.

$$\text{Hence } F = \frac{N_{OUT}}{G\,N_{IN}} = F = \frac{G(N_{IN} + N_e)}{G\,N_{IN}}$$

Rearranging gives,

$$N_e = (F - 1)\,N_{IN}$$

14. Noise Temperature

N_{IN} is the 'external' noise from the source i.e. $N_{IN} = kT_S B_n$

T_S is the equivalent noise temperature of the source (usually 290K).

We may also write $N_e = kT_e B_n$, where T_e is the equivalent noise temperature of the element i.e. with noise factor F and with source temperature T_S.

$$\text{i.e. } kT_e B_n = (F\text{-}1)\, kT_S B_n$$

$$\text{or } T_e = (F\text{-}1) T_S$$

15. Noise Figure – Noise Factor for Passive Elements

Since $F = \frac{S_{IN}}{N_{IN}} \frac{N_{OUT}}{S_{OUT}}$ and $N_{OUT} = N_{IN}$.

$$F = \frac{S_{IN}}{G\,S_{IN}} = \frac{1}{G}$$

If we let L denote the insertion loss (ratio) of the network i.e. insertion loss

$$L_{dB} = 10 \log L$$

Then

$$L = \frac{1}{G}$$ and hence for passive network

$$F = L$$

Also, since $T_e = (F\text{-}1)T_S$

Then for passive network

$$T_e = (L\text{-}1)T_S$$

Where T_e is the equivalent noise temperature of a passive device referred to its input.

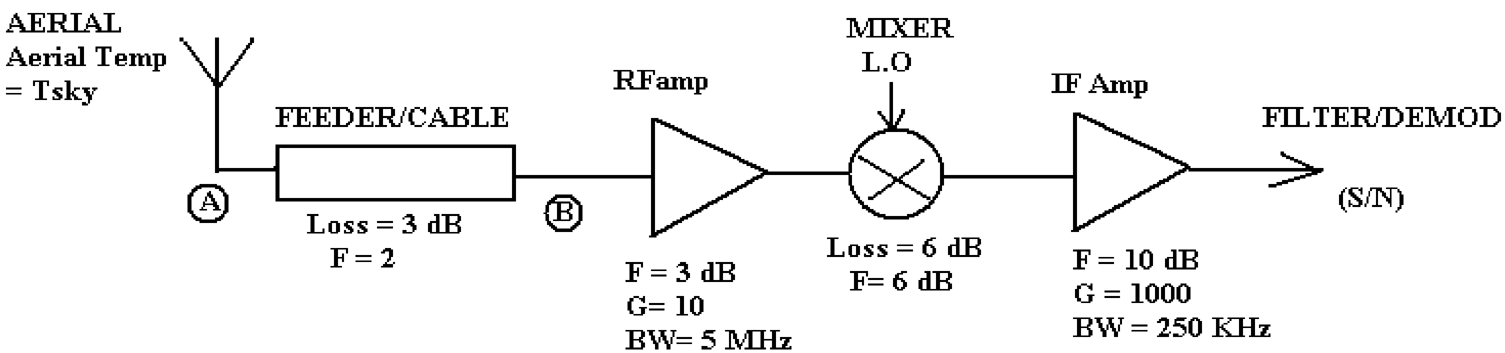
16. Review of Noise Factor – Noise Figure –Temperature

Typical values of noise temperature, noise figure and gain for various amplifiers and attenuators are given below:

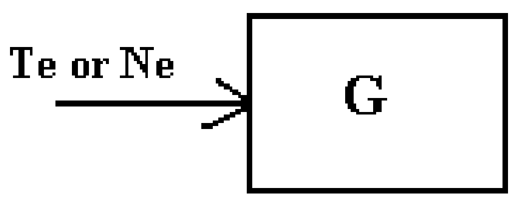
Device	**Frequency**	T_e **(K)**	F_{dB} **(dB)**	**Gain (dB)**
Maser Amplifier	9 GHz	4	0.06	20
Ga As Fet amp	9 GHz	330	303	6
Ga As Fet amp	1 GHz	110	1.4	12
Silicon Transistor	400 MHz	420	3.9	13
L C Amp	10 MHz	1160	7.0	50
Type N cable	1 GHz		2.0	2.0

17. Cascaded Network

A receiver systems usually consists of a number of passive or active elements connected in series. A typical receiver block diagram is shown below, with example



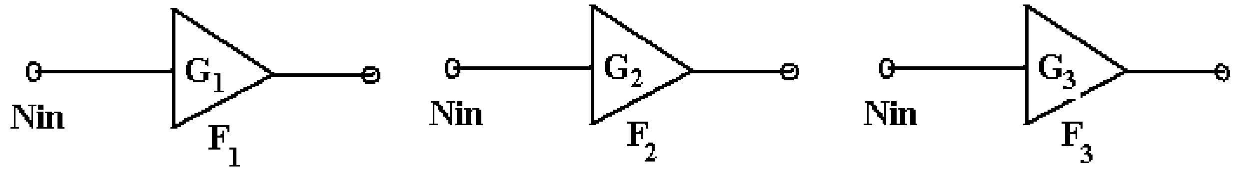
In order to determine the (S/N) at the input, the overall receiver noise figure or noise temperature must be determined. In order to do this all the noise must be referred to the same point in the receiver, for example to A, the feeder input or B, the input to the first amplifier.



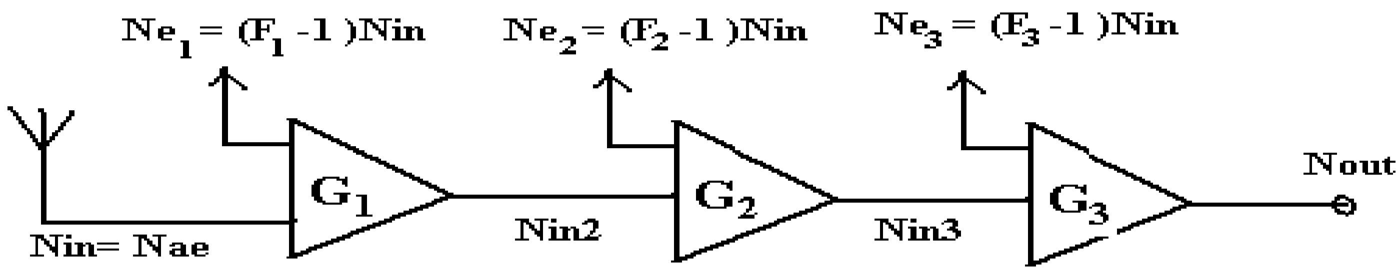
T_e or N_e is the noise referred to the input.

18. System Noise Figure

Assume that a system comprises the elements shown below,



Assume that these are now cascaded and connected to an aerial at the input, with $N_{IN} = N_{ae}$ from the aerial.



Now,
$$N_{OUT} = G_3\left(N_{IN3} + N_{e3}\right)$$
$$= G_3\left(N_{IN3} + \left(F_3 - 1\right)N_{IN}\right)$$

Since
$$N_{IN3} = G_2\left(N_{IN2} + N_{e2}\right) = G_2\left(N_{IN2} + \left(F_2 - 1\right)N_{IN}\right)$$

similarly
$$N_{IN\,2} = G_1\left(N_{ae} + \left(F_1 - 1\right)N_{IN}\right)$$

18. System Noise Figure (Cont'd)

$$N_{OUT} = G_3\left[G_2\left[G_1 N_{ae} + G_1\left(F_1 - 1\right)N_{IN}\right] + G_2\left(F_2 - 1\right)N_{IN}\right] + G_3\left(F_3 - 1\right)N_{IN}$$

The overall system Noise Factor is

$$F_{sys} = \frac{N_{OUT}}{GN_{IN}} = \frac{N_{OUT}}{G_1 G_2 G_3 N_{ae}}$$

$$= 1 + \left(F_1 - 1\right)\frac{N_{IN}}{N_{ae}} + \frac{\left(F_2 - 1\right)}{G_1}\frac{N_{IN}}{N_{ae}} + \frac{\left(F_3 - 1\right)}{G_1 G_2}\frac{N_{IN}}{N_{ae}}$$

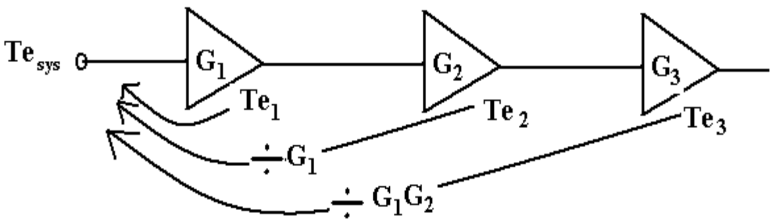
If we assume N_{ae} is $\approx N_{IN}$, i.e. we would measure and specify F_{sys} under similar conditions as F_1, F_2 etc (i.e. at 290 K), then for n elements in cascade.

$$F_{sys} = F_1 + \frac{\left(F_2 - 1\right)}{G_1} + \frac{\left(F_3 - 1\right)}{G_1 G_2} + \frac{\left(F_4 - 1\right)}{G_1 G_2 G_3} + \ldots\ldots\ldots + \frac{\left(F_n - 1\right)}{G_1 G_2 \ldots\ldots\ldots G_{n-1}}$$

The equation is called FRIIS Formula.

19. System Noise Temperature

Since $T_e = (L-1)T_S$, i.e. $F = 1 + \frac{T_e}{T_s}$



Then

$$F_{sys} = 1 + \frac{T_{e\,sys}}{T_s}$$

$\left\{ \begin{array}{l} where\, T_{e\ sys}\ is\ the\ equivalent\ Noise\ temperature\ of\ the\ system \\ and\, T_s\ is\ the\ noise\ temperature\ of\ the\ source \end{array} \right.$

and

$$\left(1 + \frac{T_{e\,sys}}{T_s}\right) = \left(1 + \frac{T_{e\,1}}{T_s}\right) + \frac{\left(1 + \frac{T_{e\,2}}{T_s} - 1\right)}{G_1} + \ldots etc$$

$$i.e.\ from\ F_{sys} = F_1 + \frac{(F_2 - 1)}{G_1} + \ldots\ldots etc$$

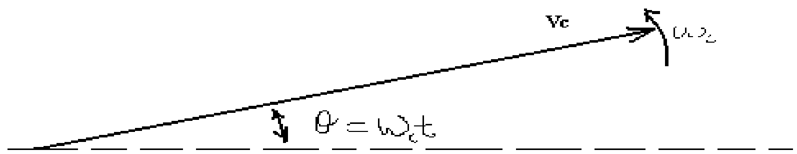
which gives

$$T_{e\,sys} = T_{e1} + \frac{T_{e\,2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \frac{T_{e4}}{G_1 G_2 G_3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

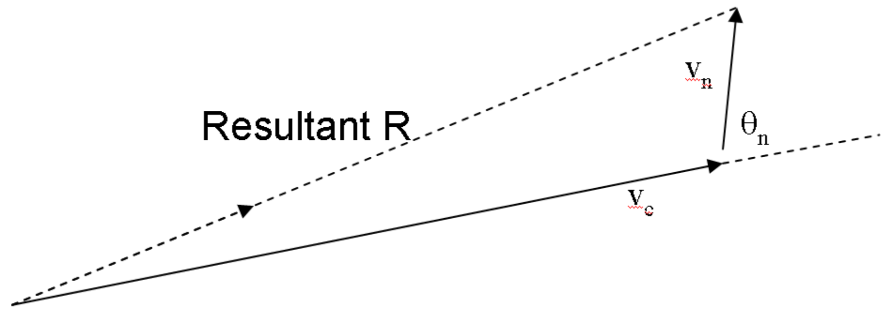
20. Algebraic Representation of Noise

Phasor Representation of Signal and Noise

The general carrier signal VcCosWct may be represented as a phasor at any instant in time as shown below:



If we now consider a carrier with a noise voltage with "peak" value superimposed we may represents this as:



Both Vn and θ_n are random variables, the above phasor diagram represents a snapshot at some instant in time.