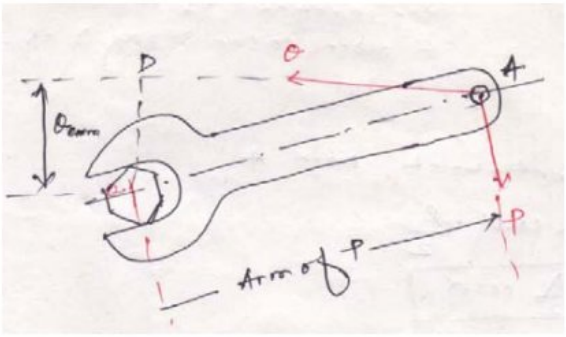
Method of moments

Moment of a force with respect to a point:



- Considering wrench subjected to two forces P and Q of equal magnitude. It is evident that force P will be more effective compared to Q, though they are of equal magnitude.
- The effectiveness of the force as regards it is the tendency to produce rotation of a body about a fixed point is called the moment of the force with respect to that point.
- Moment = Magnitude of the force × Perpendicular distance of the line of action of force.
- Point O is called moment centre and the perpendicular distance (i.e. OD) is called moment arm.
- Unit is N.m

Theorem of Varignon:

The moment of the resultant of two concurrent forces with respect to a centre in their plane is equal to the alzebric sum of the moments of the components with respect to some centre.

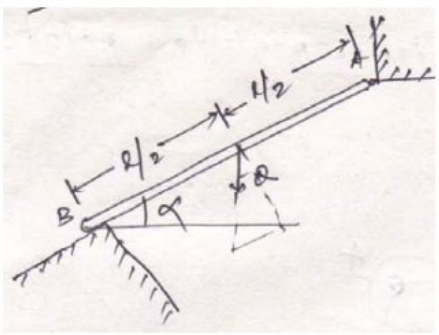
Problem 1:

A prismatic clear of AB of length l is hinged at A and supported at B. Neglecting friction, determine the reaction R_b produced at B owing to the weight Q of the bar.

Taking moment about point A,

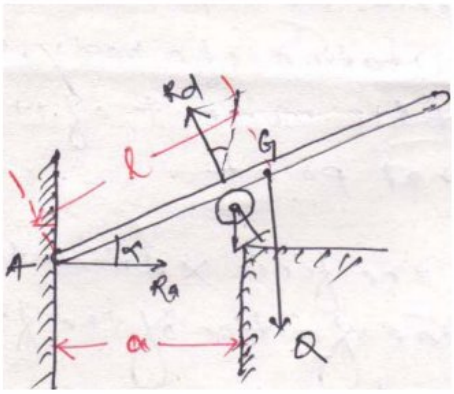
$$R_b \times l = Q\cos\alpha.\frac{l}{2}$$

$$\Rightarrow R_b = \frac{Q}{2}\cos\alpha$$



Problem 2:

A bar AB of weight Q and length 2l rests on a very small friction less roller at D and against a smooth vertical wall at A. Find the angle α that the bar must make with the horizontal in equilibrium.



B

Resolving vertically,

$R_d \cos\alpha = Q$

Now taking moment about A,

$$\frac{R_d.a}{\cos\alpha} - Q.l\cos\alpha = 0$$

$$\Rightarrow \frac{Q.a}{\cos^2\alpha} - Q.l\cos\alpha = 0$$

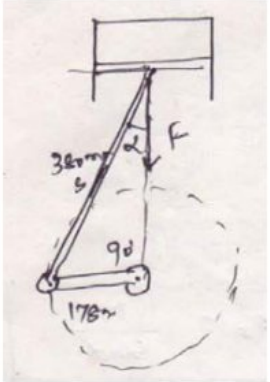
$$\Rightarrow Q.a - Q.l\cos^3\alpha = 0$$

$$\Rightarrow \cos^3\alpha = \frac{Q.a}{Q.l}$$

$$\Rightarrow \alpha = \cos^{-1}\sqrt[3]{\frac{a}{l}}$$

Problem 3:

If the piston of the engine has a diameter of 101.6 mm and the gas pressure in the cylinder is 0.69 MPa. Calculate the turning moment M exerted on the crankshaft for the particular configuration.



Area of cylinder

$$A = \frac{\pi}{4}(0.1016)^2 = 8.107\times10^{-3} m^2$$

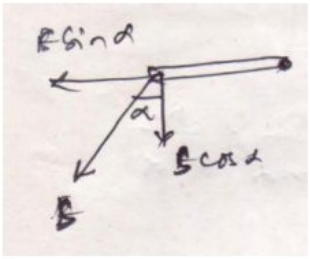
Force exerted on connecting rod,

F = Pressure × Area
$= 0.69\times10^{6} \times 8.107\times10^{-3}$
$= 5593.83$ N

Now $\alpha = \sin^{-1}\left(\frac{178}{380}\right) = 27.93°$

$$S\cos\alpha = F$$

$$\Rightarrow S = \frac{F}{\cos\alpha} = 6331.29N$$

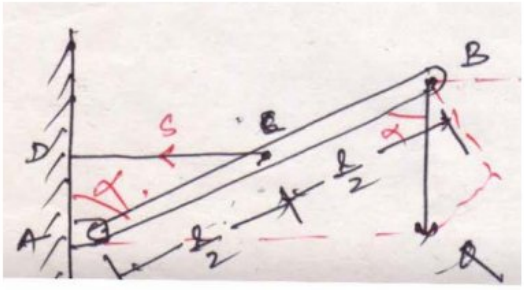


Now moment entered on crankshaft,

$$S\cos\alpha \times 0.178 = 995.7N = 1KN$$

Problem 4:

A rigid bar AB is supported in a vertical plane and carrying a load Q at its free end. Neglecting the weight of bar, find the magnitude of tensile force S in the horizontal string CD.



Taking moment about A,

$$\sum M_A = 0$$

$$S.\frac{l}{2}\cos\alpha = Q.l\sin\alpha$$

$$\Rightarrow S = \frac{Q.l\sin\alpha}{\frac{l}{2}\cos\alpha}$$

$$\Rightarrow S = 2Q.\tan\alpha$$