UNIT - 4

FREQUENCY-SHIFT KEYING

FSK is a form of constant-amplitude angle modulation similar to standard frequency modulation (FM) except the modulating signal is a binary signal that varies between two discrete voltage levels rather than a continuously changing analog waveform.

Consequently, FSK is sometimes called *binary FSK* (BFSK). The general expression for FSK is

$$v_{fsk}(t) = V_c \cos\{2\pi[f_c + v_m(t)\,\Delta f]t\} \tag{2.13}$$

where

$v_{fsk}(t)$ = binary FSK waveform
V_c = peak analog carrier amplitude (volts)
f_c = analog carrier center frequency (hertz)
Δf = peak change (shift) in the analog carrier frequency

(hertz)
$v_m(t)$ = binary input (modulating) signal (volts)

From Equation 2.13, it can be seen that the peak shift in the carrier frequency (Δf) is proportional to the amplitude of the binary input signal ($v_m[t]$), and the direction of the shift is determined by the polarity.

The modulating signal is a normalized binary waveform where a logic 1 = + 1 V and a logic 0 = -1 V. Thus, for a logic 1 input, $v_m(t) = +1$, Equation 2.13 can be rewritten as

$$v_{fsk}(t) = V_c \cos[2\pi(f_c + \Delta f)t]$$

For a logic 0 input, $v_m(t) = -1$, Equation 2.13 becomes

$$v_{fsk}(t) = V_c \cos[2\pi(f_c - \Delta f)t]$$

With binary FSK, the carrier center frequency (f_c) is shifted (deviated) up and down in the frequency domain by the binary input signal as shown in Figure 2-3.

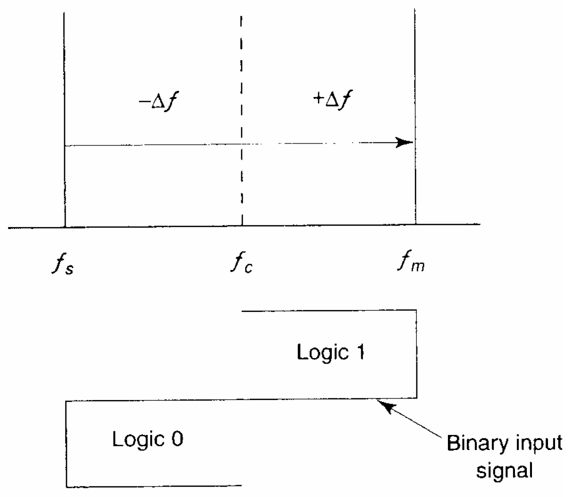


FIGURE 2-3 FSK in the frequency domain

As the binary input signal changes from a logic 0 to a logic 1 and vice versa, the output frequency shifts between two frequencies: a mark, or logic 1 frequency (f_m), and a space, or logic 0 frequency (f_s). The mark and space frequencies are separated from the carrier frequency by the peak frequency deviation (Δf) and from each other by 2 Δf.

Frequency deviation is illustrated in Figure 2-3 and expressed mathematically as

$$\Delta f = |f_m - f_s| / 2 \qquad (2.14)$$

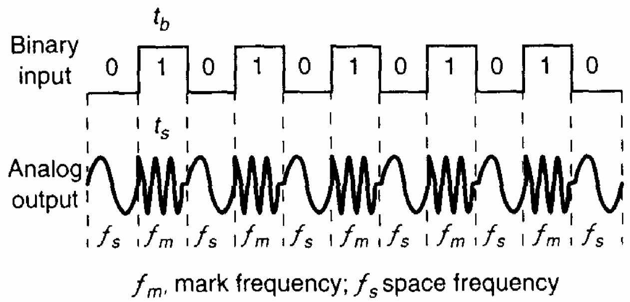
where Δf = frequency deviation (hertz)
$|f_m - f_s|$ = absolute difference between the mark and space frequencies (hertz)

Figure 2-4a shows in the time domain the binary input to an FSK modulator and the corresponding FSK output.

When the binary input (f_b) changes from a logic 1 to a logic 0 and vice versa, the FSK output frequency shifts from a mark (f_m) to a space (f_s) frequency and vice versa.

In Figure 2-4a, the mark frequency is the higher frequency ($f_c + \Delta f$) and the space frequency is the lower frequency ($f_c - \Delta f$), although this relationship could be just the opposite.

Figure 2-4b shows the truth table for a binary FSK modulator. The truth table shows the input and output possibilities for a given digital modulation scheme.



f_m, mark frequency; f_s space frequency

(a)

binary input	frequency output
0	space (f_s)
1	mark (f_m)

(b)

FIGURE 2-4 FSK in the time domain: (a) waveform: (b) truth table

FSK Bit Rate, Baud, and Bandwidth

In Figure 2-4a, it can be seen that the time of one bit *(t_b)* is the same as the time the FSK output is a mark of space frequency *(t_s)*. Thus, the bit time equals the time of an FSK signaling element, and the bit rate equals the baud.

The baud for binary FSK can also be determined by substituting N = 1 in Equation 2.11:

$$\text{baud} = f_b / 1 = f_b$$

The minimum bandwidth for FSK is given as

$$\begin{aligned} B &= |(f_s - f_b) - (f_m - f_b)| \\ &= |(f_s - f_m)| + 2f_b \end{aligned}$$

and since $|(f_s - f_m)|$ equals $2\Delta f$, the minimum bandwidth can be approximated as

$$B = 2(\Delta f + f_b) \qquad (2.15)$$

where

B= minimum Nyquist bandwidth (hertz)
Δf= frequency deviation $|(f_m - f_s)|$ (hertz)
f_b = input bit rate (bps)

Example 2-2

Determine (a) the peak frequency deviation, (b) minimum bandwidth, and (c) baud for a binary FSK signal with a mark frequency of 49 kHz, a space frequency of 51 kHz, and an input bit rate of 2 kbps.

Solution

a. The peak frequency deviation is determined from Equation 2.14:

$$\Delta f = |149\text{kHz} - 51\text{ kHz}| / 2 = 1\text{ kHz}$$

b. The minimum bandwidth is determined from Equation 2.15:

$$\begin{aligned} B &= 2(1000 + 2000) \\ &= 6\text{ kHz} \end{aligned}$$

c. For FSK, $N = 1$, and the baud is determined from Equation 2.11 as

$$\text{baud} = 2000 / 1 = 2000$$

Bessel functions can also be used to determine the approximate bandwidth for an FSK wave. As shown in Figure 2-5, the fastest rate of change (highest fundamental frequency) in a non-return-to-zero (NRZ) binary signal occurs when alternating 1s and 0s are occurring (i.e., a square wave).

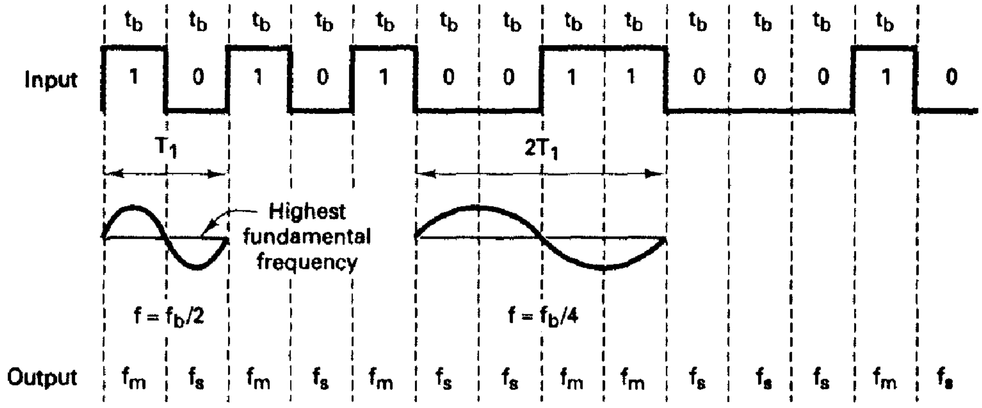


FIGURE 9-5 FSK modulator, t_b, time of one bit = $1/f_b$; f_m mark frequency; f_s, space frequency; T_1, period of shortest cycle; $1/T_1$, fundamental frequency of binary square wave; f_b, input bit rate (bps)

Since it takes a high and a low to produce a cycle, the highest fundamental frequency present in a square wave equals the repetition rate of the square wave, which with a binary signal is equal to half the bit rate. Therefore,

$$f_a = f_b / 2 \qquad (2.16)$$

where

f_a = highest fundamental frequency of the binary input signal (hertz)

f_b = input bit rate (bps)

The formula used for modulation index in FM is also valid for FSK; thus,

$$h = \Delta f / f_a \quad \text{(unitless)} \qquad (2.17)$$

where

h = FM modulation index called the h-factor in FSK

f_o = fundamental frequency of the binary modulating signal (hertz)

Δf = peak frequency deviation (hertz)

The peak frequency deviation in FSK is constant and always at its maximum value, and the highest fundamental frequency is equal to half the incoming bit rate. Thus,

$$h = \frac{\frac{|f_m - f_s|}{2}}{\frac{f_b}{2}}$$

or

$$h = \frac{|f_m - f_s|}{f_b} \qquad (2.18)$$

where
h = h-factor (unitless)
f_m = mark frequency (hertz)
f_s = space frequency (hertz)
f_b = bit rate (bits per second)

Example

Using a Bessel table, determine the minimum bandwidth for the same FSK signal described in Example 2-1 with a mark frequency of 49 kHz, a space frequency of 51 kHz, and an input bit rate of 2 kbps.

Solution The modulation index is found by substituting into Equation 2.17:

h= |49 kHz - 51 kHz| / 2 kbps = 1
= 2 kHz / 2 kbps

From a Bessel table, three sets of significant sidebands are produced for a modulation index of one. Therefore, the bandwidth can be determined as follows:

B = 2(3 x 1000)

$$= 6000 \text{ Hz}$$

The bandwidth determined in Example 2-3 using the Bessel table is identical to the bandwidth determined in Example 2-2.

Modulation Index	Carrier	Side Frequency Pairs													
m	J_0	J_1	J_2	J_3	J_4	J_5	J_6	J_7	J_8	J_9	J_{10}	J_{11}	J_{12}	J_{13}	J_{14}
0.00	1.00	—	—	—	—	—	—	—	—	—	—	—	—	—	—
0.25	0.98	0.12	—	—	—	—	—	—	—	—	—	—	—	—	—
0.5	0.94	0.24	0.03	—	—	—	—	—	—	—	—	—	—	—	—
1.0	0.77	0.44	0.11	0.02	—	—	—	—	—	—	—	—	—	—	—
1.5	0.51	0.56	0.23	0.06	0.01	—	—	—	—	—	—	—	—	—	—
2.0	0.22	0.58	0.35	0.13	0.03	—	—	—	—	—	—	—	—	—	—
2.4	0	0.52	0.43	0.20	0.06	0.02	—	—	—	—	—	—	—	—	—
2.5	−0.05	0.50	0.45	0.22	0.07	0.02	0.01	—	—	—	—	—	—	—	—
3.0	−0.26	0.34	0.49	0.31	0.13	0.04	0.01	—	—	—	—	—	—	—	—
4.0	−0.40	−0.07	0.36	0.43	0.28	0.13	0.05	0.02	—	—	—	—	—	—	—
5.0	−0.18	−0.33	0.05	0.36	0.39	0.26	0.13	0.05	0.02	—	—	—	—	—	—
5.45	0	−0.34	−0.12	0.26	0.40	0.32	0.19	0.09	0.03	0.01	—	—	—	—	—
6.0	0.15	−0.28	−0.24	0.11	0.36	0.36	0.25	0.13	0.06	0.02	—	—	—	—	—
7.0	0.30	0.00	−0.30	−0.17	0.16	0.35	0.34	0.23	0.13	0.06	0.02	—	—	—	—
8.0	0.17	0.23	−0.11	−0.29	−0.10	0.19	0.34	0.32	0.22	0.13	0.06	0.03	—	—	—
8.65	0	0.27	0.06	−0.24	−0.23	0.03	0.26	0.34	0.28	0.18	0.10	0.05	0.02	—	—
9.0	−0.09	0.25	0.14	−0.18	−0.27	−0.06	0.20	0.33	0.31	0.21	0.12	0.06	0.03	0.01	—
10.0	−0.25	0.05	0.25	0.06	−0.22	−0.23	−0.01	0.22	0.32	0.29	0.21	0.12	0.06	0.03	0.01

FSK Transmitter

Figure 2-6 shows a simplified binary FSK modulator, which is very similar to a conventional FM modulator and is very often a voltage-controlled oscillator (VCO).

The center frequency (f_c) is chosen such that it falls halfway between the mark and space frequencies.

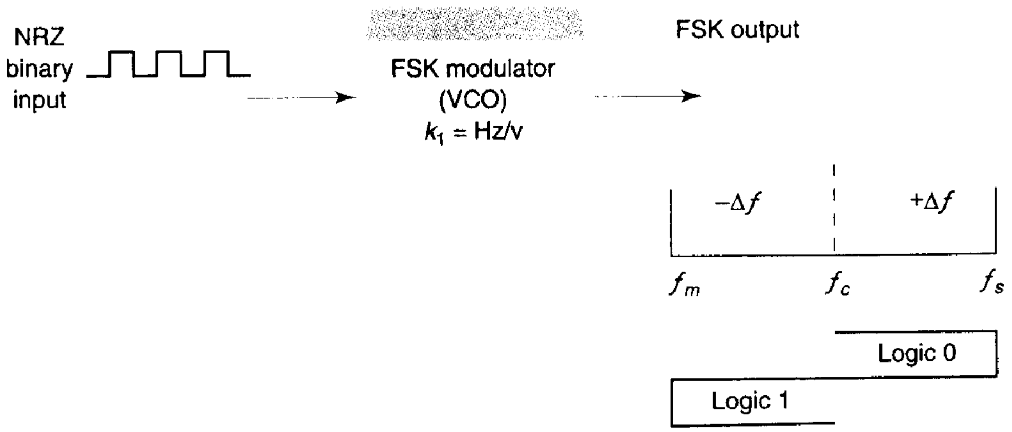


FIGURE 2-6 FSK modulator

A logic 1 input shifts the VCO output to the mark frequency, and a logic 0 input shifts the VCO output to the space frequency.

Consequently, as the binary input signal changes back and forth between logic 1 and logic 0 conditions, the VCO output shifts or deviates back and forth between the mark and space frequencies.

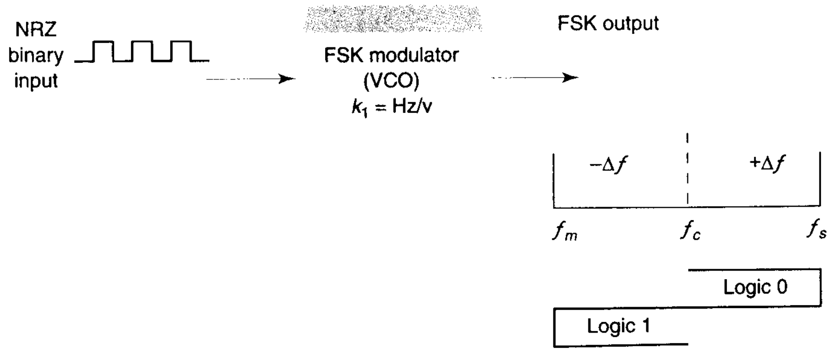


FIGURE 2-6 FSK modulator

A VCO-FSK modulator can be operated in the sweep mode where the peak frequency deviation is simply the product of the binary input voltage and the deviation sensitivity of the VCO.

With the sweep mode of modulation, the frequency deviation is expressed mathematically as

$$\Delta f = v_m(t) k_l \qquad (2\text{-}19)$$

$v_m(t)$ = peak binary modulating-signal voltage (volts)
k_l = deviation sensitivity (hertz per volt).

FSK Receiver

FSK demodulation is quite simple with a circuit such as the one shown in Figure 2-7.

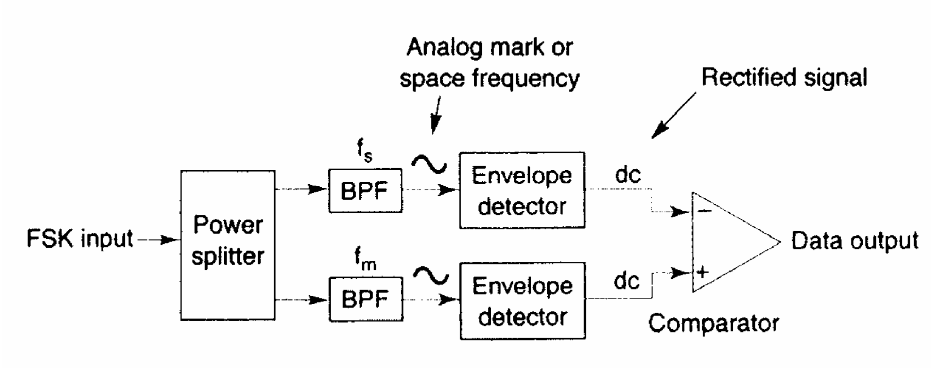


FIGURE 2-7 Noncoherent FSK demodulator

The FSK input signal is simultaneously applied to the inputs of both bandpass filters (BPFs) through a power splitter.

The respective filter passes only the mark or only the space frequency on to its respective envelope detector.

The envelope detectors, in turn, indicate the total power in each passband, and the comparator responds to the largest of the two powers.

This type of FSK detection is referred to as noncoherent detection.

Figure 2-8 shows the block diagram for a coherent FSK receiver.

The incoming FSK signal is multiplied by a recovered carrier signal that has the exact same frequency and phase as the transmitter reference.

However, the two transmitted frequencies (the mark and space frequencies) are not generally continuous; it is not practical to reproduce a local reference that is coherent with both of them. Consequently, coherent FSK detection is seldom used.

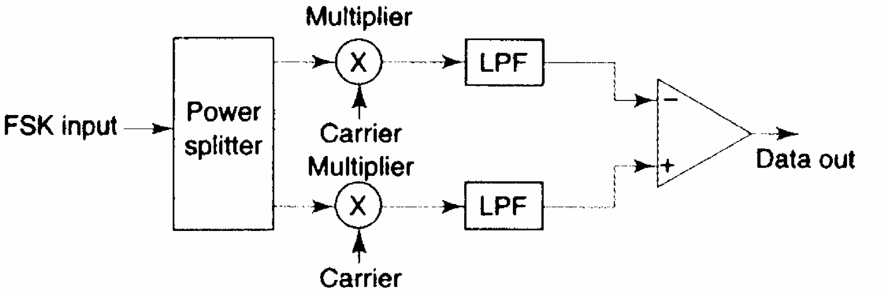


FIGURE 2-8 Coherent FSK demodulator

The most common circuit used for demodulating binary FSK signals is the *phaselocked loop* (PLL), which is shown in block diagram form in Figure 2-9.

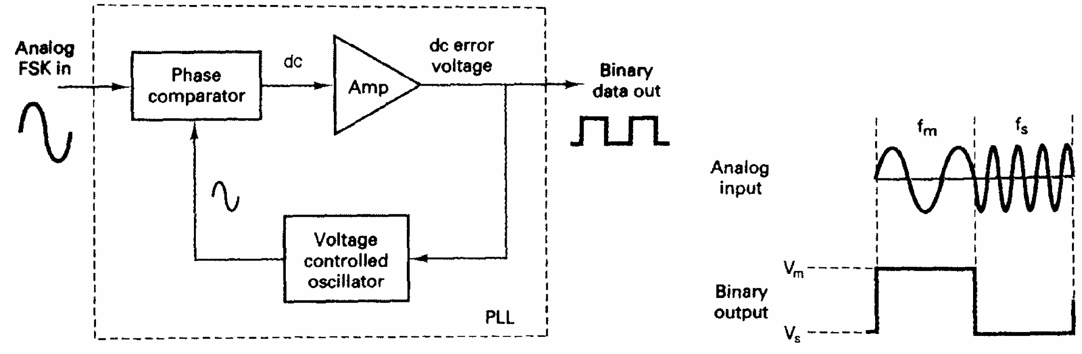


FIGURE 2-9 PLL-FSK demodulator

As the input to the PLL shifts between the mark and space frequencies, the *dc error voltage* at the output of the phase comparator follows the frequency shift.

Because there are only two input frequencies (mark and space), there are also only two output error voltages. One represents a logic 1 and the other a logic 0.

Binary FSK has a poorer error performance than PSK or QAM and, consequently, is seldom used for high-performance digital radio systems.

Its use is restricted to low-performance, low-cost, asynchronous data modems that are used for data communications over analog, voice-band telephone lines.

Continuous-Phase Frequency-Shift Keying

Continuous-phase frequency-shift keying (CP-FSK) is binary FSK except the mark and space frequencies are synchronized with the input binary bit rate.

With CP-FSK, the mark and space frequencies are selected such that they are separated from the center frequency by an exact multiple of one-half the bit rate (f_m and $f_s = n[f_b / 2])$, where n = any integer).

This ensures a smooth phase transition in the analog output signal when it changes from a mark to a space frequency or vice versa.

Figure 2-10 shows a noncontinuous FSK waveform. It can be seen that when the input changes from a logic 1 to a logic 0 and vice versa, there is an abrupt phase discontinuity in the analog signal. When this occurs, the demodulator has trouble following the frequency shift; consequently, an error may occur.

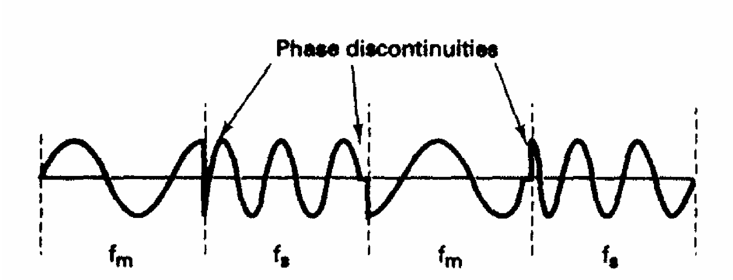


FIGURE 2-10 Noncontinuous FSK waveform

Figure 2-11 shows a continuous phase FSK waveform.

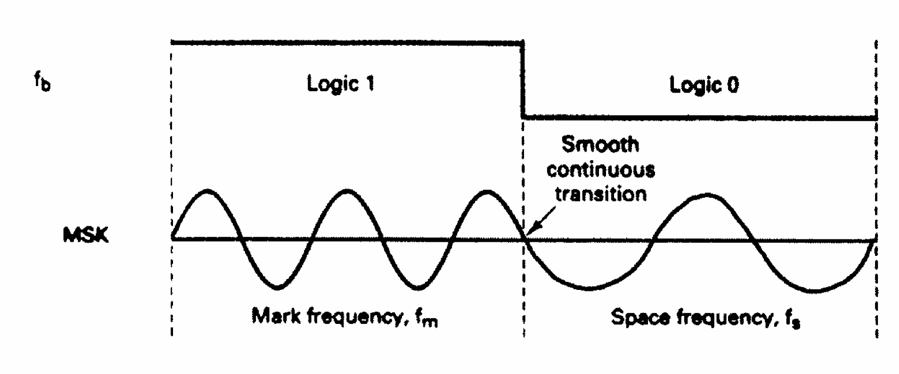


FIGURE 2-11 Continuous-phase MSK waveform

Notice that when the output frequency changes, it is a smooth, continuous transition. Consequently, there are no phase discontinuities.

CP-FSK has a better bit-error performance than conventional binary FSK for a given signal-to-noise ratio.

The disadvantage of CP-FSK is that it requires synchronization circuits and is, therefore, more expensive to implement.