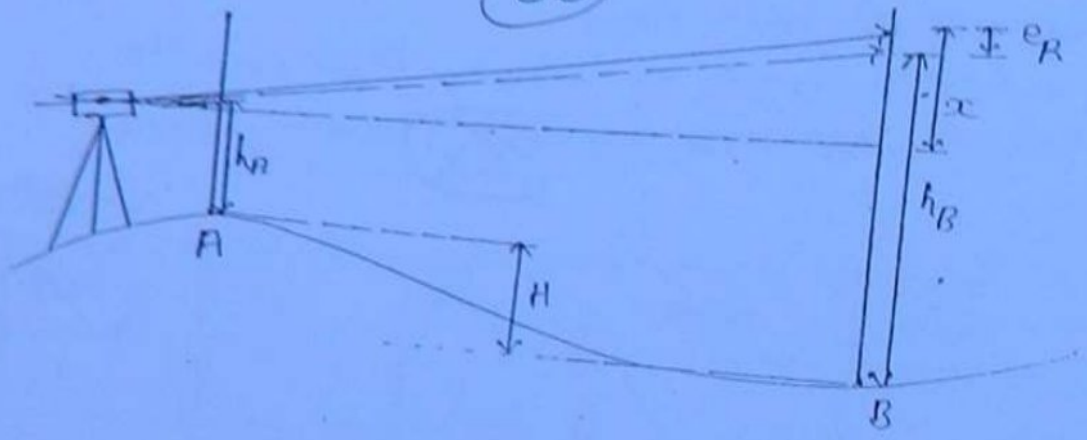
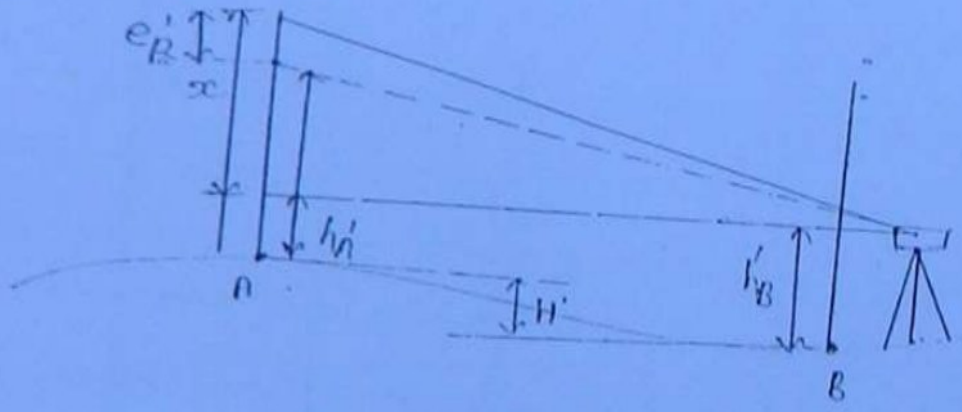
* Reciprocal Levelling :→ Reciprocal levelling is used.

① Find out any error in the levelling instrument. (Line of sight may not be in horizontal direction, even with bubble in the centre.)

② To eliminate the effect of earth curvature and refraction.





In case of reciprocal levelling, two points A and B at a distance about 100 - 200 m. is selected staff reading are noted keeping instrument first near A and then near B.

When Instrument is at A

Reading at A = h_F

Reading at $B = h_B$

When Instrument is at B

reading at $A = h'_A$

Reading at $B = h'_B$

If Instrument is Faulty

$$h_B - h_A \neq h'_B - h'_A \quad (6?)$$

When Instrument is at A

h_A = Correct reading

Correct reading at B should be $= h_B + e_R - x$

Correct difference of Level b/w A and B

$$H = (h_B + e_R - x) - h_A \quad -(1)$$

When Instrument is at B

h'_B = Correct reading

Correct reading at A should be $= h'_A + e'_R - x$

Correct difference of Level b/w B and A

$$H' = h'_B - (h'_A + e'_R - x) \quad -(2)$$

$$e'_R = e_R \text{ or } H' = H$$

Add eq. (1) and (2)

$$H + H' = h_B + e_R - h_A + h'_B - h'_A - e_R$$

$$\Rightarrow 2H = (h_B - h_A) + (h'_B - h'_A)$$

Correct difference of level b/w A and B

$$\Rightarrow \boxed{H = \frac{(h_B - h_A) + (h'_B - h'_A)}{2}}$$ (68)

Problem:→ For a reciprocal leveling following reading was taken

Instrument	Reading		difference	Correct diff
	A	B		
A	1.905 h_A	~~2.852~~ 2.76 h_B	0.91	0.97
B	0.925 h'_A ~~0.905~~	1.955 h'_B	1.03	

Distance b/w point 1 and 2 is 250 m. If R.L of A is 120.50 m. Find out correct R.L of B. Find out the error in line of sight of instrument Neglected error due to curvature and refraction.

Solution:→ Difference of reading when Instrument is at A

$\Rightarrow h_B - h_A = 2.76 - 1.905 = 0.91$

⇒ Difference of reading when Instrument is at B

$h'_B - h'_A = 1.955 - 0.925 = 1.03$

⇒ Correct difference of level b/w A and B

$$H = \frac{(h_B - h_A) + (h'_B - h'_A)}{2} = \frac{0.91 + 1.03}{2}$$

$H = 0.97$ m

⇒ Correct Reading at A = 1.85
when Instrument is at A)

At B should be = 1.85 + 0.97 = 2.82 m

where reading taken = 2.76 < 2.82

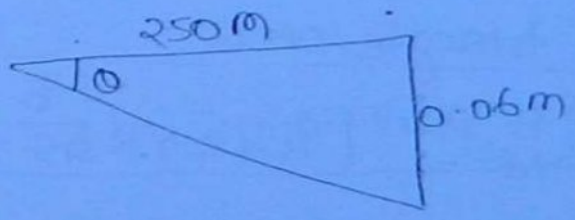
So line of Sight is inclined downward

R.L of A = 120.50 m

R.L of B = 120.50 m
− 0.97 m
119.53 m

(B is at lower elevation than A)

Error in line of Collimation
= 2.82 − 2.76
= 0.06



$$\tan\theta = \frac{0.06}{250} = 2.4 \times 10^{-4}$$

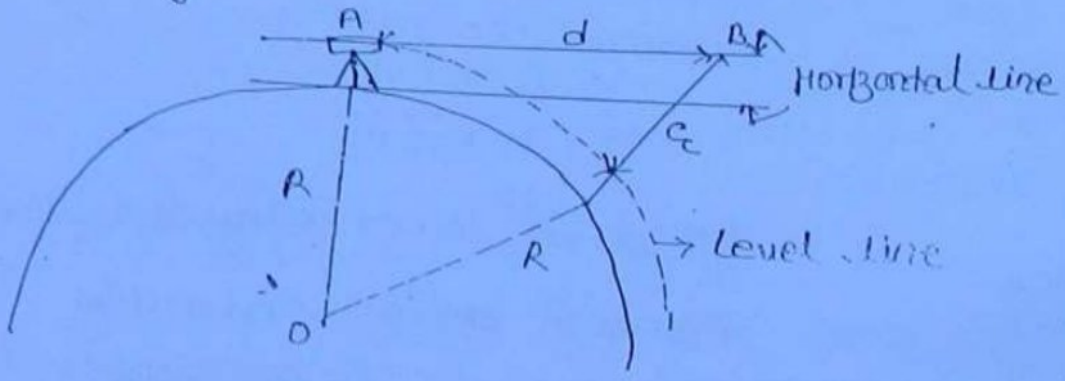
$$\theta = \frac{1}{4166.67}$$

(#) Correction due to curvature and refraction :→

(1) Correction due to Curvature (Earth Curvature) :→

→ Level line ⇒ A line parallel to earth surface (curved line)

→ Horizontal line ⇒ Tangent line at earth surface at any point.



In triangle OAB

$\Rightarrow R^2 + d^2 = (R + C_c)^2$

$\Rightarrow R^2 + d^2 = R^2 + C_c^2 + 2RC_c$

$\Rightarrow d^2 = C_c(2R + C_c)$

$\because 2R + C_c \approx 2R$

⇒ Correction due to curvature

$$\boxed{C_c = \frac{d^2}{2R}}$$

∴ R = Radius of earth or Radius of curvature (6370 km)

Correction due to earth curvature

$$C_c = \frac{d^2}{2 \times 6370} \times 1000 = \text{meter}$$

$$\Rightarrow \boxed{C_c = 0.07849\, d^2 \text{ m}} \quad (7)$$

here d = distance in Km.

(#) for staff reading

Error = (+) ve

Correction = (-) ve

(#) For reduced levels calculated using wrong staff read.

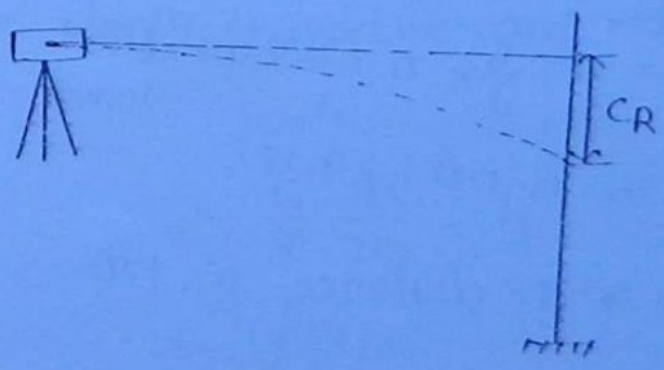
error = (-) ve

Correction = (+) ve

Generally

$$\boxed{C_c = (-)\frac{d^2}{2R} = 0.07849\, d^2}$$

(2) Correction due to refraction :→



This Correction is required due to refraction. The value is ($\frac{1}{7}$) of curvature

C_R = $\frac{1}{7}$ x Correction due to curvature

$\Rightarrow C_R = \frac{1}{7} \times \frac{d^2}{2R}$ (72)

$\Rightarrow C_R = \frac{d^2}{14R}$

$\Rightarrow C_R = \frac{1}{7} \times 0.07849\, d^2$

$\Rightarrow \boxed{C_R = 0.01121\, d^2}$

C_R is always (+) ve

) Combined Correction :⇒

Due to curvature and refraction

$= -\left(\frac{d^2}{2R}\right) + \frac{1}{7}\left(\frac{d^2}{2R}\right)$

$= -\frac{6}{7} \times \frac{d^2}{2R}$

$= -\frac{6}{7} \times 0.07849\, d^2$

$= 0.06729\, d^2$

* d = distance in km.

Note

$$C_c = (-)\frac{d^2}{2R} = \ominus 0.07849\, d^2$$

$$C_R = (+)\frac{1}{7} \times \frac{d^2}{2R} = \oplus 0.01121\, d^2$$

$$C = (-)\frac{6}{7} \times \frac{d^2}{2R} = (-)\ 0.06728\, d^2$$

$R = 6370$

④ Distance of Visible horizon :→

A person at h height from Sea level can see the point at Sea surface upto a distance d.

Combined formula for Correction due earth curvature and refraction is used

hence $C = h$

$$C = \frac{6}{7}\frac{d^2}{2R} = h$$

$$d = \sqrt{\frac{14}{6}Rh}$$

$$C = 0.06728\, d^2$$

$$d = \sqrt{\frac{h}{0.06729}}$$

$$d = 3.855\sqrt{h}$$

If h = in meter
d - in km

(74)

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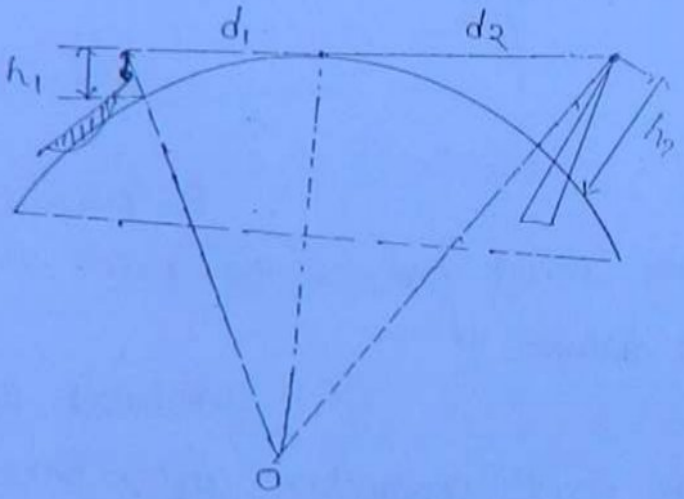
Problem → (b) An observer standing on the deck of a ship just see a light house. The top of light house is 49m above sea level and the height of observaber eye is 9m above sea level. Find the distance of observber from Light house.

Solution →



Distance of observation to light house = $d_1 + d_2$

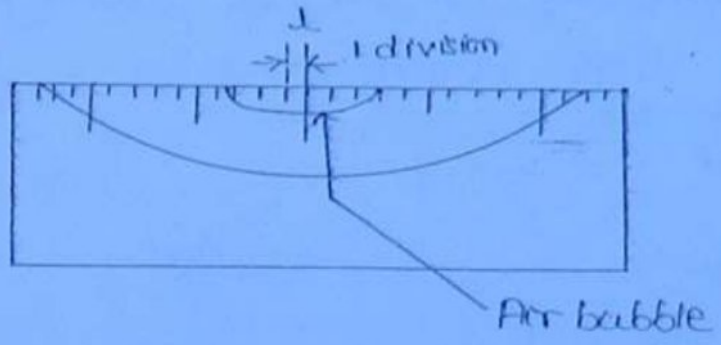
$$= 3.855\sqrt{h_1} + 3.855\sqrt{h_2}$$

$$= 3.855(\sqrt{9} + \sqrt{49})$$

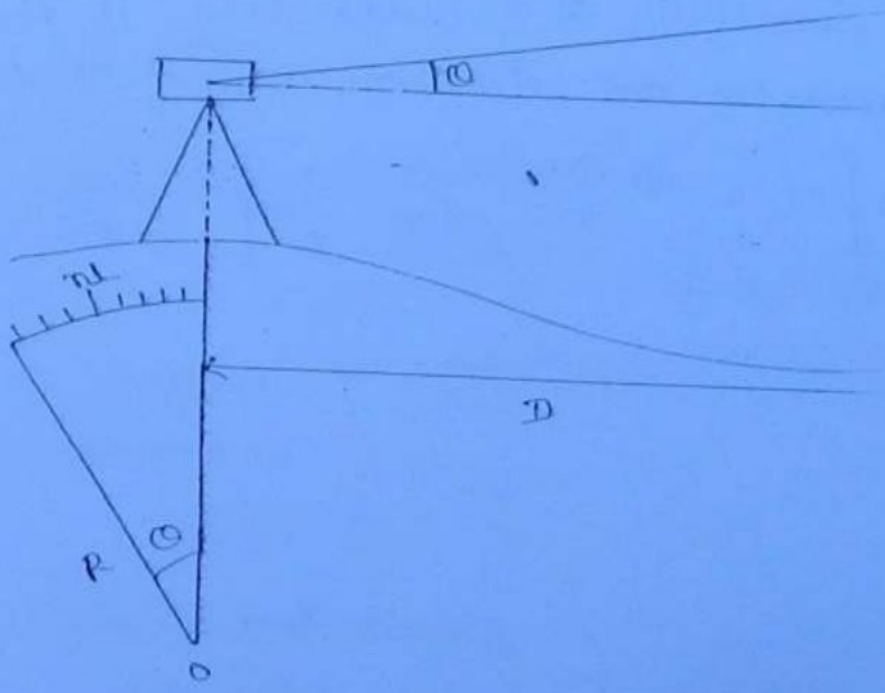
$$= 3.855 \times (3+7)$$

$$= 38.55 \text{ km ans}$$

☆ Sensitiveness of a bubble Tube :→



(75)



Experiment :→

① Fix the instrument at a location take bubble in the centre at on a staff kept

② Now rotated the telescope such that bubble n division.

If l = length of one division.

Total moment of bubble = nl

(3) Radius of curvature of bubble tube = R

(4) Staff reading after rotation = S_2

(5) Staff intercept $S = S_2 - S_1$

total angle

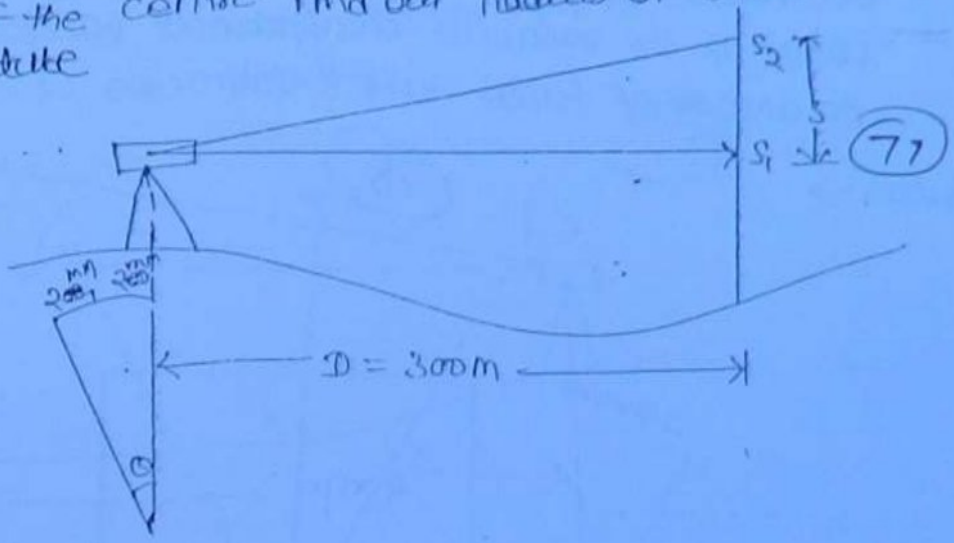
$$\theta = \frac{S}{D} = \frac{nl}{R} \quad \text{—(A)}$$

Sensitivity of a bubble tube is the angle of rotation for one division moment of bubble

$$\Rightarrow \alpha = \frac{\theta}{n} = \frac{S}{nD} = \frac{l}{R} \quad \text{—(B)}$$

Problem :→ If a bubble tube has Sensitivity of 11/01/201 25 Sec length of one division of bubble tube is 2mm. Find out the error in staff reading on a staff kept at 300m distance caused due to bubble 2-division out of the centre. Find out Radius of curvature of bubble tube

Solution:-



Sensitivity of bubble tube

$$\psi = 25 \text{ Sec} = \frac{25}{60\times60} \times \frac{\pi}{180} = \frac{25}{206265} \text{ rad.}$$

$n = 2$

length of one division $l = 2mm$

Sensitivity

$$\psi = \frac{\theta}{n} = \frac{S}{nD} = \frac{l}{R}$$

Error in staff reading

$$S = n\psi D = 2\times\left(\frac{25}{206265}\right)\times 300$$

$$S = 0.0727 m$$

$$\boxed{S = 7.27 m}$$ Ans

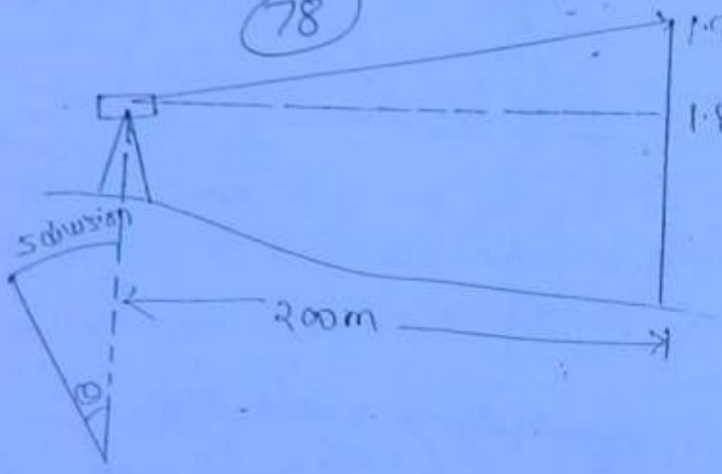
$\Rightarrow \psi = l/R \quad R = l/\psi = 8 \frac{2mm}{25/206265}$

Radius of curvature of the $\boxed{R = 16.502 m}$ mm

Problem:→ The reading taken on a staff kept at 200 m from an instrument with bubble at centre was 1.85m. Bubble is then moved by 5 division out of the centre and an staff reading observed was 1.986 m. Find the sensitivity of bubble tube & what is the radius of curvature of bubble tube. Length of one division of bubble tube is 3mm

Solution:→

78



$S_1 = 1.85\,m$

$S_2 = 1.98\,m$ (after 5 division movement)

$S = S_2 - S_1$

$S = 1.98 - 1.85$

$\boxed{S = 0.136\,m}$

$n = 5$

$l = 3mm$

Sensitivity of bubble tube $\gamma = \frac{\theta}{n} = \frac{S}{nD} = \frac{l}{R}$

$\Rightarrow \gamma = \frac{S}{nD} = \frac{0.136}{5 \times 200} = \theta = 1.36 \times 10^{-4} = 1.36 \times 10^{-4} \times \frac{60 \times 60 \times 180}{\pi}$

$= 28.05$ deg

Radius of curvature

$R = \frac{l}{S\gamma} = \frac{3}{0.136 \times 1.36 \times 10^{-4}} \Rightarrow \boxed{22058.5\ mm}$