$$\sum M_A = 0$$
$$S_b \times l = P \times \frac{l}{4} + Q \times \frac{l}{2}$$
$$\Rightarrow S_b = \frac{P}{4} + \frac{Q}{2}$$
$$\Rightarrow S_b = \frac{89}{4} + \frac{44.5}{2}$$
$$\Rightarrow S_b = 44.5$$
$$\therefore S_a = 133.5 - 44.5$$
$$\Rightarrow S_a = 89N$$

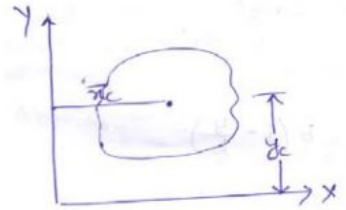
Centre of gravity

Centre of gravity: It is that point through which the resultant of the distributed gravity force passes regardless of the orientation of the body in space.

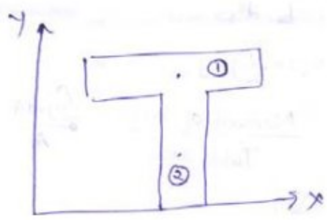
- As the point through which resultant of force of gravity (weight) of the body acts.

Centroid: Centrroid of an area lies on the axis of symmetry if it exits.

Centre of gravity is applied to bodies with mass and weight and centroid is applied to plane areas.



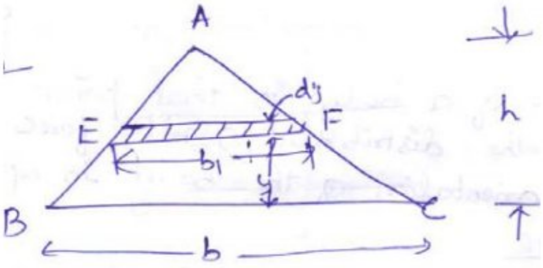
$$x_c = \sum A_i x_i$$
$$y_c = \sum A_i y_i$$



$$x_c = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$
$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$x_c = y_c = \frac{\text{Moment of area}}{\text{Total area}}$$
$$x_c = \frac{\int x.dA}{A}$$
$$y_c = \frac{\int y.dA}{A}$$

Problem 1: Consider the triangle ABC of base 'b' and height 'h'. Determine the distance of centroid from the base.



Let us consider an elemental strip of width 'b_1' and thickness 'dy'.

$$\Delta AEF \sim \Delta ABC$$

$$\therefore \frac{b_1}{b} = \frac{h-y}{h}$$

$$\Rightarrow b_1 = b\left(\frac{h-y}{h}\right)$$

$$\Rightarrow b_1 = b\left(1-\frac{y}{h}\right)$$

Area of element EF (dA) = $b_1 \times dy$

$$= b\left(1-\frac{y}{h}\right)dy$$

$$y_c = \frac{\int y.dA}{A}$$

$$= \frac{\int_0^h yb\left(1-\frac{y}{h}\right)dy}{\frac{1}{2}b.h}$$

$$= \frac{b\left[\frac{y^2}{2}-\frac{y^3}{3h}\right]_0^h}{\frac{1}{2}b.h}$$

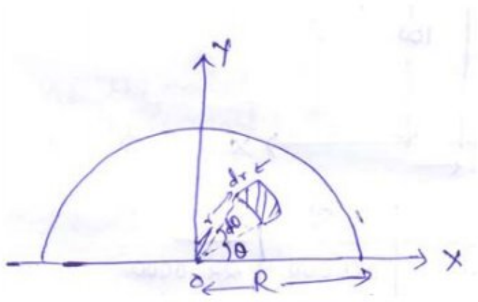
$$= \frac{2}{h}\left[\frac{h^2}{2}-\frac{h^3}{3}\right]$$

$$= \frac{2}{h}\times\frac{h^2}{6}$$

$$= \frac{h}{3}$$

Therefore, y_c is at a distance of h/3 from base.

Problem 2: Consider a semi-circle of radius R. Determine its distance from diametral axis.



Due to symmetry, centroid 'y_c' must lie on Y-axis.

Consider an element at a distance 'r' from centre 'o' of the semicircle with radial width dr.

Area of element = (r.dθ)×dr

Moment of area about x = $\int y.dA$

$$= \int_0^{\pi}\int_0^{R} (r.d\theta).dr \times (r.\sin\theta)$$

$$= \int_0^{\pi}\int_0^{R} r^2 \sin\theta .dr.d\theta$$

$$= \int_0^{\pi}\int_0^{R} (r^2.dr).\sin\theta .d\theta$$

$$= \int_0^{\pi} \left[\frac{r^3}{3}\right]_0^R .\sin\theta .d\theta$$

$$= \int_0^{\pi} \frac{R^3}{3}.\sin\theta .d\theta$$

$$= \frac{R^3}{3}\left[-\cos\theta\right]_0^{\pi}$$

$$= \frac{R^3}{3}[1+1]$$

$$= \frac{2}{3}R^3$$

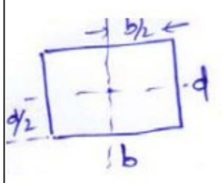
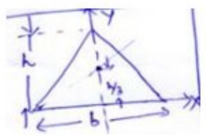
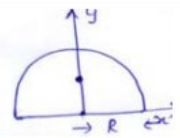
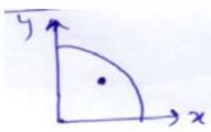
$$y_c = \frac{\text{Moment of area}}{\text{Total area}}$$

$$= \frac{\frac{2}{3}R^3}{\frac{\pi R^2}{2}}$$

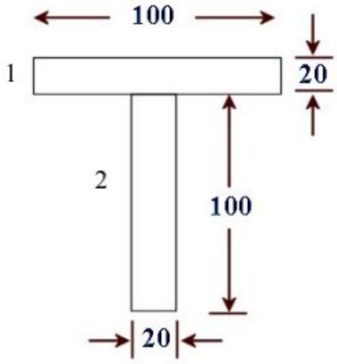
$$= \frac{4R}{3\pi}$$

Therefore, the centroid of the semicircle is at a distance of $\frac{4R}{3\pi}$ from the diametric axis.

Centroids of different figures

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Rectangle		$\frac{b}{2}$	$\frac{d}{2}$	bd
Triangle		0	$\frac{h}{3}$	$\frac{bh}{2}$
Semicircle		0	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{4}$

Problem 3: Find the centroid of the T-section as shown in figure from the bottom.

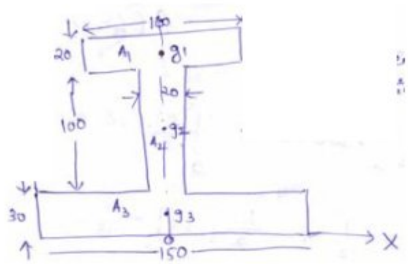


Area (A_i)	x_i	y_i	$A_i\ x_i$	$A_i\ y_i$
2000	0	110	10,000	22,0000
2000	0	50	10,000	10,0000
4000			20,000	32,0000

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{32,0000}{4000} = 80$$

Due to symmetry, the centroid lies on Y-axis and it is at distance of 80 mm from the bottom.

Problem 4: Locate the centroid of the I-section.



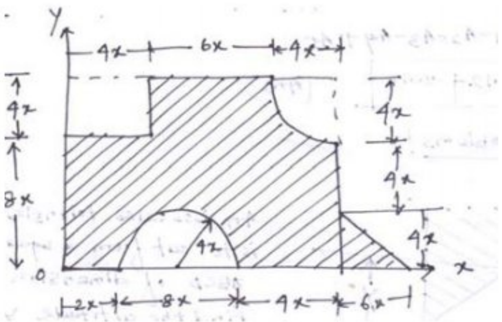
As the figure is symmetric, centroid lies on y-axis. Therefore, $\overline{x} = 0$

Area (A_i)	x_i	y_i	$A_i\ x_i$	$A_i\ y_i$
2000	0	140	0	280000
2000	0	80	0	160000
4500	0	15	0	67500

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = 59.71mm$$

Thus, the centroid is on the symmetric axis at a distance 59.71 mm from the bottom.

Problem 5: Determine the centroid of the composite figure about x-y coordinate. Take x = 40 mm.



A_1 = Area of rectangle = 12x.14x=168x^2
A_2 = Area of rectangle to be subtracted = 4x.4x = 16 x^2

A_3 = Area of semicircle to be subtracted = $\frac{\pi R^2}{2} = \frac{\pi(4x)^2}{2} = 25.13x^2$

A_4 = Area of quatercircle to be subtracted = $\frac{\pi R^2}{4} = \frac{\pi(4x)^2}{4} = 12.56x^2$

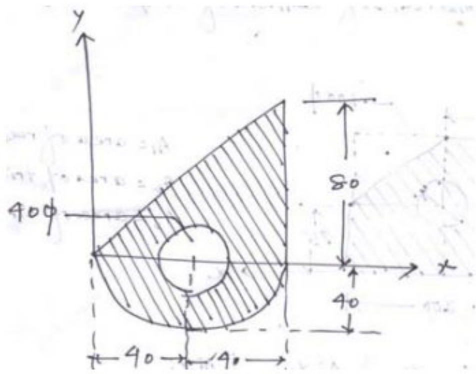
A_5 = Area of triangle = $\frac{1}{2} \times 6x \times 4x = 12x^2$

Area (A_i)	**x_i**	**y_i**	**$A_i x_i$**	**$A_i y_i$**
A_1 = 268800	7x = 280	6x =240	75264000	64512000
A_2 = 25600	2x = 80	10x=400	2048000	10240000
A_3 = 40208	6x =240	$\frac{4\times 4x}{3\pi}$=67.906	9649920	2730364.448
A_4 = 20096	$10x+\left(4x-\frac{4\times 4x}{3\pi}\right)$ = 492.09	$8x+\left(4x-\frac{4\times 4x}{3\pi}\right)$ = 412.093	9889040.64	8281420.926
A_5 = 19200	$14x+\frac{6x}{3}=16x$ = 640	$\frac{4x}{3}=53.33$	12288000	1023936

$$x_c = \frac{A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4 + A_5x_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 326.404mm$$

$$y_c = \frac{A_1y_1 - A_2y_2 - A_3y_3 - A_4y_4 + A_5y_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 219.124mm$$

Problem 6: Determine the centroid of the following figure.



A_1 = Area of triangle = $\frac{1}{2} \times 80 \times 80 = 3200m^2$

A_2 = Area of semicircle = $\frac{\pi d^2}{8} - \frac{\pi R^2}{2} = 2513.274m^2$

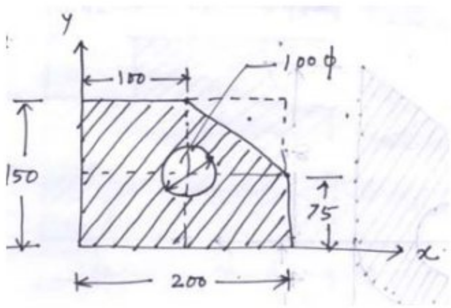
A_3 = Area of semicircle = $\frac{\pi D^2}{2} = 1256.64m^2$

Area (A_i)	x_i	y_i	A_i x_i	A_i y_i
3200	2×(80/3)=53.33	80/3 = 26.67	170656	85344
2513.274	40	$\frac{-4\times 40}{3\pi} = -16.97$	100530.96	-42650.259
1256.64	40	0	50265.6	0

$$x_c = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A_1 + A_2 + A_3} = 49.57mm$$

$$y_c = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A_1 + A_2 - A_3} = 9.58mm$$

Problem 7: Determine the centroid of the following figure.



A_1 = Area of the rectangle
A_2 = Area of triangle
A_3 = Area of circle

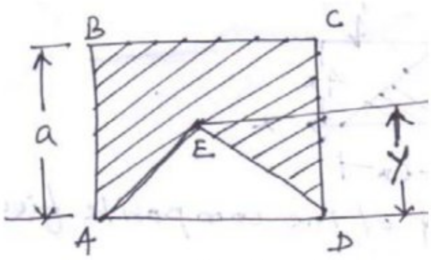
Area (A_i)	x_i	y_i	A_i x_i	A_i y_i
30,000	100	75	3000000	2250000
3750	100+200/3 = 166.67	75+150/3 =125	625012.5	468750
7853.98	100	75	785398	589048.5

$$x_c = \frac{\sum A_i x_i}{\sum A_i} = \frac{A_1x_1 - A_2x_2 - A_3x_3}{A_1 - A_2 - A_3} = 86.4mm$$

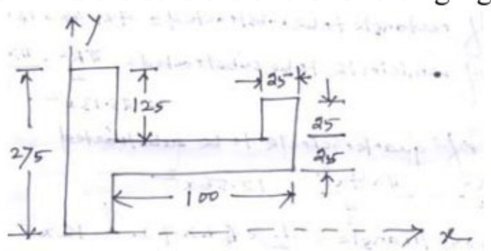
$$y_c = \frac{\sum A_i y_i}{\sum A_i} = \frac{A_1y_1 - A_2y_2 - A_3y_3}{A_1 - A_2 - A_3} = 64.8mm$$

Numerical Problems (Assignment)

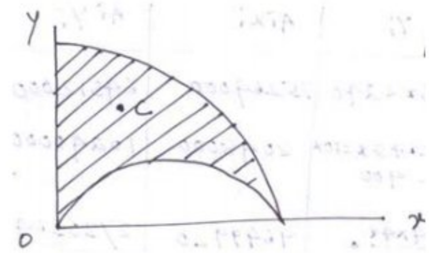
1. An isosceles triangle ADE is to cut from a square ABCD of dimension ‘a’. Find the altitude ‘y’ of the triangle so that vertex E will be centroid of remaining shaded area.



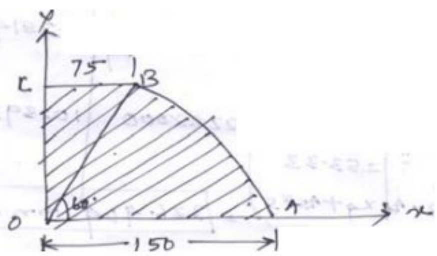
2. Find the centroid of the following figure.



3. Locate the centroid C of the shaded area obtained by cutting a semi-circle of diameter ‘a’ from the quadrant of a circle of radius ‘a’.



4. Locate the centroid of the composite figure.



Module –I