Momentum and Forces in Fluid Flow

- We have all seen moving fluids exerting forces. The lift force on an aircraft is exerted by the air moving over the wing. A jet of water from a hose exerts a force on whatever it hits.

- In fluid mechanics the analysis of motion is performed in the same way as in solid mechanics - by use of Newton's laws of motion.

- i.e., *F = ma which is used in the analysis of solid mechanics to relate applied force to acceleration.*

- *In* fluid mechanics it is not clear what mass of moving fluid we should use so we use a different form of the equation.

$$\sum F = ma = \frac{d(mV)_s}{dt}$$

Momentum and Forces in Fluid Flow

- Newton's 2nd Law can be written:
- *The Rate of change of momentum of a body is equal to the resultant force acting on the body, and takes place in the direction of the force.*

$$\sum F = \frac{d(mV)_s}{dt}$$

$\sum F =$ **Sum of all external forces on a body of fluid or system s**

$mV =$ **Momentum of fluid body in direction s**

- *The symbols F and V represent vectors and so the change in momentum must be in the same direction as force.*

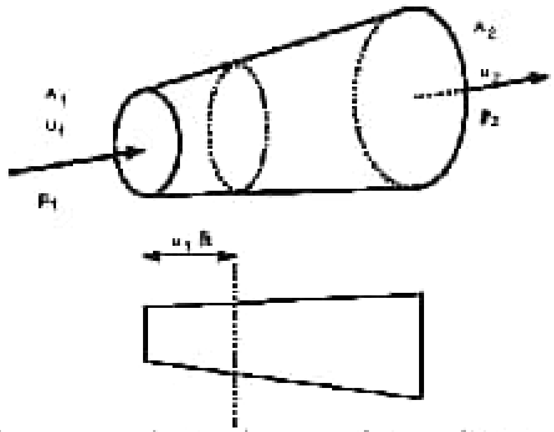
$$\sum F dt = d(mV)_s$$

It is also termed as impulse momentum principle

Momentum and Forces in Fluid Flow

- Let's start by assuming that we have *steady flow which is non-uniform flowing in a stream tube.*

- In time δt *a volume of the fluid moves from the inlet a distance u* δt *, so the volume entering the* streamtube in the time δt *is*



A streamtube in three and two-dimensions

volume entering the stream tube = area x distance $= A_1 u_1 \delta t$

mass entering stream tube = volume x density $= \rho_1 A_1 u_1 \delta t$

momentum of fluid entering stream tube = mass x velocity $= (\rho_1 A_1 u_1 \delta t) u_1$

 momentum of fluid leaving stream tube $= (\rho_2 A_2 u_2 \delta t) u_2$

Momentum and Forces in Fluid Flow

- Now, according to Newton's 2nd Law the force exerted by the fluid is equal to the rate of change of momentum. So
- Force=rate of change of momentum

$$\sum F = \frac{\rho_2 A_2 u_2 \delta t u_2 - \rho_1 A_1 u_1 \delta t u_1}{\delta t} = \frac{\rho_2 A_2 u_2 \delta t u_2}{\delta t} - \frac{\rho_1 A_1 u_1 \delta t u_1}{\delta t}$$

$$\sum F = \frac{\rho_2 (A_2 u_2 \delta t) u_2}{\delta t} - \frac{\rho_1 (A_1 u_1 \delta t) u_1}{\delta t} = \rho_2 (Q_2) u_2 - \rho_1 (Q_1) u_1$$

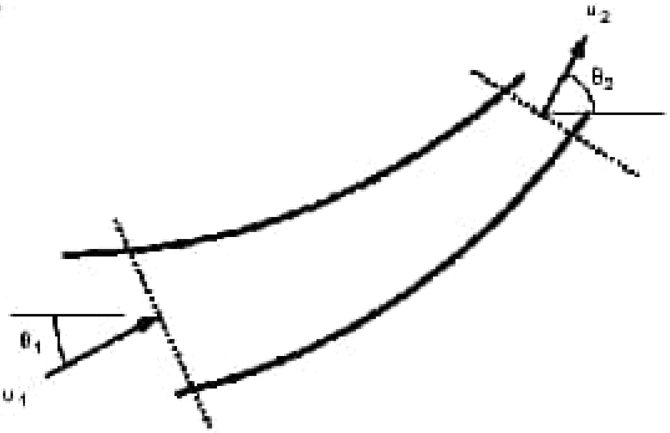
- We know from continuity of incompressible flow, $\rho = \rho_1 = \rho_2$ & $Q = Q_1 = Q_2$

$$F = \rho Q[u_2 - u_1] = m[u_2 - u_1]$$

This analysis assumed that the inlet and outlet velocities were in the same direction - i.e. a one dimensional system. What happens when this is not the case?

Momentum and Forces in Fluid Flow

- Consider the two dimensional system in the figure below:
- At the inlet the velocity vector, *u1* , *makes an angle, θ1* , with the x-axis, while at the outlet *u2 make an* angle θ 2.
- In this case we consider the forces by resolving in the directions of the co-ordinate axes.



Two dimensional flow in a streamtube

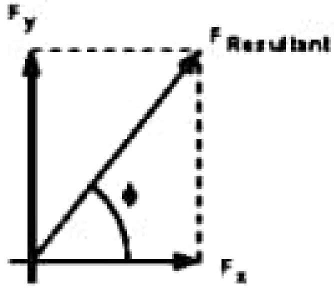
- **The force in the x-direction**

$$F_x = \text{Rate of change of momentum in x - direction}$$
$$= \text{Rate of change of mass} \times \text{change in velocity in x - direction}$$
$$= \dot{m}(u_2 \cos\theta_2 - u_1 \cos\theta_1)$$
$$= \dot{m}(u_{2x} - u_{1x})$$
$$= \rho Q(u_2 \cos\theta_2 - u_1 \cos\theta_1)$$
$$= \rho Q(u_{2x} - u_{1x})$$

Momentum and Forces in Fluid Flow

- **The force in the y-direction**

$$F_y = \dot{m}(u_2 \sin\theta_2 - u_1 \sin\theta_1)$$
$$= \dot{m}(u_{2_y} - u_{1_y})$$
$$= \rho Q(u_2 \sin\theta_2 - u_1 \sin\theta_1)$$
$$= \rho Q(u_{2_y} - u_{1_y})$$



- The resultant force can be determined by combining Fx and Fy vectorially as

$$F_{resultant} = \sqrt{F_x^2 + F_y^2}$$

- And the angle at which F acts is given by

$$\phi = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

Momentum and Forces in Fluid Flow

- For a three-dimensional (x, y, z) system we then have an extra force to calculate and resolve in the z direction.
- This is considered in exactly the same way.

- In summary we can say: The total force the fluid = rate of change of momentum through the control volume

$$F = \dot{m}(u_{out} - u_{in})$$
$$= \rho Q(u_{out} - u_{in})$$

Momentum and Forces in Fluid Flow

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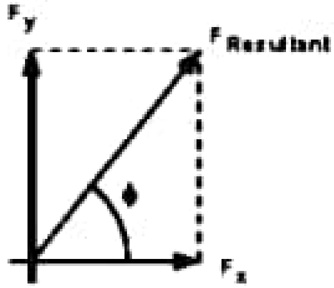

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Momentum and Forces in Fluid Flow

- Note that we are working with vectors so F is in the direction of the velocity. This force is made up of three components:

- F_R = *Force exerted on the fluid by any solid body touching the control volume*
- F_B = *Force exerted on the fluid body (e.g. gravity)*
- F_P = *Force exerted on the fluid by fluid pressure outside the control volume*

- So we say that the total force, F_T, *is given by the sum of these forces:*

$$F_T = F_R + F_B + F_P$$

- The force exerted **by** the fluid **on** the solid body touching the control volume is opposite to F_R. *So the* reaction force, R, is given by

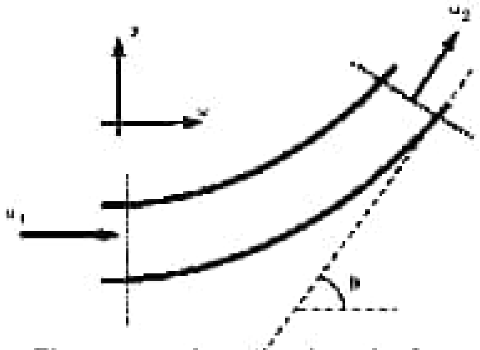
$$R = -F_R$$

Application of the Momentum Equation

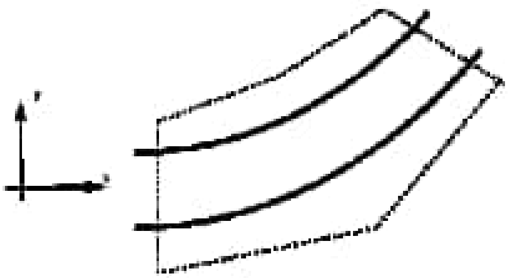
- In common application of the momentum principle, we use it to find forces that flowing fluid exert on structures open to the atmosphere like gate and overflow spillways

- In the following section, we will consider the application of momentum principle for the following cases.
  - 1. Force due to the flow of fluid round a pipe bend.
  - 2. Force on a nozzle at the outlet of a pipe.
  - 3. Impact of a jet on a plane surface.
  - 4. Force due to flow round a curved vane.

Force due to the flow of fluid round a pipe bend



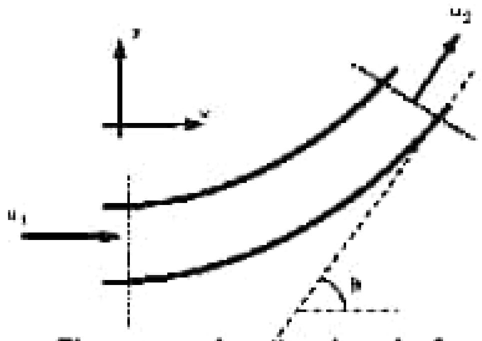
Flow round a pipe bend of constant cross-section



Control volume

- **Coordinate system**: It is convenient to choose the co-ordinate axis so that one is pointing in the direction of the inlet velocity.
- In the above figure the x-axis points in the direction of the inlet velocity.
- Let's compute, total force, pressure force, body force and resultant force

Force due to the flow of fluid round a pipe bend



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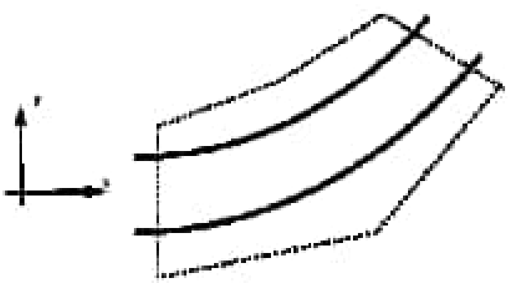


Control volume

- **1.Total Force:**

In x-direction

$$F_{T_x} = \rho Q(u_{2_x} - u_{1_x})$$

$$u_{1_x} = u_1$$

$$u_{2_x} = u_2 \cos\theta$$

$$F_{T_x} = \rho Q(u_2 \cos\theta - u_1)$$

In y-direction

$$F_{T_y} = \rho Q(u_{2_y} - u_{1_y})$$

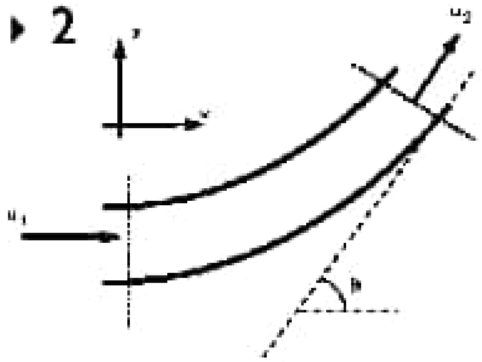
$$u_{1_y} = u_1 \sin 0 = 0$$

$$u_{2_y} = u_2 \sin\theta$$

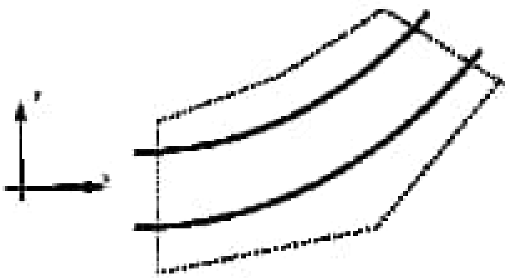
$$F_{T_y} = \rho Q u_2 \sin\theta$$

Force due to the flow of fluid round a pipe bend

- 2



Flow round a pipe bend of constant cross-section



Control volume

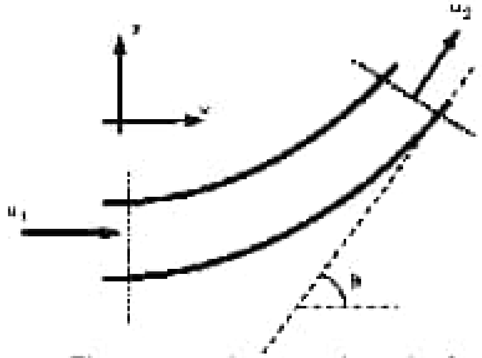
- **2. Pressure force**

$$F_p = \text{pressure force at 1 - pressure force at 2}$$

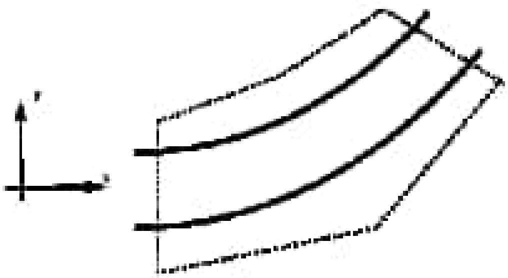
$$F_{p_x} = p_1 A_1 \cos 0 - p_2 A_2 \cos\theta = p_1 A_1 - p_2 A_2 \cos\theta$$

$$F_{p_y} = p_1 A_1 \sin 0 - p_2 A_2 \sin\theta = -p_2 A_2 \sin\theta$$

Force due to the flow of fluid round a pipe bend



Flow round a pipe bend of constant cross-section

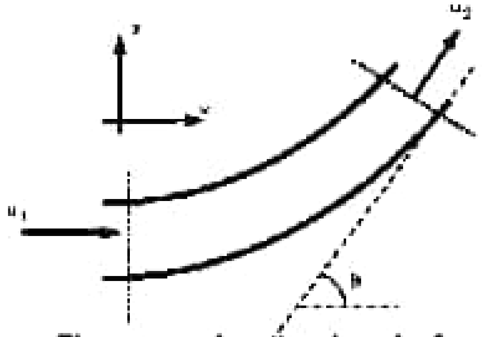


Control volume

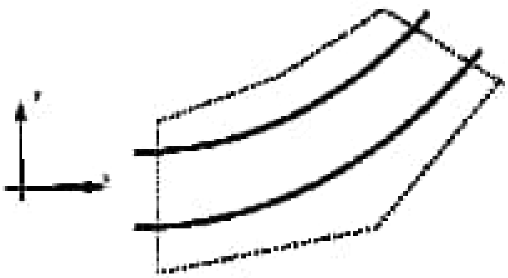
- **3. Body force:**

There are no body forces in the x or y directions. The only body force is that exerted by gravity (which acts into the paper in this example - a direction we do not need to consider).

Force due to the flow of fluid round a pipe bend



Flow round a pipe bend of constant cross-section



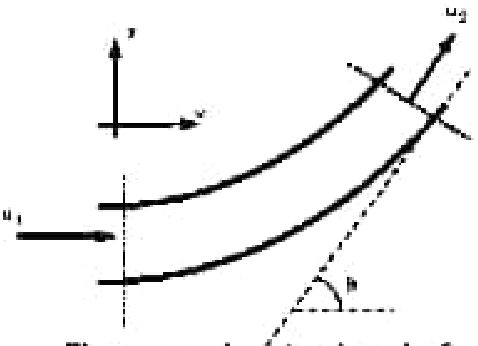
Control volume

- **Resultant force**

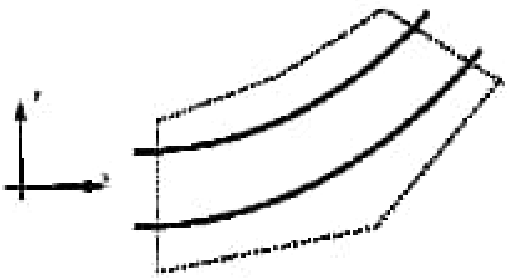
$$F_{T_x} = F_{R_x} + F_{P_x} + F_{B_x}$$
$$F_{T_y} = F_{R_y} + F_{P_y} + F_{B_y}$$

$$F_{R_x} = F_{T_x} - F_{P_x} - 0 = \rho Q(u_2 \cos\theta - u_1) - p_1 A_1 + p_2 A_2 \cos\theta$$
$$F_{R_y} = F_{T_y} - F_{P_y} - 0 = \rho Q u_2 \sin\theta + p_2 A_2 \sin\theta$$

Force due to the flow of fluid round a pipe bend



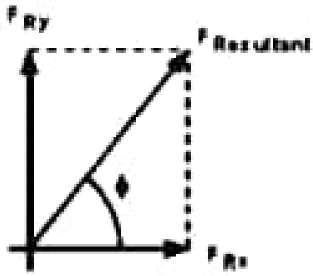
Flow round a pipe bend of constant cross-section



Control volume

- **Resultant force and direction**

$$F_R = \sqrt{F_{R_x}^2 - F_{R_y}^2} \qquad \phi = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right)$$



- **Finally, the force on bent is same magnitude but in opposite direction**

$$R = -F_R$$

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