20/11/2019

Principles of Dynamics:

Newton's law of motion:

first law: Everybody continues in its state of rest or of uniform motion in a straight line except in so far as it may be compelled by force to change that state.

Second Law:

The acceleration of a given particle is proportional to the force applied to it and takes place in the direction of the straight line in which the force acts.

Third law To every action there is always an equal and contrary reaction or the mutual actions of any two bodies are always equal and oppositely directed.

General Equation of Motion of a Particle:

$$\boxed{ma = f}$$

Differential equation of Rectilinear motion:

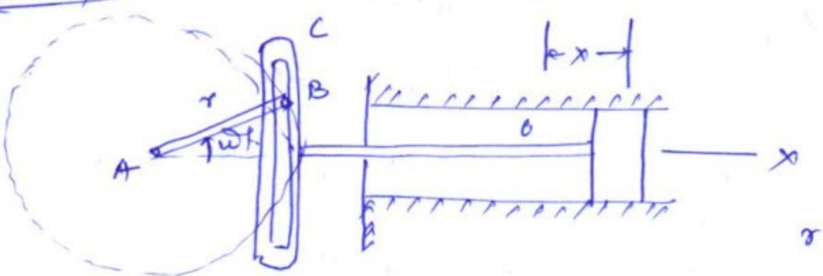
Differential form of equation for rectilinear motion can be expressed as

$$\boxed{\frac{W}{g}\ddot{x} = X}$$

where $\ddot{x}$ = acceleration

X = Resultant acting force.

Example



For the engine shown in fig., the combined wt. of piston and piston rod $W = 450N$, crank radius $r = 250mm$ and uniform speed of rotation $n = 120$ rpm. Determine the magnitude of resultant force acting in piston (a) at extreme position and (b) at the middle position piston has a simple harmonic motion represented displacement-time equation

$$x = r\cos\omega t \quad \text{---(1)}$$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi \times 120}{60} = 4\pi \text{ rad/s.}$$

$$\dot{x} = -r\omega \sin\omega t$$

$$\ddot{x} = -r\omega^2 \cos\omega t \quad \text{---(2)}$$

Differential equation of motion

$$\frac{W}{g}\ddot{x} = X$$

$$\Rightarrow -\frac{W}{g} r\omega^2 \cos\omega t = X$$

$$\Rightarrow X = -\frac{450}{9.81} \times 0.25 (4\pi)^2 \cos(4\pi t)$$

for extreme position

$\cos\omega t = -1$

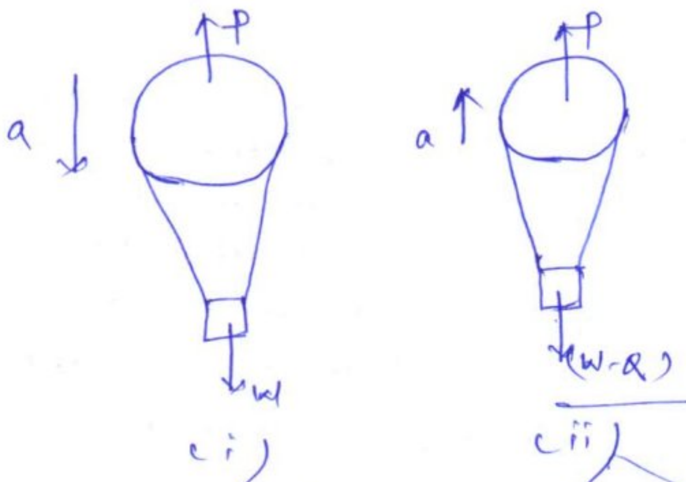
so $\boxed{X = 1810\text{N.}}$

~~for extre~~ middle position $\cos\omega t = 0$.

so Resultant force $= 0$.

E-2 A ballon of gross wt W is falling vertically downward with constant acceleration a. What amount of ballast Q must be thrown out in order to give ballon an equal upward acceleration a.

P = buoyant force.



(i) considering 1st case when ballon is falling,

$$\frac{W}{g}a = W - P \quad \text{---(1)}$$

(ii) $$\frac{W-Q}{g}a = P - (W-Q) \quad \text{---(2)}$$

Eq(1) + Eq(2)

$$\frac{Q}{g}\cdot a = W + W - Q = 2W + Q$$

$$\Rightarrow Q\left(\frac{a}{g} - 1\right) = 2W$$

$$\Rightarrow Q = \frac{2Wg}{(a-g)}$$

2

$$\frac{Wa}{g} = (W-P)$$

$$\frac{(W-Q)a}{g} = P-(W-Q)$$

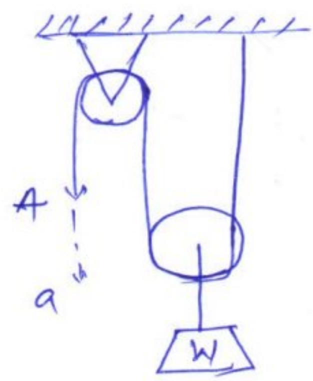
$$\frac{Wa+(W-Q)a}{g} = W-\cancel{P}+\cancel{P}-(W-Q) = Q$$

$$\Rightarrow \frac{Wa+Wa-Qa}{g} = Q$$

$$\Rightarrow 2Wa = Qg+Qa$$

$$\Rightarrow \boxed{Q = \frac{2Wa}{(g+a)}}$$

Q.1 A wt. $W = 4450N$ is supported in a vertical plane by string and pulleys arranged shown in fig. If the free end A of ~~oth~~ the string is pulled vertically downward with constant acceleration $a = 18\,m/s^2$ find tension S in the string.



Differential equation of motion for the system is

$$2S - W = \frac{W}{g} \times \frac{a}{2}$$

$$\Rightarrow 2S = W + \frac{Wa}{2g}$$

$$= \cancel{\frac{W}{2}\left(2+\frac{a}{2g}\right)}$$

$$= W\left(1+\frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W}{2}\left(1+\frac{a}{2g}\right)$$

$$= \frac{4450}{2}\left(1+\frac{18}{2\times 9.81}\right) = \boxed{4266.28\ N.}$$

$$\frac{Wa}{g} = (W - P)$$

$$\frac{(W-Q)a}{g} = P - (W-Q)$$

$$\frac{Wa + (W-Q)a}{g} = W - \cancel{P} + \cancel{P} - (W-Q) = Q$$

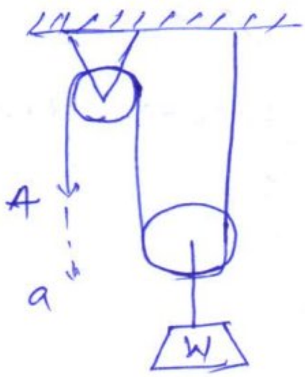
$$\Rightarrow \frac{Wa + Wa - Qa}{g} = Q$$

$$\Rightarrow 2Wa = Qg + Qa$$

$$\Rightarrow \boxed{Q = \frac{2Wa}{(g+a)}}$$

Q.1 A wt. W = 4450N is supported in a vertical plane by string and pulleys arranged shown in fig. If the free end A of ~~sth~~ the string is pulled vertically downward with constant acceleration $a = 18\ m/s^2$ find tension S in the string.

Differential equation of motion for the system is



$$2S - W = \frac{W}{g} \times \frac{a}{2}$$

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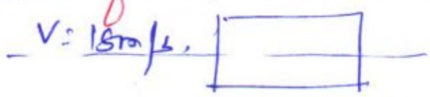
$$= \cancel{\frac{W}{2}\left(2 + \frac{a}{2g}\right)}$$

$$= W\left(1 + \frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W}{2}\left(1 + \frac{a}{2g}\right)$$

$$= \frac{4450}{2}\left(1 + \frac{18}{2 \times 9.81}\right) = \boxed{4266.28\ N.}$$

Q.2 An elevator of gross wt $W = 4450N$ starts to move upward direction with a constant acceleration and acquires a velocity $v = 18m/s$, after travelling a distance $= 1.80m$. find tensile force S in the cable during it's motion.

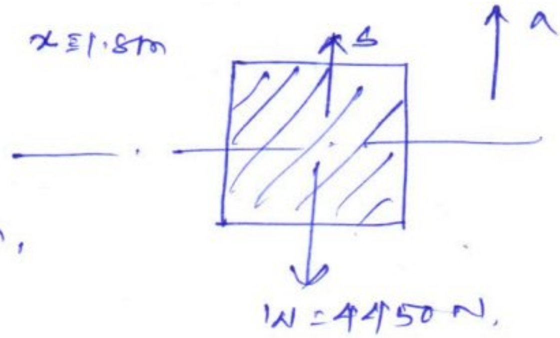


$W = 4450N$.

$V = 18m/s$.

initial velocity $u = 0$

distance travelled $x = 1.8m$,



$$S - W = \frac{W}{g} \cdot a$$

$$\Rightarrow S = W + \frac{W}{g} a = W\left(1 + \frac{a}{g}\right) \quad \text{---(1)}$$

Now applying equation of kinematics

$$v^2 - u^2 = 2as$$

$$\Rightarrow 18^2 - 0 = 2a \times 1.8$$

$$\Rightarrow a = \frac{18^2}{2 \times 1.8} = \boxed{90\ m/s^2}$$

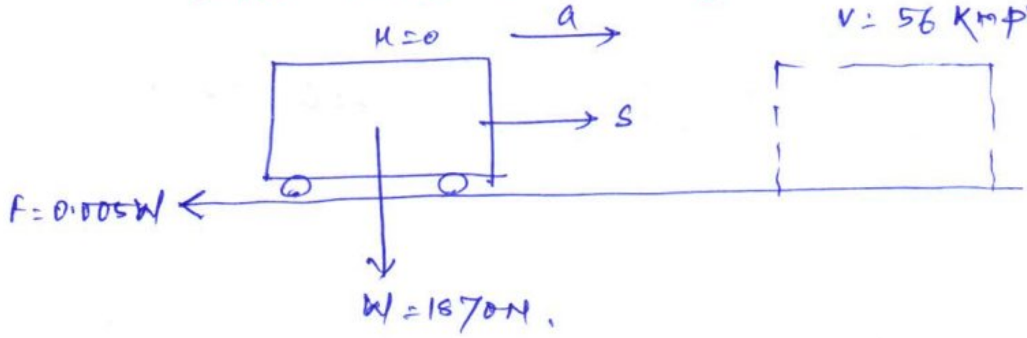
substituting the values of a in eq. (1)

$$S = 4450\left(1 + \frac{90}{9.81}\right) = \boxed{45275.7\ N.}$$

A-1

Q.3 A train weighing 1870N without the locomotive starts to move with constant acceleration along a straight track and in first 60s acquires a velocity of 56 Kmph. Determine the tension S in draw bar both locomotive and train if the air resistance is 0.005 times the wt. of the train.

$V = 56\ Kmph = 15.56\ m/s$.



$S - F = \frac{W}{g} \cdot a$

$\Rightarrow S = 0.005W + \frac{Wa}{g}$ —(1)

from eq. of kinematics,

$v = u + at$

$\Rightarrow a = \left(\frac{15.56 - 0}{60}\right) = 0.26 \, m/sec^2$

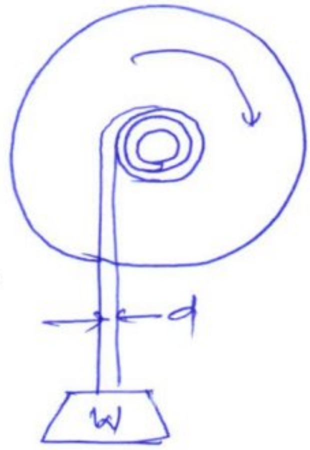
substituting the value of a in eq. (1)

$S = W\left(0.005 + \frac{a}{g}\right)$

$= 1870\left(0.005 + \frac{0.26}{9.81}\right) = \boxed{58.9 \text{ kN.}}$

A-2
Q.4

A wt. W is attached to the end of a small flexible rope of dia. d = 6.25mm. and is raised vertically by winding the rope on a reel. if the reel is turned uniformly at a rate of 2 rps. what will be the tension in rope.



dia of rope d = 6.25mm = 0.00625m.

No of revolutions N = 2rps.

let x = initial radius of reel.

t = time taken for N revolutions.

Net radius after t sec.

$R = [x + (Nt d)]$

Now mean velocity $v = R\omega$

$\omega = 2\pi N$.

$\therefore v = (x + Ntd)\, 2\pi N$

acceleration of rope = $a = \frac{dv}{dt}$

$a = \frac{d}{dt}\left[2\pi N x + 2\pi N^2 t d\right] = 2\pi N^2 d$

$S - W = \frac{W}{g} \cdot a \Rightarrow S = W + \frac{Wa}{g} = W\left(1 + \frac{a}{g}\right)$

$\Rightarrow S = W\left(1 + \frac{2\pi N^2 d}{g}\right)$ ~~W(1+~~

$$\Rightarrow S = W\left(1 + \frac{2\pi \times 2^2 \times 0.00625}{9.81}\right)$$

$=$

Ans-3

Q.5

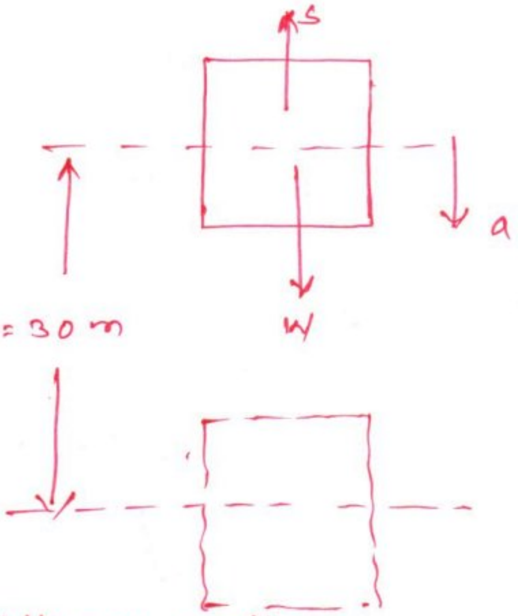
A mine cage of wt $W = 8.9$ kN starts from rest and moves downward with constant acceleration travelling a distance $s = 30$ m in 10 sec. find the tensile force in the cable.

Wt. of cage $W = 8.9$ kN.

initial velocity $u = 0$.

distance travelled $s = 30$ m

time $t = 10$ sec.



$$s = \cancel{ut}^{0} + \frac{1}{2}at^2$$

$$\Rightarrow 30 = \frac{1}{2}a \times 10^2$$

$$\Rightarrow t = \frac{60}{10^2} = \boxed{0.6\ m/sec^2}$$

Differential equation of rectilinear motion

$$W - S = \frac{W}{g} \cdot a$$

$$\Rightarrow S = W - \frac{W}{g}a = W\left(1 - \frac{a}{g}\right)$$

$$= 8.9\left(1 - \frac{0.6}{9.81}\right)$$

$$\Rightarrow \boxed{S = 8.35\ kN.}$$ (Ans)