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# D'Alembert's Principle

Differential equation of motion (rectilinear) can be written as

$$X - m\ddot{x} = 0 \quad \text{---} (1)$$

Where $X$ = Resultant of all applied force in the direction of motion

$m$ = mass of the particle

The above equation may be treated as equation of dynamic equilibrium. To express this equation, in addition to the real force acting on the particle a fictitious force $m\ddot{x}$ is required to be considered. This force is equal to the product of mass of the particle and it's acceleration and directed in opposit direction, and is called the inertia force of the particle.

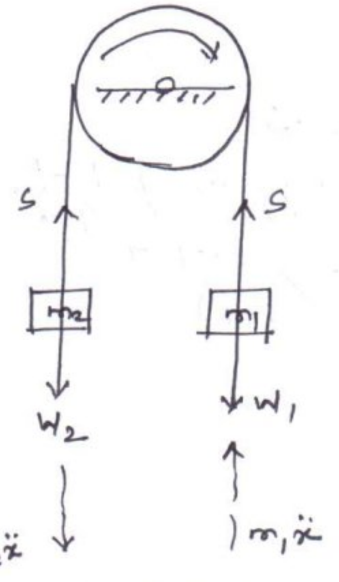
$$\boxed{-\sum m\ddot{x} = -\ddot{x}\sum m = -\frac{W}{g}\ddot{x}}$$

Where $W$ = total weight of the body

so the equation of dynamic equilibrium can be expressed as:

$$\boxed{\sum X_i + \left(-\frac{W}{g}\ddot{x}\right) = 0} \quad \text{---} (2)$$

## Example 1




for the example shown considering the motion of pulley as shown by the arrow mark. where upward acceleration $\ddot{x}_2$ for $W_2$ and downward acceleration $\ddot{x}_1$ for $W_1$

- corresponding inertia forces and their direction are indicated by dotted line.
- By adding inertia forces to the real forces (such as $W_1, W_2$ and tension in strings S) we obtain, for each particle, a system of forces in equilibrium.

The equilibrium equation for the entire system without S

$$W_2 + m_2\ddot{x} = W_1 - m_1\ddot{x}$$

$$\Rightarrow (m_1 + m_2)\ddot{x} = (W_1 - W_2) \Rightarrow \boxed{\ddot{x} = \frac{W_1 - W_2}{(W_1 + W_2)} \cdot g}$$

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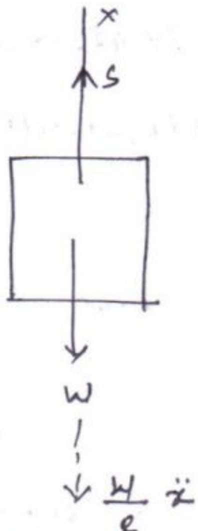
Example 2

A ~~body~~ is moving in upward direction by a rope.

So the equation of dynamic equilibrium considering the real and inertia forces.

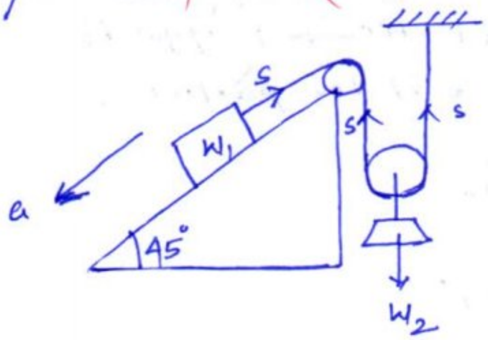


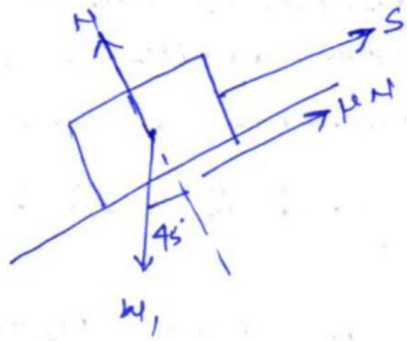

$S - W - \frac{W}{g}a = 0$, so tensile force in rope

$\Rightarrow \boxed{S = W\left(1 + \frac{a}{g}\right)}$

Q.1 Find tensions in the string during motion of the system if $W_1 = 900N$, $W_2 = 450N$. The $\mu$ betn the inclined plane and block $W_1 = 0.2$

X




When $W_1$ moves downward in the inclined plane with an acceleration $a$, then acceleration of $W_2 = \frac{a}{2}$

Considering dynamic equilibrium of $W_1$, from D' Alembert's principle

$$\left(W_1 \sin 45° - \mu N - S\right) - \frac{W_1}{g}a = 0$$

$$\Rightarrow \frac{W_1}{g}a = W_1 \sin 45° - \mu N - S$$

$$= W_1 \sin 45 - \mu W_1 \cos 45 - S$$

$$\Rightarrow a = \left(900 \times \frac{1}{\sqrt{2}} - 0.2 \times 900 \times \frac{1}{\sqrt{2}} - S\right)\frac{9.81}{900}$$

$$= (636.4 - 127.28 - S)\,0.0109$$

$$\Rightarrow a = \cancel{5.549}\ 6.93676 - 1.387352 - 0.0109S \quad \text{---(1)}$$

Similarly ~~from~~ for weight $W_2$

$$2S - W_2 - \frac{W_2}{g}\frac{a}{2} = 0$$

$$\Rightarrow \cancel{\frac{W_2 a}{2g}} = W_2\left(1 + \frac{a}{2g}\right) = 2S$$

$$\Rightarrow S = \frac{450}{2}\left(1 + \frac{a}{19.62}\right) = 225 + 11.46a \quad \text{---(2)}$$

Substituting the value of S in eq. (1)

~~a = 5.549 = 2.4525 − 0.125a~~ $\Rightarrow a =$

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$a = 6.93676 - 1.387352 - 0.0109\,(225 + 11.46\,a)$

$= \cancel{6.93}\ 5.549408 - 2.4525 - 0.124914\,a$

$= 3.096908 - 0.124914\,a$

$\Rightarrow \boxed{a = 2.75\ m/s^2}$

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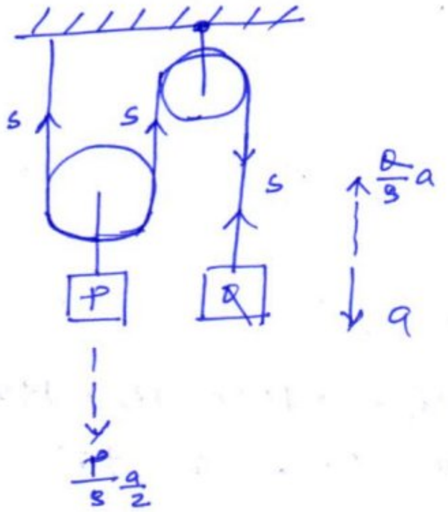
**Q.2** Two weights P and Q are connected by the arrangement shown in fig. Neglecting friction and inertia of pulley and cord find the acceleration a of wt. Q

Assume P = 178 N, Q = 133.5 N.



Applying D'Alembert's principle for Q

$Q - S - \frac{Q}{g} a = 0$

$\Rightarrow S = Q\left(1 - \frac{a}{g}\right)$ — (1)

$= 133.5\left(1 - \frac{a}{9.81}\right)$

Applying D'Alembert's principle to P

~~P - 2S = P~~

$2S - P - \frac{Pa}{2g} = 0$

$\Rightarrow 2S = P\left(1 + \frac{a}{2g}\right)$

$\Rightarrow S = \frac{P}{2}\left(1 + \frac{a}{2g}\right)$ — (2)

$= \frac{178}{2}\left(1 + \frac{a}{19.62}\right)$

$133.5\left(1 - \frac{a}{9.81}\right) = 89\left(1 + \frac{a}{19.62}\right)$

$\Rightarrow 133.5 - 13.608\,a = 89 + 4.536\,a$

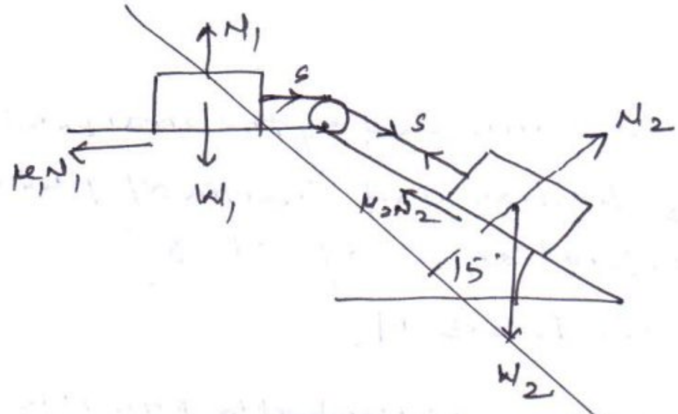
$\Rightarrow 18.144\,a = 44.5$

$\Rightarrow \boxed{a = 2.45\ m/s^2}$ (Ans)

~~**Q.3** Assuming the car in the fig. to have a velocity of 6 m/s find shortest distance s in which it can be stopped with constant deceleration without disturbing the block. Data: c = 0.6 m, h = 0.9 m~~

~~$\mu = 0.5$~~

Q.3 Two blocks of wt $W_1 = 150N$ and $W_2 = 500N$ are connected by an inextensible string. Find the acceler of the blocks and tension in the string. $\mu_1 = 0.1$, $\mu_2 = 0.$




for block 1

$$S - \mu_1 N_1 = 0$$

$$\Rightarrow S = \mu_1 W_1 = 0.1 \times 150 = 15N.$$

for block 2

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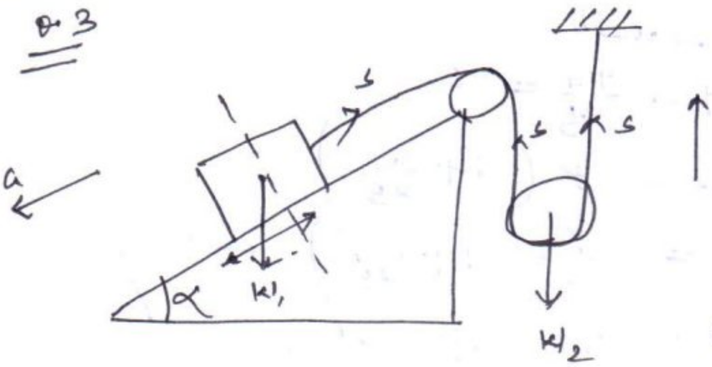
Q.3




$W_1 = 890N$ $W_2 = 445N$.
$\mu = 0.2$ $\alpha = 45°$
find S.

considering equilibrium of $W_1$ and applying D'Alembert's principle

$$W_1 \sin 45° - \mu N_1 - S - \frac{W_1}{g} a = 0$$

$$\Rightarrow S = W_1 \sin 45 - \mu N - \frac{W_1}{g} a$$

$$= \frac{890}{\sqrt{2}} - 0.2 \times 890 \times \frac{1}{\sqrt{2}} - \frac{890}{9.81} a$$

$$= 629.32 - 125.865 - 90.72a$$

$$\boxed{S = 503.455 - 90.72a} \quad \text{---(1)}$$

Applying D'Alembert's principle for $W_2$

$$2S - W_2 - \frac{W_2}{g}\frac{a}{2} = 0$$

$$\Rightarrow 2S = W_2\left(1 + \frac{a}{2g}\right)$$

$$\Rightarrow S = \frac{W_2}{2}\left(1 + \frac{a}{2g}\right) = \frac{445}{2}\left(1 + \frac{a}{19.62}\right) = 222.5 + 11.34a \quad \text{---(2)}$$

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equating (1) and (2)

$503.455 - 90.72a = 222.5 + 11.34a$

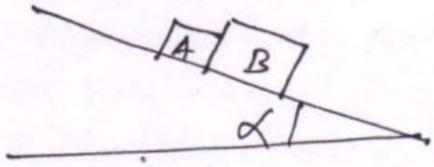
$\Rightarrow 102.0604\,a = 280.955$

$\Rightarrow \boxed{a = 2.75\ m/s^2}$

so $S = 222.5 + 11.34 \times 2.75$

$= \boxed{253.71\ N.}$

Q.4

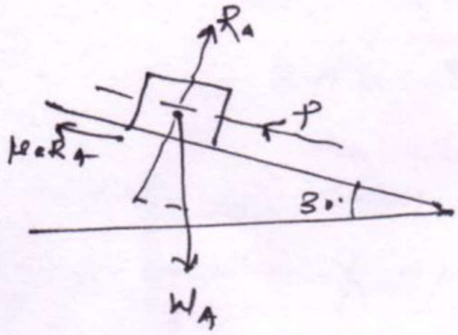


$W_A = 44.5N$ $\quad W_B = 89N$

$\alpha = 30°$ $\quad \mu_a = 0.15$

$\mu_B = 0.3$

find pressure P. betn blocks.



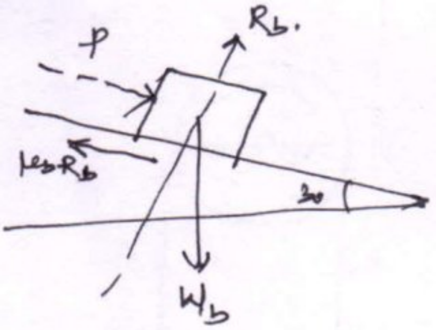
$W_a \sin 30 - P - \mu_a R_a - \frac{W_a}{g} a = 0$

$\Rightarrow P = W_a \sin 30 - \mu_a R_a - \frac{W_a}{g} a$

$= 44.5 \times \frac{1}{2} - 0.15 \times 44.5 \times \cos 30 - \frac{44.5}{9.81} a$

$= 22.25 - 5.78 - 4.53a$

$= 16.47 - 4.53a \quad —(1)$



$P + W_b \sin 30 - \mu_b R_b - \frac{W_b}{g} a = 0$

$\Rightarrow P = -\frac{W_b}{2} + 0.3 \times 89 \cos 30 + \frac{89}{9.81} a$

$= -\frac{89}{2} + 23.122 + 9.07a$

$= -21.378 + 9.07a \quad —(2)$

$16.47 - 4.53a = -21.378 + 9.07a$

$\Rightarrow 13.6a = 37.848$

$\Rightarrow a = 2.78\ m/s^2$

$P = 3.57\ N.$