

## Introduction: →

Earth: → Earth is an oblate spheroid.

(Flattened on poles)

(3)

Diameter - polar axis = 12714 km.

Equatorial axis = 12756.75 km.

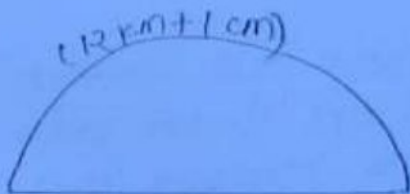
difference = 42.75 km (0.34% less)

## Types: →

① plain Surveying: → at least curvature is not considered  
(suitable for small area)

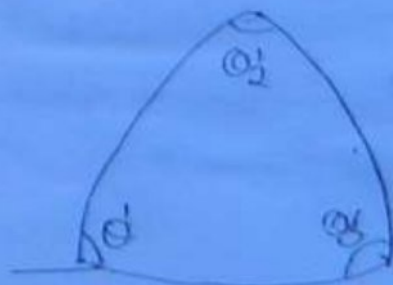
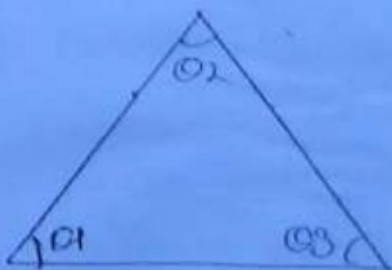
② Geodetic Surveying: → If earth curvature is considered  
(suitable for large area)

①



difference for 12 km length = 1 cm

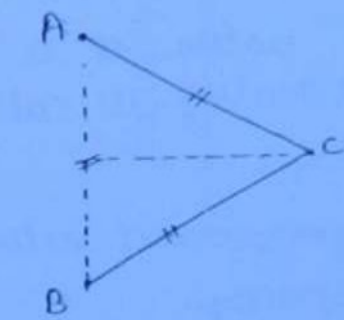
②



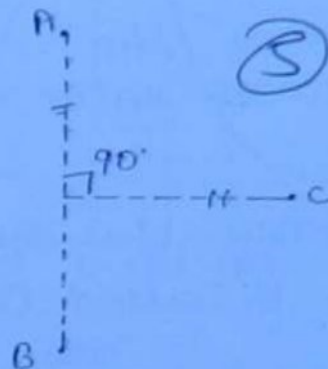
Difference of total angle  
 $(\alpha_1' + \alpha_2' + \alpha_3') - (\alpha_1 + \alpha_2 + \alpha_3) = 1 \text{ Second} = 0^{\circ} 0' 1''$

③ Principle of Surveying: →

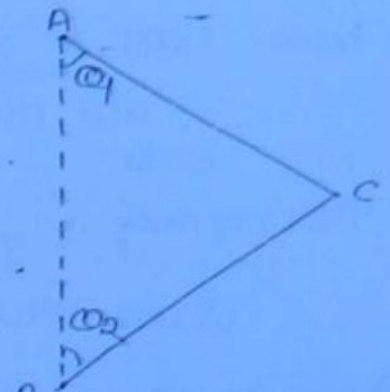
① Location of a point by measurement from two points of reference.



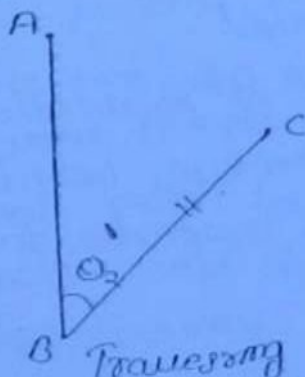
Chain Survey



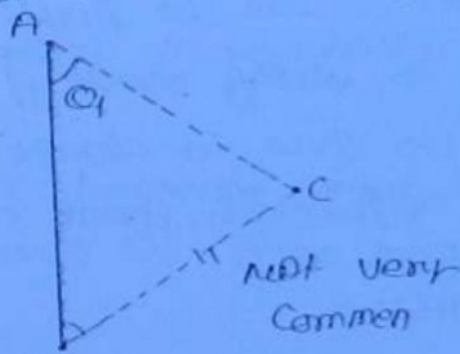
Offset Method



Compass Survey



Traversing



Not very Common

④ Working from whole to part: → first major control points are set and distance are measured with higher accuracy then minor details can be taken even with less precision. Error involved in ~~can be taken~~ own measurement will not be accumulated.

\* Accuracy and Errors: →

① Definitions

① Accuracy: → Degree of precision obtained in measurement of a quantity is called accuracy. By using precise instrument correct measurement and correct manner of taking measurement.

⑤ Precision: →

⑥

Degree of perfection used in the measurement is called precision.

⑦ True Error: → Difference b/w true value of a quantity and measurement value of a quantity is called true error.

⑧ Discrepancy: → Difference b/w two measured value of same quantity is called Discrepancy.

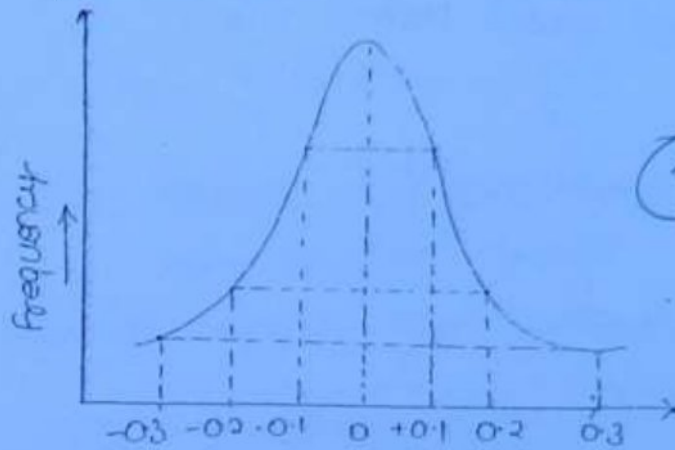
\* Source of errors: →

- ① Instrument :- Due to faulty instrument.
- ② Personal → Wrong reading/writing of measurement.
- ③ Nature → Due to change in temperature, humidity, refraction, local attraction magnetic declination.

\* Kind of errors: →

- ① Mistakes: → Human error's due to less knowledge, carelessness, confusion in experience.
- ② Systematic Error: → (Cumulative error)  
Always have same size and direction under same condition of measurement may be either (+)ive or (-)ive
- ③ Accidental Error's → (Compensating error): →  
These error's occur's some times in one direction and some times in opposite time and (-)ive error's compensate each others.

## A Theory of probability :->



-> Accidental errors followed a definite rule method law of probability.

As per this law according to possibility distribution curve of error.

- ① Small errors have higher frequency than large errors.
- ② Positive and negative errors of same magnitude have same frequency.

A True value :-> Exact value of a quantity.  
(Almost impossible to measure)

A Most probable value :-> The value of measurement which has chances of being the correct of a quantity than other measurement is called most probable value.

A principle of least square :->

Most probable value of quantity is for which sum of a square of a residual error is min.

Ex :-> when all measurement have equal weight  $x_1, x_2, x_3, \dots, x_n$  (equal weight = 1.0)

Residual error:→

If most probable value =  $x$

$x = x_1$

$x = x_2$

-----

(8)

Square  $(x-x_1)^2$

$(x-x_2)^2$

-----

As per principle of least square

$y = (x-x_1)^2 + (x-x_2)^2 + \dots = \min^m$

$\Rightarrow \frac{dy}{dx} = 2(x-x_1) + 2(x-x_2) + \dots = 0$

$n \cdot x = (x_1 + x_2 + \dots + x_n) = 0$

$n \cdot x = (x_1 + x_2 + \dots + x_n) = 0$

$x = \left( \frac{x_1 + x_2 + \dots + x_n}{n} \right) = \text{mean of all}$

Case (ii) Unequal weights

Squares	measurement	weight	M.P.V	Residual error
$(x-x_1)^2 \times w_1$	$x_1$	$w_1$	↑	$x-x_1$
$(x-x_2)^2 \times w_2$	$x_2$	$w_2$		$x-x_2$
-----	-----	-----	$x$	-----
$(x-x_n)^2 \times w_n$	$x_n$	$w_n$	↓	$(x-x_n)$

Sum of square of residual error.

$y = w_1(x-x_1)^2 + w_2(x-x_2)^2 + \dots = \min^m$

$\frac{dy}{dx} = 2w_1(x-x_1) + 2w_2(x-x_2) + \dots = 0$

M.P.V  $\Rightarrow x = \frac{w_1x_1 + w_2x_2 + \dots}{w_1 + w_2 + \dots} = \frac{\sum wx}{\sum w}$

★ The probable error of a single observation.

$$E_s = \pm 0.6745 \sqrt{\frac{\sum V^2}{(n-1)}} \quad (9)$$

$V$  = difference b/w any single observation and mean of the series.

★ The probable error of the mean:—

$$E_m = \pm 0.6745 \sqrt{\frac{\sum V^2}{n(n-1)}}$$

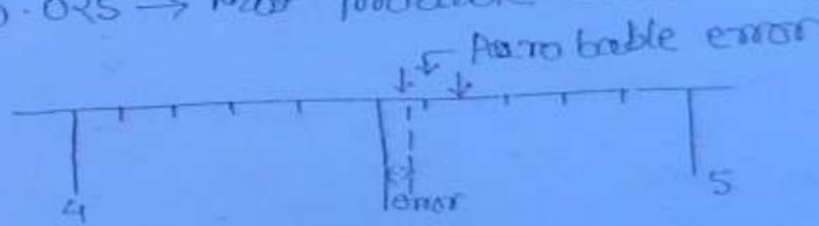
$$E_m = \frac{E_s}{\sqrt{n}}$$

★ Significant Figures: → |max. error| / Most probable error: →

4.6

error 0.05 ← max. error

0.025 → Most probable error



→ No. of Significant figures shows the accuracy of measurement

→ If there are  $n$  significant figure in a measurement (n-1) figure are called - certain figure

least figure is called - Uncertain figure.

→ There are two types of error

Example	Max. error	Probable error
4.6	0.05	0.025
5.86	0.005	0.0025

\* Accumulation of error  $\rightarrow$

(10)

(1) For max. error  $\rightarrow$

Total error = Algebraic Sum of all errors.

$$e_T = \pm e_1 \pm e_2 \pm e_3$$

(2) For probable error  $\rightarrow$

Sum = Root mean Square value

$$e_T = \pm \sqrt{e_1^2 + e_2^2 + e_3^2 + \dots}$$

\* Error's in computed Result  $\rightarrow$

(a) Addition  $\rightarrow$

IF  $x$  and  $y$  are two measured value

$$S = x + y$$

$$\boxed{ds = dx + dy} \quad \text{--- (1)}$$

If max. error's are  $\pm \delta_x$  and  $\pm \delta_y$

max. error

$$S \pm \delta_s = (x \pm \delta_x) + (y \pm \delta_y)$$

$$\Rightarrow S \pm \delta_s = (x + y) \pm (\delta_x + \delta_y)$$

$$\delta_s = (\delta_x + \delta_y) \text{ max. error in } S$$

Probable error  $\rightarrow$

are  $e_x$  and  $e_y$ , sum of probable error's

$$e_s = \sqrt{e_x^2 + e_y^2}$$

$$\text{Range of } S = (S + e_s) \text{ to } (S - e_s)$$

(ii) Sub-Traction:  $\rightarrow$

$$S = x - y$$
$$ds = dx - dy$$

(11)

Max. error if  $\pm \delta x$  and  $\pm \delta y$  are max. error

For (1)  $\rightarrow \delta_s = \delta(+\delta x) - (-\delta y)$

$$\delta_s = +(\delta x + \delta y)$$
$$\text{or } = (-\delta x) - (+\delta y)$$
$$= -(\delta x + \delta y)$$

$$\delta_s = \pm (\delta x + \delta y)$$

Range of  $s$

$$s = (s + \delta_s) \text{ to } (s - \delta_s)$$

Probable error

$$e_s = \pm \sqrt{(e_x)^2 + (e_y)^2}$$

$$\text{Range} = (s + e_s) \text{ to } (s - e_s)$$

(iii) Multiplication:  $\rightarrow$

$$S = x \cdot y$$

$$ds = x dy + y dx$$

Max. error

Respective error of  $x$  and  $y \rightarrow \delta x$  and  $\delta y$

for  $s$  in  $x \rightarrow y \cdot \delta x$

in  $y \rightarrow x \cdot \delta y$

$$\delta_s = x \cdot \delta y + y \cdot \delta x \quad \text{--- (1)}$$

$$\text{Range } (s + \delta_s) \text{ to } (s - \delta_s)$$

Probable error:  $\rightarrow$

Respective error of  $x$  and  $y \rightarrow e_x$  and  $e_y$

for  $s$ , error in  $x = y \cdot e_x$

$$y = x \cdot e_y$$



$$\text{Sum} = e_s = \sqrt{(y \cdot e_x)^2 + (x \cdot e_y)^2} \quad (12)$$

$$\Rightarrow \frac{e_s}{s} = \frac{1}{xy} \cdot \sqrt{(y \cdot e_x)^2 + (x \cdot e_y)^2}$$

$$\Rightarrow \frac{e_s}{s} = \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2}$$

$$\Rightarrow e_s = s \times \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2}$$

(4) Division:  $\rightarrow s = x/y$

$$ds = \frac{y \cdot dx - x \cdot dy}{y^2}$$

$$ds = \left(\frac{dx}{y}\right) - \left(\frac{x}{y^2}\right) dy$$

max error: - Respective error of  $x$  and  $y \rightarrow \delta x$  and  $\delta y$

$$\text{For } \delta x \rightarrow \frac{1}{y} \delta x$$

$$\text{in } y \rightarrow \frac{x}{y^2} \delta y$$

$$\therefore \boxed{\delta s = \frac{\delta x}{y} + \frac{x}{y^2} \delta y}$$

Range (S+ds) to (S-ds)

Probable error: -

Respective error of  $x$  and  $y \rightarrow e_x$  and  $e_y$

$$\text{For } s \text{ error in } x = \frac{1}{y} e_x$$

$$y = \frac{x}{y^2} e_y$$

$$\text{Sum } e_s = \sqrt{\left(\frac{e_x}{y}\right)^2 + \left(x \cdot \frac{e_y}{y^2}\right)^2}$$

$$\Rightarrow \frac{e_s}{s} = \frac{y}{x} \sqrt{\left(\frac{e_x}{y}\right)^2 + \left(\frac{x}{y^2} e_y\right)^2}$$

$$\Rightarrow \boxed{\frac{e_s}{s} = \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2}} \quad \text{--- B}$$

Q. A quantity  $S$  is equal to sum of two measured quantities  $x$  and  $y$ .

$$S = 5.68 + 3.648$$

(13)

Find out probable error, max. error, most probable limits, max. limits of  $S$ .

$$x = 5.68$$

$$0.005 \delta x$$

$$0.0025 \delta y$$

$$y = 3.648$$

$$0.0005 \delta y$$

$$0.00025 \delta y$$

max. error

probable error.

Solution:  $\rightarrow$  (1) max. error

$$\text{(For } S) \delta S = \delta x + \delta y$$

$$\delta S = 0.005 + 0.0005$$

$$\delta S = 0.0055$$

$$\text{Range max. range} = S + \delta S \text{ to } (S - \delta S)$$

$$= 9.328 + 0.0055 \text{ to } \frac{9.328}{-0.0055}$$

$$9.3225$$

(2) most probable error: -

$$e_s = \sqrt{e_x^2 + e_y^2}$$

$$e_s = \sqrt{(0.0025)^2 + (0.00025)^2}$$

$$e_s = 0.002512$$

probable range of  $S$

$$= S + e_s \text{ to } S - e_s$$

$$= 9.328 \text{ to } 9.328$$

$$= 9.33051 \quad - 0.002512$$

$$= 9.3255 \text{ Ans}$$

Ques 3) Calculate the max. & probable error & range of a computed quantity.

(14)

$$S = \frac{9.58}{4.6}$$

Solution: →

		max <sup>m</sup>	Probable
x	9.58	$\Delta x = 0.005$	$e_x = 0.0025$
y	4.60	$\Delta y = 0.05$	$e_y = 0.025$

$$S = 2.0826$$

$$ds = \frac{y dx - x dy}{y^2}$$

$$ds = \frac{dx}{y} - \frac{x}{y^2} dy$$

max<sup>m</sup> error

$$\Delta s = \frac{\Delta x}{y} + \frac{\Delta y}{y^2} x$$

$$\Delta s = \frac{0.005}{4.6} + \frac{9.58 \times 0.05}{4.6^2}$$

$$\Delta s = 0.0237$$

Range of s

$$\begin{array}{r} 2.0826 \\ + 0.0237 \\ \hline 2.1063 \end{array} \quad \begin{array}{r} - 0.0237 \\ \hline 2.0589 \end{array}$$

Probable error

$$\Rightarrow \frac{e_s}{s} = \sqrt{\left(\frac{e_x}{x}\right)^2 + \left(\frac{e_y}{y}\right)^2}$$

$$\Rightarrow e_s = 2.0826 \sqrt{\left(\frac{0.0025}{9.58}\right)^2 + \left(\frac{0.025}{4.6}\right)^2}$$

$$\Rightarrow e_s = +0.0113$$

Range of s

$$\begin{array}{r} 2.0826 \\ + 0.0113 \\ \hline 2.0939 \end{array} \quad \begin{array}{r} 2.0826 \\ - 0.0113 \\ \hline 2.0713 \end{array}$$

Ans

Fundamental definition :→  
(Linear measurement)

Scale :→ scale is ratio of distance plotted in drawing to distance on the ground.

EX = 1 cm = 1 km  
1 cm = 1000 m  
1 cm = 1000 x 100 cm

R.F =  $\frac{1}{100000}$  = Representative fraction

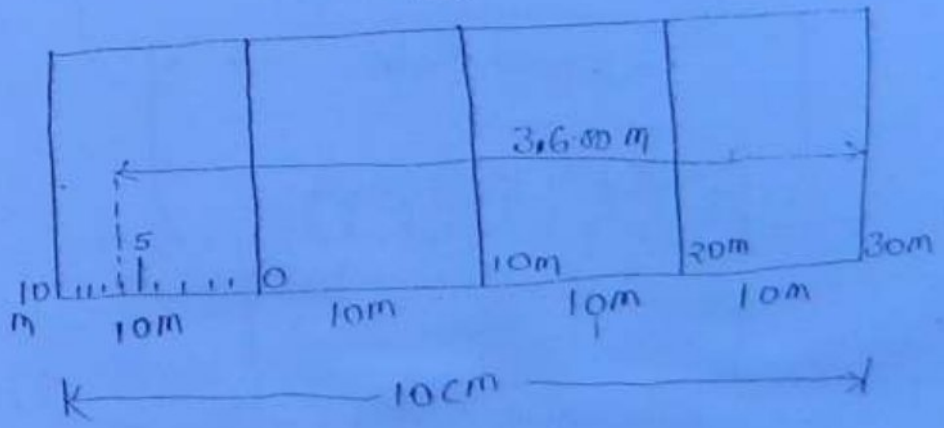
Types :→

- ① plane scale.
- ② Diagonal scale.
- ③ Vernier scale.
- ④ Chord scale.

① plane scale :→ It measures upto two dimension only

A scale 1 cm = 4 m to be prepared.

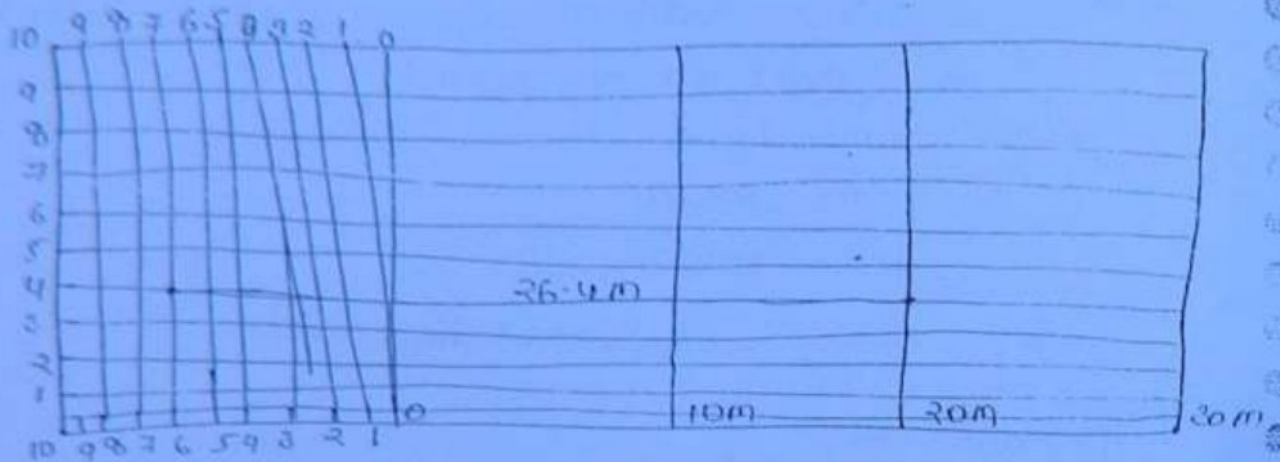
R.F =  $\frac{1}{400}$



→ In this case, 10m and meter are the two dimension that can be measured.

② Diagonal Scale  $\rightarrow$  It can be measured up to 3 dimensions. Similar triangle theory is used in diagonal scale.

(16)

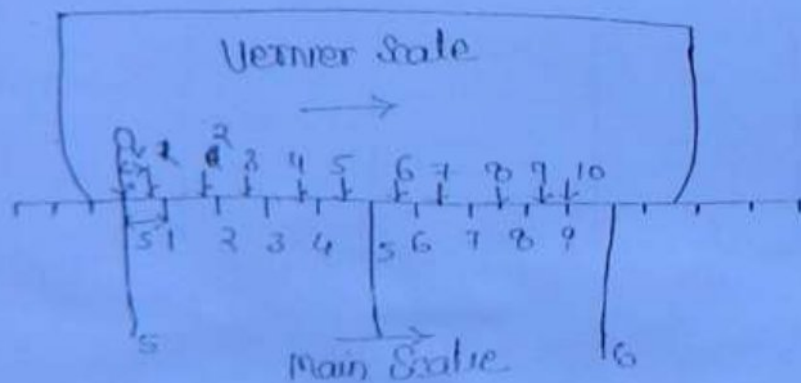


In three dimensions in above case

- ① 10m
- ② meter
- ③ decimeter

③ Vernier Scale  $\rightarrow$

① Direct Vernier Scale  $\rightarrow$  In this case also up to 2 dimensions can be read.



In case of direct Vernier scale

- ① Vernier scale moves in same direction as of main scale.
- ②  $(n-1)$  parts of main scale is equal to  $n$  divisions (units) of Vernier scale.

$$n \cdot V = (n-1)S$$

$S =$  main scale  
 $V =$  vernier scale

(17)

$$V = \frac{(n-1) \cdot S}{n}$$

Least count  $\rightarrow$  minimum value that can be used using a scale is called least count

$$L.C. = S - V$$

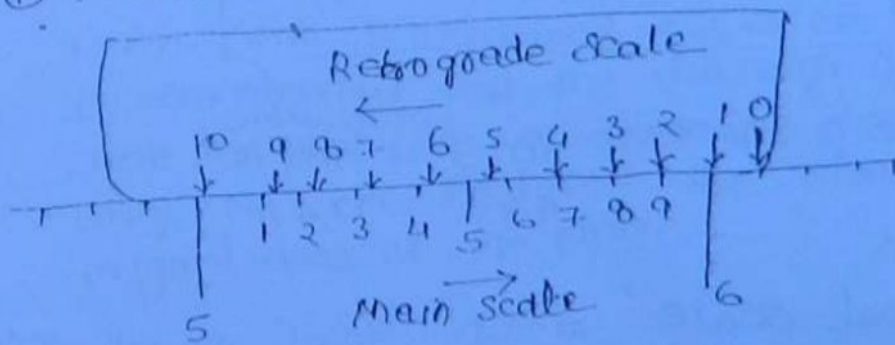
$$L.C. = S - \frac{n-1}{n} S$$

$$L.C. = \frac{nS - nS + S}{n}$$

$$L.C. = \frac{S}{n}$$

Least count of this vernier scale.

(6) Retrograde Scale  $\rightarrow$  In case of Retrograde scale  
(1) Vernier scale moves in opposite direction as the main scale.



(3)  $(n+1)$  parts of main scale is equal  $n$  division (parts) of vernier scale.

$$n \cdot V = (n+1)S$$

$$V = \frac{(n+1) \cdot S}{n}$$

Least count  $\rightarrow$  minimum value that can be read using a scale is called least count

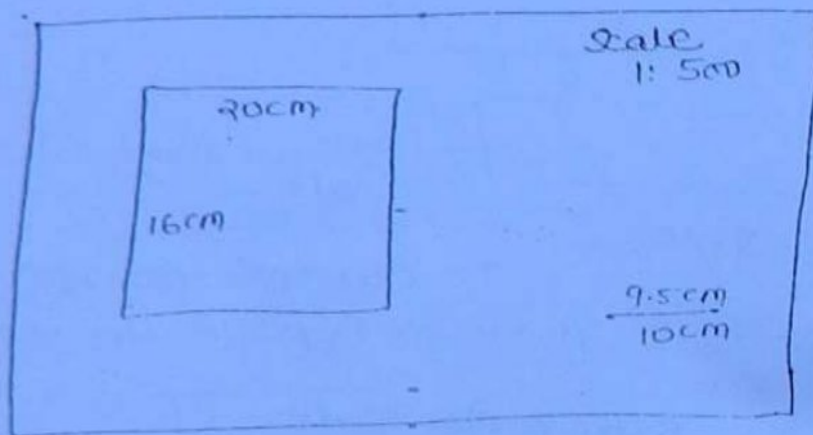
$$L.C. = V - S$$

$$L.C. = \frac{(n+1)}{n} \times S - S$$

$$L.C. = \frac{ns + S - ns}{n}$$

$$L.C. = \frac{S}{n}$$

18  
A Shrink Scale  $\rightarrow$   
(and Shrinkage factor)



$$1: 500$$
$$1\text{cm} = 500\text{ cm}$$
$$1\text{cm} = 5\text{m}$$

This line is represent -  $20 \times 5 = 100\text{ m}$  (Scale value)  
 $16 \times 5 = 80\text{ m}$

original scale

$$1: 500$$

$$10\text{cm} = 5000\text{ cm}$$

Now

present - Scale

$$9.5\text{ cm} = 5000\text{ cm}$$

$$1\text{ cm} = \frac{5000}{9.5}\text{ cm}$$

$$1\text{ cm} = 526.316\text{ cm} \quad \boxed{1: 526.316}$$

→ Shrinkage factor :- It is the ratio of Shrink length to original length of the line.

$$SF = \frac{\text{Shrink length}}{\text{original length}} \quad \text{--- (19)}$$

→ Shrink scale = Shrinkage factor  $\times$  Original scale.

Ex. For given example.

$$SF = \frac{9.5}{10}$$

$$SF = 0.95$$

$$\begin{aligned} \text{Shrink scale} &= \text{Shrinkage factor} \times \text{Original scale} \\ &= 0.95 \times \frac{1}{500} \\ &= \frac{19}{526.316} \end{aligned}$$

problem:-> Area of a plan in a drawing plotted to a scale 1 cm = 50 m is measured 250 sq. cm by planimeter. It was observed that the drawing has shrunk and line originally 10 cm drawn on drawing measured only 9.20 cm. Find out the shrink scale and original area of the plan.

Solution:-> Shrink length = 9.2 cm

original length = 10 cm

$$\text{Shrinkage factor} = \frac{\text{Shrink length}}{\text{original length}}$$

$$= \frac{9.2}{10}$$

$$= 0.92$$

original scale = 1 cm = 50 m

1 cm = 5000 cm



$$= 1/5000$$

Shrinkage = S.F.  $\times$  0.5

$$= 0.92 \times \frac{1}{5000}$$

(20)

$$= \frac{1}{5434.98}$$

Scale 1cm = 54.35m

$$\text{Original area} = 250 \text{ cm}^2 \times (54.35)^2$$

$$= 738490.6 \text{ m}^2$$

Ans:-

★ Error due to wrong length of chain / Tape:  $\rightarrow$

$L$  = length (designated) length of Tape / chain

$L'$  = wrong length of Tape / chain [actual length]

$J'$  = <sup>wrong</sup> measured (written) length of a line

$J$  = True of line measured

$$\boxed{\begin{array}{l} \text{True} \times \text{True} = \text{wrong} \times \text{wrong} \\ J \times L = L' \times J' \end{array}}$$

True length of line  $J = \left(\frac{L'}{L}\right) \times J'$

Ex:  $\rightarrow$  If a 30m chain is actually 30.20 m long, what will be the actual length of a line which is measured 3052m using above tape.

Solution:  $\rightarrow$

$$L = 30 \text{ m}$$

$$L' = 30.20 \text{ m}$$

$$J' = 3052 \text{ m}$$

$$J = ?$$

$$L \times J = L' \times J'$$

$$J = \left(\frac{L'}{L}\right) \times J'$$

$$J = \frac{30.20}{30} \times 3052$$

$$= \boxed{3072.35 \text{ m}}$$

## Formula

①  $L = \left(\frac{L'}{L}\right) \times L'$  for length

② For Area

$$A = \left(\frac{L'}{L}\right)^2 \times A'$$

(21)

③ For Volume

$$V = \left(\frac{L'}{L}\right)^3 \times V'$$

## ★ Tape Correction : →

① Correction due to standardization. [Due to wear & tear]  
length of chain [Tape].

Total correction required

$$Ca = \frac{\text{Total length of line (L')} \times c}{L}$$

$$Ca = \frac{L' \times c}{L}$$

★  $Ca$  = Total correction

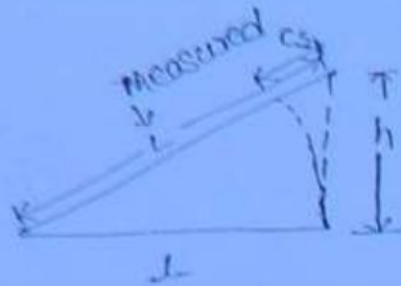
$c$  = Correction required per chain length

$L$  = Designated length of tape / chain

② Measured length	③ Written / noted length	④ Error	⑤ Correction	① Case
More 30.50	less 30.0	(-)ve -0.50m	(+)ve +0.50m	Tape is long
less 20.50m	More 20.0m	(+)ve +0.50m	(-)ve -0.50m	Tape is short

(B) Correction due to slope ( $C_s$ )

(22)



In case of chaining along a sloping area we need to measure the horizontal distance.

$$\text{Slope correction } (C_s) = L - l$$

$$C_s = L - (\sqrt{L^2 - h^2})$$

$$C_s = L - L \left[ 1 - \frac{h^2}{L^2} \right]^{1/2}$$

$$C_s = L - L + \frac{h^2}{2L} + \dots$$

$$\boxed{C_s = \frac{h^2}{2L}}$$

Error always positive (+ve) correction is always (-)ve.

(C) Correction due to alignment  $\Rightarrow$  ( $C_{al}$ )  $\Rightarrow$



If  $h$  is error in alignment.

$L$  = length of line measured

Correction due to alignment

$$C_{al} = \frac{h^2}{2L} \leftarrow \text{Same as slope correction}$$

(Error always (+)ve) [Correction always (-)ve]

(d) Correction due to Temperature:  $\rightarrow$  The correction required  $C_T$

(23)

$$C_T = L (T_m - T_0) \alpha$$

here  $L$  = length of line measured

$T_m$  = Temperature at the time of measurement

$T_0$  = Temperature at the time of Standardization

Case (1) - IF  $T_m$  is more than  $T_0$

$$(T_m > T_0)$$

Tape length is increase.

$$\text{Error} = (+)ve$$

$$\text{Correction} = (-)ve$$

Case (2) = IF  $(T_m < T_0)$

Tape length is decrease

$$\text{Error} = (+)ve$$

$$\text{Correction} = (-)ve$$

(e) Full Correction:  $\rightarrow$

If  $L$  = length of line measured.

$P_m$  = Pull applied at the time of measurement.

$P_0$  = Pull applied at the time of Standardization

Pull Correction

$$C_P = \frac{(P_m - P_0) \cdot L}{AE}$$

$A$  = c/s area of Tape/chain

$E$  = Young's modulus of elasticity of Tape/chain

Case ①  $P_m > P_0$  length increases

Error = (+) ve

Correction = (-) ve

(24)

Case ②  $P_m < P_0$

length is less

Error = (+) ve

Correction = (-) ve

④ Sag correction: →



$$\text{Sag correction } C_{\text{sag}} = \frac{W^2 L}{24 P_m^2} = \frac{(wL)^2 L}{24 P_m^2}$$

✓  $W$  = Total weight of chain

$= wL$

$w$  = weight per meter

$L$  = length of chain / Tape

$P_m$  = Pull applied at the time of measured.

Error - always = (+) ve

Correction - always = (-) ve

⊕ Normal Tension: →

(23)

If  $(P_m > P_0)$  Pull correction is (+)ve  
Sag correction is (-)ve

Pull correction and sag correction neutralize each other

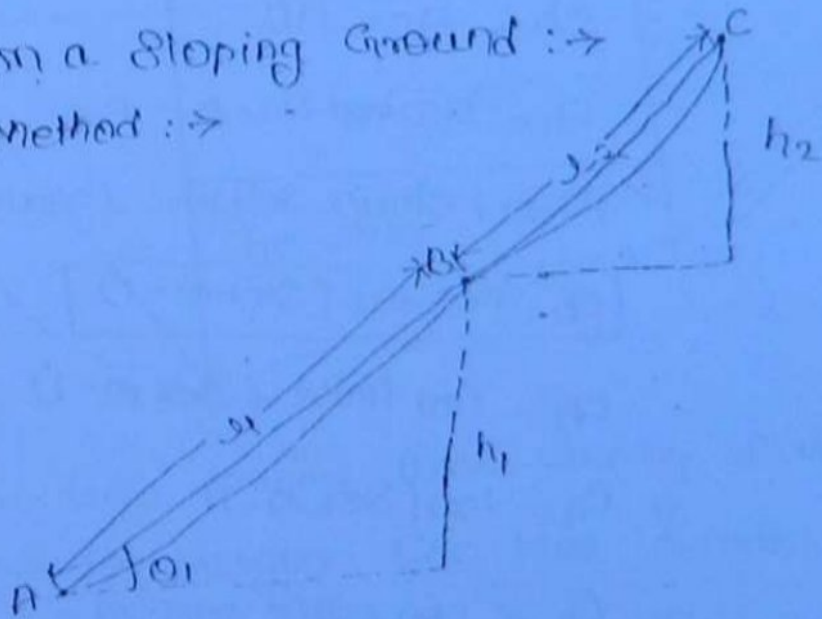
→ the value of pull ( $P_m$ ) for which pull correction is equal to sag correction is called Normal tension

$$\frac{(P_m - P_0)L}{AE} = \frac{w^2 L}{24 P_m^2}$$

This eq. can be solved for total error for ( $P_m$ )

⊕ Chaining on a Sloping Ground: →

⊕ Indirect Method: →



Horizontal distance A to C

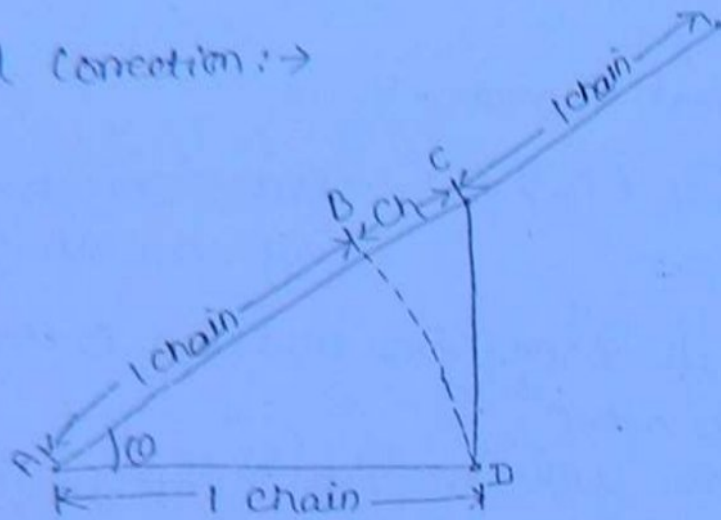
$$= l_1 \cos \theta_1 + l_2 \cos \theta_2 + \dots$$

⊕ Measuring difference of level: →

$$\text{Horizontal distance} = \sqrt{l_1^2 - h_1^2} + \sqrt{l_2^2 - h_2^2} + \dots$$

③ Hypotenusal Correction: →

(26)



Hypotenusal correction  $C_h = BC$

$$\frac{AD}{AC} = \cos \theta$$

$$C_h = AC - AB$$

$$C_h = AD \sec \theta - AB$$

$$C_h = 1 \text{ chain } \sec \theta - 1 \text{ chain}$$

$$C_h = 1 \text{ chain } (\sec \theta - 1)$$

$$C_h = 100 \text{ links } (\sec \theta - 1)$$

$$\Rightarrow C_h = 100 (\sec \theta - 1)$$

$$\Rightarrow C_h = 100 \times \frac{\theta^2}{2}$$

$$\Rightarrow C_h = 50 \theta^2 \text{ links}$$

This correction is applied after each chain, and next chaining is started after correction.