EULER'S EQUATION OF MOTION

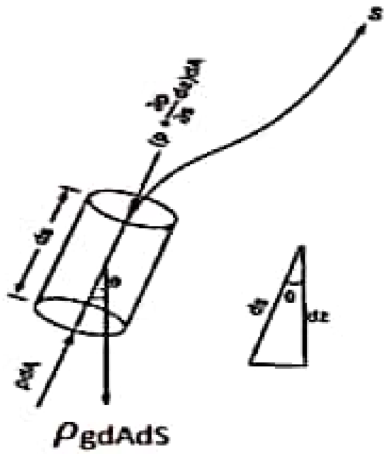
This is equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of a fluid element along a stream-line as:

Consider a stream-line in which flow is taking place in s-direction as shown in Fig. Consider a cylindrical element of cross-section dA and length *ds*. The forces acting on the cylindrical element arc:

I. Pressure force pdA in the direction of flow.

2. Pressure force $(p+\frac{\partial p}{\partial s}ds)\,dA$ opposite to the direction of flow.

3. Weight of element ρgdAds.



forces on a fluid fluid

Let θ is the angle between the direction of flow and the line of action of the weight of element. The resultant force on the fluid element in the direction of s must be equal to the mass of fluid element x acceleration in the direction s.

$$pdA - (p+\frac{\partial p}{\partial s}ds)dA - \rho g dA ds \cos\theta$$

$$= \rho g dA ds \times a_s \qquad \text{--------- (1)}$$

where, a_s is the acceleration in the direction of *s*

Now $a_s = \frac{dv}{dt}$, *where v is a function of s and t,*

$$\frac{dv}{ds}\frac{ds}{dt} + \frac{dv}{dt} = \frac{vdv}{ds} + \frac{dv}{dt} \qquad \{\frac{ds}{dt} = v\}$$

If the flow is steady,

$$\frac{dv}{dt} = 0$$

$$a_s = \frac{vdv}{ds},$$

Substituting the value of as in equation (1) and simplifying the equation, we get

$$-\frac{\partial p}{\partial s} dsdA - \rho g\, dAds \cos\theta = \rho dAds \times \frac{dv}{ds}$$

Dividing by ρdsdA,

$$-\frac{\partial p}{\rho \partial s} - g\cos\theta = \frac{vdv}{ds} \quad \text{(Or)}$$

$$\frac{\partial p}{\rho \partial s} + g\cos\theta + \frac{vdv}{ds} = 0$$

But from Fig. 6.1 (b), we have $\cos\theta = \frac{dz}{ds}$

$$\frac{1}{\rho}\frac{dp}{ds} + g\frac{dz}{ds} + \frac{vdv}{ds} = 0 \quad \text{(or)}$$

$$\frac{dp}{\rho} + g\,dz + vdv = 0 \quad \text{-------- (2)}$$

Equation (2) is known as Euler's equation of motion.

BERNOULLI'S EQUATION FROM EULER'S EQUATION

Bernoulli's equation is obtained by integrating the Euler's equation of motion (2) as

$$\int \frac{dp}{\rho} + \int g dz + \int vdv = \text{constant}$$

If flow is incompressible, ρ is constant and

$$\frac{p}{\rho} + gz + V^2/2 = \text{constant}$$

$$\frac{p}{\rho g} + z + V^2/2g = \text{constant}$$

$$\frac{p}{\rho g} + V^2/2g + z = \text{constant}$$

The above equation is a Bernoulli's equation in which,

$\frac{p}{\rho g}$ = pressure energy per unit weight of fluid or pressure head.

$V^2/2g$ = kinetic energy per unit weight or kinetic head.

z = potential energy per unit weight or potential head.

ASSUMPTIONS BERNOULLI'S EQUATION:

The following are the assumptions made in the derivation of Bernoulli's equation:

(i) The fluid is ideal, i.e., viscosity is zero

(ii) The flow is steady

(iii) The flow is incompressible

(iv) The flow is irrotational.

Water is flowing through a pipe of 5 cm diameter under a pressure of 29.43 N/cm² (gauge) and with mean velocity of 2.0 m/s. Find the total head or total energy per unit weight of the water at a cross-section, which is 5 m above the datum line.

Solution: Given:

Diameter of pipe = 5 cm = 0.5 m

Pressure, p = 29.43 N/cm² = 29.43×10^4 N/m²

Velocity, v = 2.0 m/s

Datum head, z = 5 m

Total head = [pressure head + kinetic head + datum head]

Pressure head $= \frac{p}{\rho g} = \frac{29.43 \times 10^4}{1000 \times 9.81} = 30\text{m}$

Kinetic head $= \frac{v^2}{2g} = \frac{2 \times 2}{2 \times 9.81} = 0.204\text{ m}$

Total head $= \frac{p}{\rho g} + \frac{v^2}{2g} + z = 30 + 0.204 + 5$

= 35.204 m. Ans

A pipe, through which water is flowing, is having diameters, 20 cm and 10 cm at the cross-sections 1 and 2 respectively. The velocity of water at section 1 is given 4.0 m/s. Find the velocity head at sections 1 and 2 and also rate of discharge.

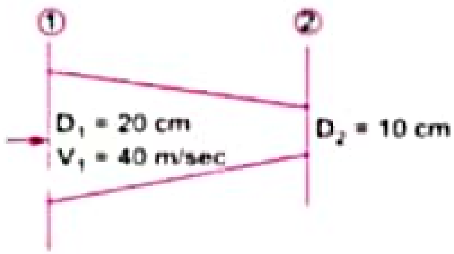
D_1 = 20 cm = 0.2 m

Area, $A_1 = \frac{\pi}{4} D_1^2 = \frac{\pi}{4}(0.2^2) = 0.0314\text{ m}^2$

$V_1 = 4.0$ m/s

$D_2 = 0.1$ m

$A_1 = \frac{\pi}{4} D_2^{\,2} = (0.1^2) = 0.0785\ m^2$



Solution

(i)Velocity head at section 1,

$$\frac{v_1^{\,2}}{2g} = \frac{4.0 \times 4.0}{2 \times 9.81} = 0.815\ m$$

(ii) Velocity head at section 2, $\frac{v_2^{\,2}}{2g}$

To find V_2, apply continuity equation at 1 and 2,

$A_1V_1 = A_2V_2$ (or)

$$V_2 = \frac{A_1V_1}{V_2} = \frac{0.314}{.00785} \times 4.0$$

$$V_2 = 16.0\ m/s$$

Velocity head at section 2, $\frac{v_2^{\,2}}{2g} = \frac{16.0 \times 16.0}{2 \times 9.81} = 83.047\ m$

(iii)Rate of discharge, (Q) = A_1V_1 (or) A_2V_2

$= 0.0314 \times 4.0 = 0.1256\ m^3/s$

$Q = 0.1256\ m^3/s$