Q An open circular tank of 20 cm dia and 100 cm long contains water upto a ht of 60 cm the tank is rotated about its vertical axis at 300 r.p.m. find the depth of parabola formed at the free surface of water.

Solution Given that

Dia of cylinder = 20 cm

$$R = \frac{20}{2} = 10 \text{ cm}$$

Height of liquid $H = 60$ cm

$N = 300$ r.p.m

Angular velocity $\omega = \frac{2\pi N}{60}$

$$= \frac{2 \times \pi \times 300}{60} = 31.41 \text{ rad/sec}$$

Let the depth of parabola $= z$

$$z = \frac{\omega^2 r_2^2}{2g} \quad (r_2 = R)$$

$$= \frac{\omega^2 R^2}{2g} = \frac{(31.41)^2 \times (6)^2}{2 \times 9.81}$$

$$= 50.28 \text{ cm}$$

Q An open circular cylinder of 15 cm dia and 100 cm long contains water upto a ht of 80 cm, find the maximum speed at which the cylinder is to be rotated about its vertical axis so that no water spills.

Soln Rad = 15 cm

$$R = \frac{15}{2} = 7.5 \text{ cm}$$

Length of cylinder is rotated at angular speed of $\omega$ rad/sec, when the water is about to spill, then using

Rise of liquid at ends = fall of liquid at centre

= Length - initial height

= 100 - 80 = 20 cm

Ht of parabola = 20 + 20 = 40 cm

$z = 40$ cm

$$z = \frac{\omega^2 R^2}{2g}$$

$$40 = \frac{\omega^2 (7.5)^2}{2 \times 9.81}$$

$$\omega^2 = \frac{40 \times e \times 981}{7.5 \times 7.5}$$

$$= 1395.2$$

$$N\omega = \frac{2\pi N}{60}$$

$$N = \frac{60 \times \omega}{2\pi}$$

$$N = \frac{60 \times 37.35}{2 \times \pi} = 356.66 \text{ r.p.m}$$

$$\boxed{N = 356.66 \text{ rpm}}$$

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Q A cylindrical vessel 12cm in dia and 30cm deep is filled with water upto the top. the vessel is open at the top. find the quantity of liquid left in the vessel. when it is rotated about its vertical axis with a speed of (a) 300 r.p.m (b) 600 rpm

Soln Given that

Dia = 12cm

Radius = 6cm

initial ht of water = 30cm

initial volum of vlate = Area × initial Ht of vlate

$= \frac{\pi}{4} \times 12^2 \times 30 = 3392.9 \text{ cm}^3$

N = 300 r.p.m

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 300}{60} = 31.41$ rad/sec

Ht of Parabola $z = \frac{\omega^2 R^2}{2g} = \frac{(31.41)^2 \times 6^2}{2 \times 981}$

$= 18.10$ cm

An Vessel is initially full of vlater, vlill be Spilled if it is rotated, Volume of vlate Spilled is equal to the volume of Paraboloid

Volume of paraboloid = [Area of Cross section × Ht of parabola] ÷ 2

$= \frac{\pi}{4} D^2 \times \frac{z}{2} = \frac{\pi}{4} \times 12^2 \times \frac{18.10}{2} = 1023.53 \text{ cm}^3$

Volume of vlate left = initial Volume − Volume of vlate Spilled

$= 3392.9 - 1023.53 = 2369.37 \text{ cm}^3$

$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 600}{60} = 62.82$ rad/sec

$z = \frac{\omega^2 R^2}{2g} = \frac{(62.82)^2}{2 \times 981} = 72.40$ cm.