

Module

2

AC to DC Converters

# Lesson

13

## Operation and Analysis of the Three Phase Fully Controlled Bridge Converter

## Instructional Objectives

On completion the student will be able to

- Draw the circuit diagram and waveforms associated with a three phase fully controlled bridge converter.
- Find out the average, RMS values and the harmonic spectrum of the output voltage / current waveforms of the converter.
- Find out the closed form expression of the output current and hence the condition for continuous conduction.
- Find out the displacement factor, distortion factor and the power factor of the input current as well as its harmonic spectrum.
- Analyze the operation of higher pulse number converters and dual converter.
- Design the triggering circuit of the three phase fully controlled bridge converter.

## 13.1 Introduction

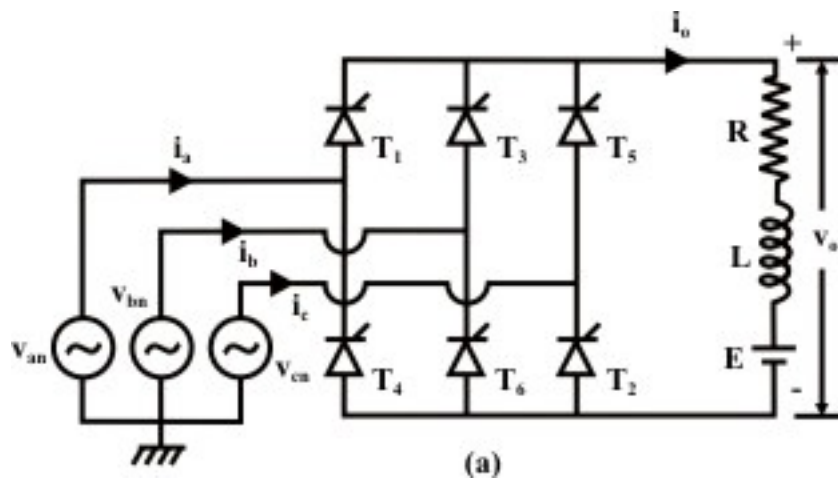
The three phase fully controlled bridge converter has been probably the most widely used power electronic converter in the medium to high power applications. Three phase circuits are preferable when large power is involved. The controlled rectifier can provide controllable output dc voltage in a single unit instead of a three phase autotransformer and a diode bridge rectifier. The controlled rectifier is obtained by replacing the diodes of the uncontrolled rectifier with thyristors. Control over the output dc voltage is obtained by controlling the conduction interval of each thyristor. This method is known as phase control and converters are also called “phase controlled converters”. Since thyristors can block voltage in both directions it is possible to reverse the polarity of the output dc voltage and hence feed power back to the ac supply from the dc side. Under such condition the converter is said to be operating in the “inverting mode”. The thyristors in the converter circuit are commutated with the help of the supply voltage in the rectifying mode of operation and are known as “Line commutated converter”. The same circuit while operating in the inverter mode requires load side counter emf. for commutation and are referred to as the “Load commutated inverter”.

In phase controlled rectifiers though the output voltage can be varied continuously the load harmonic voltage increases considerably as the average value goes down. Of course the magnitude of harmonic voltage is lower in three phase converter compared to the single phase circuit. Since the frequency of the harmonic voltage is higher smaller load inductance leads to continuous conduction. Input current wave shape become rectangular and contain 5<sup>th</sup> and higher order odd harmonics. The displacement angle of the input current increases with firing angle. The frequency of the harmonic voltage and current can be increased by increasing the pulse number of the converter which can be achieved by series and parallel connection of basic 6 pulse converters. The control circuit become considerably complicated and the use of coupling transformer and / or interphase reactors become mandatory.

With the introduction of high power IGBTs the three phase bridge converter has all but been replaced by dc link voltage source converters in the medium to moderately high power range. However in very high power application (such as HV dc transmission system, cycloconverter drives, load commutated inverter synchronous motor drives, static scherbius drives etc.) the basic B phase bridge converter block is still used. In this lesson the operating principle and characteristic of this very important converter topology will be discussed in source depth.

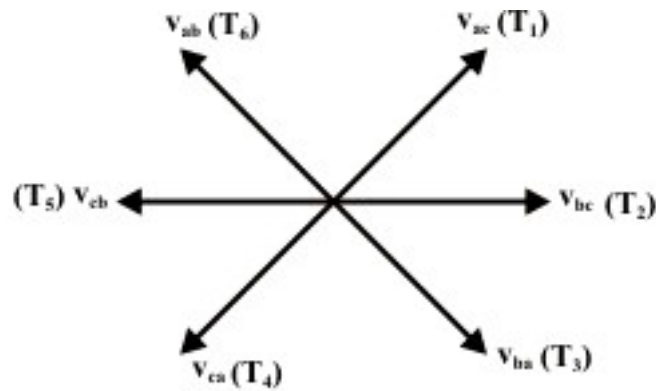
## 13.2 Operating principle of 3 phase fully controlled bridge converter

A three phase fully controlled converter is obtained by replacing all the six diodes of an uncontrolled converter by six thyristors as shown in Fig. 13.1 (a)



Device Mode	$V_{T1}$	$V_{T2}$	$V_{T3}$	$V_{T4}$	$V_{T5}$	$V_{T6}$	$V_o$
$T_1T_2$	0	0	$v_{ba}$	$v_{ca}$	$v_{ca}$	$v_{cb}$	$v_{ac}$
$T_2T_3$	$v_{ab}$	0	0	$v_{ca}$	$v_{cb}$	$v_{cb}$	$v_{bc}$
$T_3T_4$	$v_{ab}$	$v_{ac}$	0	0	$v_{cb}$	$v_{ab}$	$v_{ba}$
$T_4T_5$	$v_{ac}$	$v_{ac}$	$v_{bc}$	0	0	$v_{ab}$	$v_{ca}$
$T_5T_6$	$v_{ac}$	$v_{bc}$	$v_{bc}$	$v_{ba}$	0	0	$v_{cb}$
$T_6T_1$	0	$v_{bc}$	$v_{ba}$	$v_{ba}$	$v_{ca}$	0	$v_{ab}$
NONE	-	-	-	-	-	-	E

(b)



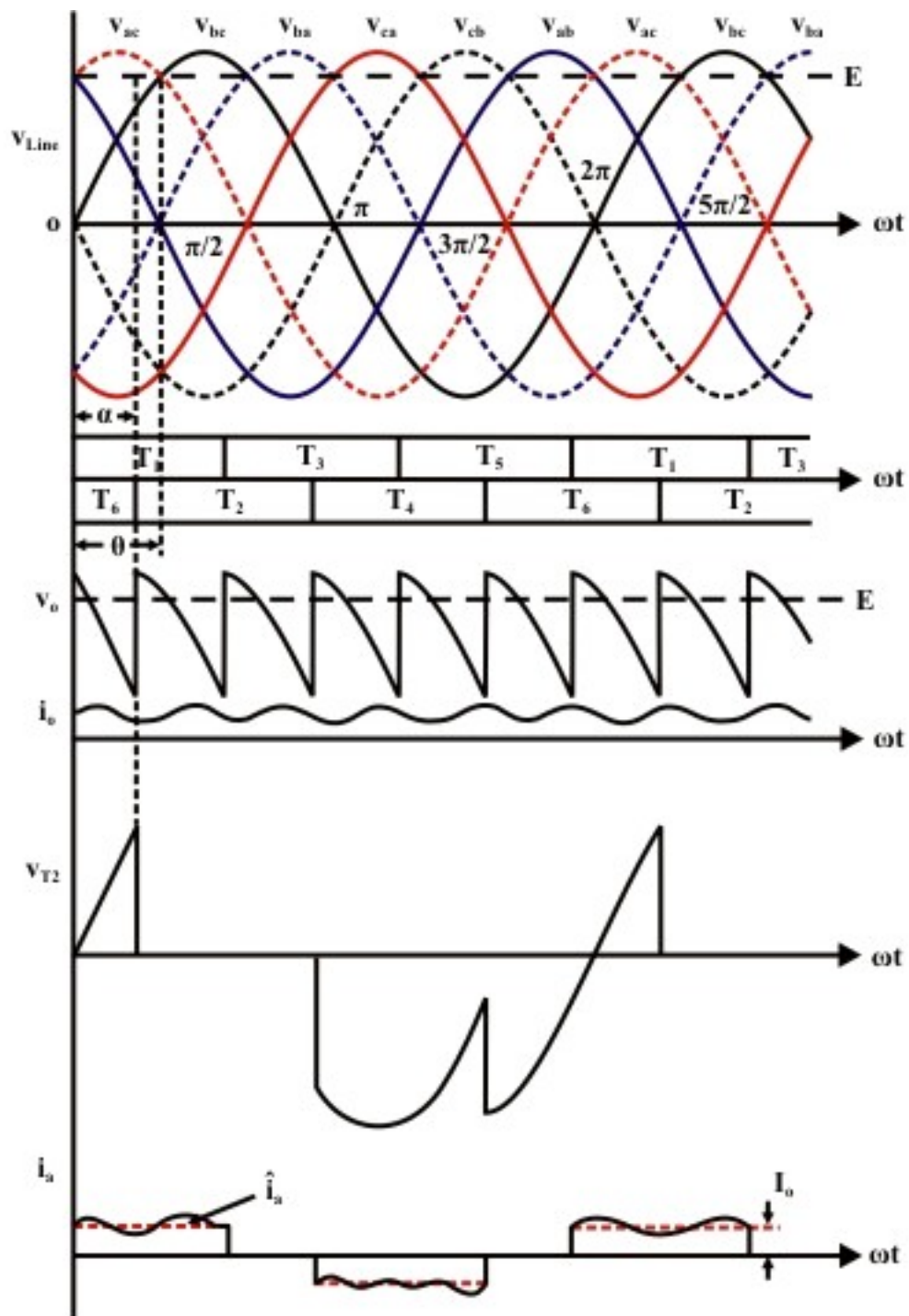
(c)

**Fig. 13.1: operation of a three phase full controlled bridge converter**  
**(a) circuit diagram,**  
**(b) conduction table,**  
**(c) phaser diagram of line voltages.**

For any current to flow in the load at least one device from the top group ( $T_1, T_3, T_5$ ) and one from the bottom group ( $T_2, T_4, T_6$ ) must conduct. It can be argued as in the case of an uncontrolled converter only one device from these two groups will conduct.

Then from symmetry consideration it can be argued that each thyristor conducts for  $120^\circ$  of the input cycle. Now the thyristors are fired in the sequence  $T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4 \rightarrow T_5 \rightarrow T_6 \rightarrow T_1$  with  $60^\circ$  interval between each firing. Therefore thyristors on the same phase leg are fired at an interval of  $180^\circ$  and hence can not conduct simultaneously. This leaves only six possible conduction mode for the converter in the continuous conduction mode of operation. These are  $T_1T_2, T_2T_3, T_3T_4, T_4T_5, T_5T_6, T_6T_1$ . Each conduction mode is of  $60^\circ$  duration and appears in the sequence mentioned. The conduction table of Fig. 13.1 (b) shows voltage across different devices and the dc output voltage for each conduction interval. The phasor diagram of the line voltages appear in Fig. 13.1 (c). Each of these line voltages can be associated with the firing of a thyristor with the help of the conduction table-1. For example the thyristor  $T_1$  is fired at the end of  $T_5T_6$  conduction interval. During this period the voltage across  $T_1$  was  $v_{ac}$ . Therefore  $T_1$  is fired  $\alpha$  angle after the positive going zero crossing of  $v_{ac}$ . Similar observation can be made about other thyristors. The phasor diagram of Fig. 13.1 (c) also confirms that all the thyristors are fired in the correct sequence with  $60^\circ$  interval between each firing.

Fig. 13.2 shows the waveforms of different variables (shown in Fig. 13.1 (a)). To arrive at the waveforms it is necessary to draw the conduction diagram which shows the interval of conduction for each thyristor and can be drawn with the help of the phasor diagram of fig. 13.1 (c). If the converter firing angle is  $\alpha$  each thyristor is fired " $\alpha$ " angle after the positive going zero crossing of the line voltage with which it's firing is associated. Once the conduction diagram is drawn all other voltage waveforms can be drawn from the line voltage waveforms and from the conduction table of fig. 13.1 (b). Similarly line currents can be drawn from the output current and the conduction diagram. It is clear from the waveforms that output voltage and current waveforms are periodic over one sixth of the input cycle. Therefore this converter is also called the "six pulse" converter. The input current on the other hand contains only odds harmonics of the input frequency other than the triplex ( $3^{rd}, 9^{th}$  etc.) harmonics. The next section will analyze the operation of this converter in more details.



**Fig. 13.2: Waveforms of three phase fully controlled converter 13.2.1 Analysis of the converter in the rectifier mode.**

### Exercise 13.1

Fill in the blank(s) with the appropriate word(s)

- The three phase fully controlled bridge converter is obtained by replacing six \_\_\_\_\_ of an uncontrolled converter by six \_\_\_\_\_.

- ii) The pulse number of a three phase fully controlled bridge converter is \_\_\_\_\_.
- iii) In a three phase fully controlled converter each device conducts for an interval of \_\_\_\_\_ degrees.
- iv) In a three phase fully controlled converter operating in continuous conduction there are \_\_\_\_\_ different conduction modes.
- v) The output voltage of a three phase fully controlled converter operating in the continuous conduction mode consists of segments of the input ac \_\_\_\_\_ voltage.
- vi) The peak voltage appearing across any device of a three phase fully controlled converter is equal to the \_\_\_\_\_ input ac \_\_\_\_\_ voltage.
- vii) The input ac current of a three phase fully controlled converter has a \_\_\_\_\_ step waveform.
- viii) The input ac current of a three phase fully controlled converter contains only \_\_\_\_\_ harmonics but no \_\_\_\_\_ harmonic.
- ix) A three phase fully controlled converter can also operate in the \_\_\_\_\_ mode.
- x) Discontinuous conduction in a three phase fully controlled converter is \_\_\_\_\_.

**Answers:** (i) diodes, thyristors; (ii) six; (iii) 120; (iv) six; (v) line; (vi) peak, line; (vii) six; (viii) odd, tripler; (ix) inverting; (x) rare.

### 13.2.1 Analysis of the converter in the rectifier mode

The output voltage waveform can be written as

$$v_0 = V_0 + \sum_{K=1,2}^{\alpha} V_{AK} \cos 6 K\omega t + \sum_{K=1,2}^{\alpha} V_{BK} \sin 6 K\omega t \quad (13.1)$$

$$\begin{aligned} V_0 &= \frac{3}{\pi} \int_{\alpha}^{\alpha+\frac{\pi}{3}} v_0 \, d\omega t = \frac{3\sqrt{2}}{\pi} V_L \int_{\alpha}^{\alpha+\frac{\pi}{3}} \sin\left(\omega t + \frac{\pi}{3}\right) d\omega t \\ &= \frac{3\sqrt{2}}{\pi} V_L \cos\alpha \end{aligned} \quad (13.2)$$

$$\begin{aligned} V_{AK} &= \frac{6}{\pi} \int_{\alpha}^{\alpha+\frac{\pi}{3}} v_0 \cos 6 K\omega t \, d\omega t \\ &= \frac{6}{\pi} \int_{\alpha}^{\alpha+\frac{\pi}{3}} \sqrt{2} V_L \sin\left(\omega t + \frac{\pi}{3}\right) \cos 6 \omega t \, d\omega t \\ &= \frac{3\sqrt{2}}{\pi} V_L \left[ \frac{\cos(6K+1)\alpha}{6K+1} - \frac{\cos(6K-1)\alpha}{6K-1} \right] \end{aligned} \quad (13.3)$$



$$\begin{aligned}
V_{BK} &= \frac{6}{\pi} \int_{\alpha}^{\alpha+\frac{\pi}{3}} v_0 \sin 6K\omega t \, d\omega t \\
&= \frac{6}{\pi} \int_{\alpha}^{\alpha+\frac{\pi}{3}} \sqrt{2} V_L \sin\left(\omega t + \frac{\pi}{3}\right) \sin 6\omega t \, d\omega t \\
&= \frac{3\sqrt{2}}{\pi} V_L \left[ \frac{\sin(6K+1)\alpha}{6K+1} - \frac{\sin(6K-1)\alpha}{6K-1} \right] \\
V_{ORMS} &= \sqrt{\frac{3}{\pi} \int_{\alpha}^{\alpha+\frac{\pi}{3}} v_0^2 \, d\omega t} = V_L \left[ 1 + \frac{3\sqrt{3}}{4\pi} \cos 2\alpha \right]^{\frac{1}{2}}
\end{aligned} \tag{13.4}$$

The input phase current  $i_a$  is expressed as

$$\begin{aligned}
i_a &= i_0 & \alpha \leq \omega t \leq \alpha + \frac{\pi}{3} \\
i_a &= -i_0 & \alpha + \frac{2\pi}{3} \leq \omega t \leq \alpha + \frac{4\pi}{3} \\
i_a &= i_0 & \alpha + \frac{5\pi}{3} \leq \omega t \leq \alpha + 2\pi \\
i_a &= 0 & \text{otherwise}
\end{aligned}$$

From Fig. 13.2 it can be observed that  $i_0$  itself has a ripple at a frequency six times the input frequency. The closed form expression of  $i_0$ , as will be seen later is some what complicated. However, considerable simplification in the expression of  $i_a$  can be obtained if  $i_0$  is replaced by its average value  $I_0$ . This approximation will be valid provided the ripple on  $i_0$  is small, i.e, the load is highly inductive. The modified input current waveform will then be  $i_a$  which can be expressed in terms of a fourier series as

$$i_a \approx \hat{i}_a = \frac{I_{A0}}{2} + \sum_{n=1}^{\infty} I_{An} \cos n\omega t + \sum_{n=1}^{\infty} I_{Bn} \sin n\omega t \tag{13.5}$$

Where

$$I_{A0} = \frac{1}{2\pi} \int_{\alpha}^{\alpha+2\pi} i_a \, d\omega t = 0 \tag{13.6}$$

$$\begin{aligned}
I_{An} &= \frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} i_a \cos n\omega t \, d\omega t & n \neq 0 \\
&= \frac{4I_0}{n\pi} \cos \frac{n\pi}{6} \sin \frac{n\pi}{2} \cos n\alpha
\end{aligned} \tag{13.7}$$

$$\begin{aligned}
\therefore I_{An} &= (-1)^K \frac{2\sqrt{3}I_0}{(6K \pm 1)\pi} \sin\left(K\pi \pm \frac{\pi}{2}\right) \cos(6K \pm 1)\alpha \\
&\text{for } n = 6K \pm 1, K = 0, 1, 2, 3 \dots
\end{aligned} \tag{13.8}$$

$$I_{An} = 0 \quad \text{otherwise.}$$

$$\begin{aligned}
I_{Bn} &= \frac{1}{\pi} \int_{\alpha}^{\alpha+2\pi} i_a \sin n\omega t \, d\omega t \\
&= \frac{4I_0}{n\pi} \cos \frac{n\pi}{6} \sin n\alpha \sin \frac{n\pi}{2}
\end{aligned} \tag{13.9}$$

$$\begin{aligned}
\therefore I_{Bn} &= (-1)^K \frac{2\sqrt{3}I_0}{(6K \pm 1)\pi} \sin\left(K\pi \pm \frac{\pi}{2}\right) \sin(6K \pm 1)\alpha \\
&\text{for } n = 6K \pm 1, K = 0, 1, 2, \dots
\end{aligned} \tag{13.10}$$

$$I_{Bn} = 0 \quad \text{otherwise.}$$

$$\therefore i_a = \sum_{K=0}^{\infty} \frac{(-1)^K 2\sqrt{3}I_0}{(6K \pm 1)\pi} \sin\left(K\pi \pm \frac{\pi}{2}\right) \cos[(6K \pm 1)(\omega t - \alpha)] \tag{13.11}$$

in particular  $i_{a1}$  = fundamental component of  $i_a$

$$= \frac{2\sqrt{3}}{\pi} I_0 \cos(\omega t - \alpha) \tag{13.12}$$

$$\text{From Fig. 13.2} \quad v_{an} = \frac{\sqrt{2}V_L}{\sqrt{3}} \cos \omega t \tag{13.13}$$

$$\therefore \text{ displacement angle } \phi = \alpha. \quad \therefore \text{ displacement factor} = \cos \alpha \tag{13.14}$$

$$\text{distortion factor} = \frac{I_{a1}}{I_a} = \left(\frac{\sqrt{6}}{\pi}\right)^{I_0} \bigg/ \frac{\sqrt{2}}{\sqrt{3}} I_0 = \frac{3}{\pi} \tag{13.15}$$

$$\therefore \text{ Power factor} = \text{Displacement factor} \times \text{Distortion factor} = \frac{3}{\pi} \cos \alpha \tag{13.16}$$

The closed form expression for  $i_0$  in the interval  $\alpha \leq \omega t \leq \alpha + \frac{\pi}{3}$  can be found as follows

in this interval

$$Ri_0 + L \frac{di_0}{dt} + E = v_0 = \sqrt{2}V_L \sin\left(\omega t + \frac{\pi}{3}\right) \tag{13.17}$$

$$i_0 = I_1 e^{-\frac{(\omega t - \alpha)}{\tan \phi}} + \frac{\sqrt{2}V_L}{Z} \sin\left(\omega t + \frac{\pi}{3} - \phi\right) - \frac{E}{R} \tag{13.18}$$

$$\text{Where} \quad Z = \sqrt{R^2 + \omega^2 L^2}, \quad \tan \phi = \frac{\omega L}{R}$$

$$\therefore R = Z \cos \phi, \quad E = \sqrt{2}V_L \sin \theta \quad (\text{from Fig. 13.2}) \tag{13.19}$$

$$\therefore i_0 = I_1 e^{-\frac{(\omega t - \alpha)}{\tan \phi}} + \frac{\sqrt{2}V_L}{Z} \left[ \sin\left(\omega t + \frac{\pi}{3} - \phi\right) - \frac{\sin \theta}{\cos \phi} \right] \quad (13.20)$$

Since  $i_0$  is periodic over  $\pi/3$

$$\begin{aligned} i_0 \Big|_{\omega t = \alpha} &= i_0 \Big|_{\omega t = \alpha + \frac{\pi}{3}} \\ \therefore I_1 + \frac{\sqrt{2}V_L}{Z} \left[ \sin\left(\alpha + \frac{\pi}{3} - \phi\right) - \frac{\sin \theta}{\cos \phi} \right] \\ &= I_1 e^{-\frac{\pi}{3 \tan \phi}} + \frac{\sqrt{2}V_L}{Z} \left[ \sin\left(\alpha + \frac{2\pi}{3} - \phi\right) - \frac{\sin \theta}{\cos \phi} \right] \end{aligned} \quad (13.21)$$

$$\text{OR } I_1 = \frac{\sqrt{2}V_L}{Z} \frac{\sin(\phi - \alpha)}{1 - e^{-\frac{\pi}{3 \tan \phi}}} \quad (13.22)$$

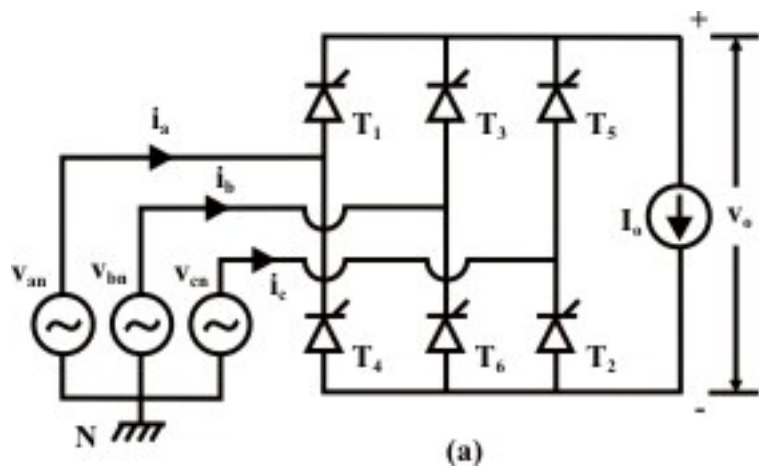
$$\therefore i_0 = \frac{\sqrt{2}V_L}{Z} \left[ \frac{\sin(\phi - \alpha)}{1 - e^{-\frac{\pi}{3 \tan \phi}}} e^{-\frac{(\omega t - \alpha)}{\tan \phi}} + \sin\left(\omega t + \frac{\pi}{3} - \phi\right) - \frac{\sin \theta}{\cos \phi} \right] \quad (13.23)$$

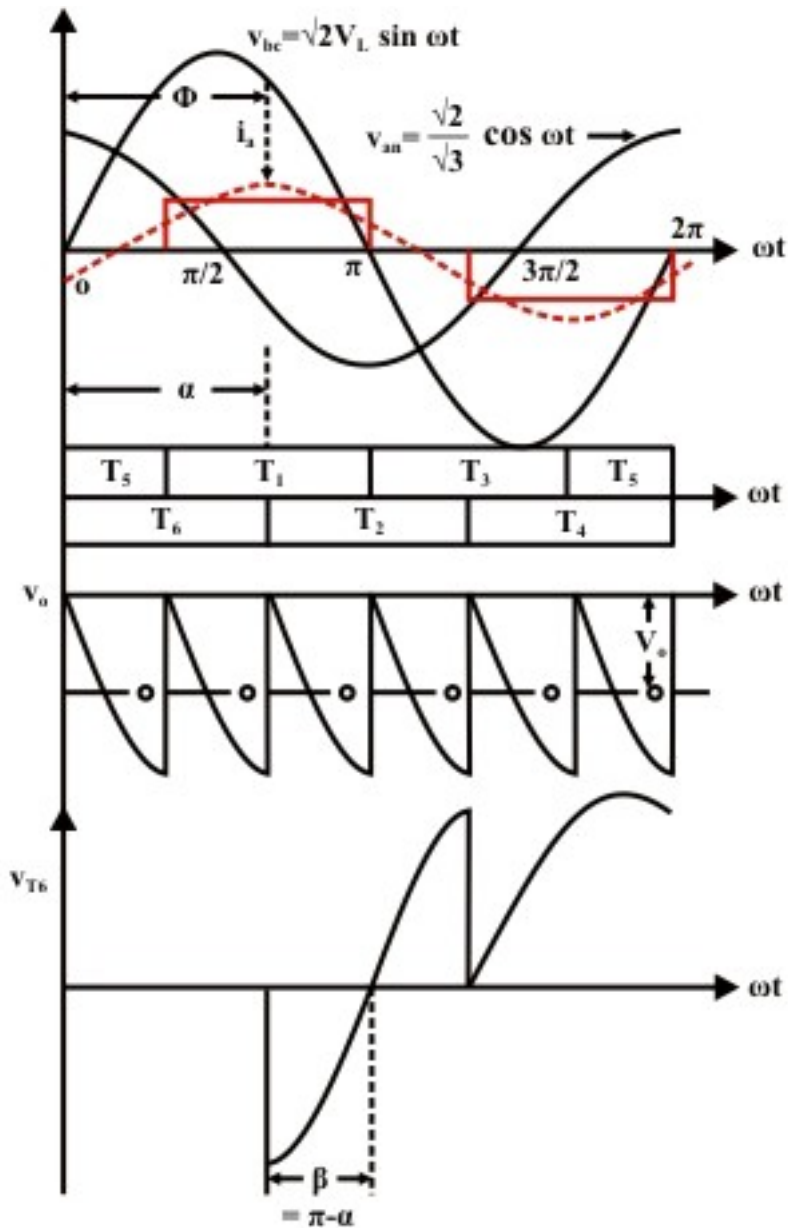
To find out the condition for continuous conduction it is noted that in the limiting case of continuous conduction.

$i_0 \Big|_{\min=0}$ , Now if  $\theta \leq \alpha + \frac{\pi}{3}$  then  $i_0$  is minimum at  $\omega t = \alpha$ .  $\therefore$  Condition for continuous conduction is  $i_0 \Big|_{\omega t = \alpha} \geq 0$ . However discontinuous conduction is rare in these conversions and will not be discussed any further.

### 13.2.2 Analysis of the converter in the inverting mode.

In all the analysis presented so far it has been assumed that  $\alpha < 90^\circ$ . It follows from equation 13.2 that the output dc voltage will be positive in this case and power will be flowing from the three phase ac side to the dc side. This is the rectifier mode of operation of the converter. However if  $\alpha$  is made larger than  $90^\circ$  the direction of power flow through the converter will reverse provided there exists a power source in the dc side of suitable polarity. The converter in that case is said to be operating in the inverter mode. It has been explained in connection with single phase converters that the polarity of EMF source on the dc side [Fig. 13.1(a)] would have to be reversed for inverter mode of operation. Fig. 13.3 shows the circuit connection and wave forms in the inverting mode of operation where the load current has been assumed to be continuous and ripple free.





**Fig. 13.3(b): Inverter mode of operation of the three phase fully controlled bridge converter**  
**(a) circuit diagram**  
**(b) waveforms.**

Analysis of the converter in the inverting mode is similar to its rectifier mode of operation. The same expressions hold for the dc and harmonic compounds in the output voltage and current. The input supply current Fourier series is also identical to Equation 13.8. In particular

$$V_0 = \frac{3\sqrt{2}}{\pi} V_L \cos \alpha \quad (13.24)$$

$$i_{a1} = \frac{2\sqrt{3}}{\pi} I_0 \cos(\omega t - \alpha) \quad (13.25)$$

For values of  $\alpha$  in the range  $90^\circ < \alpha < 180^\circ$  it is observed from Fig. 13.3(b) that the average dc voltage is negative and the displacement angle  $\phi$  of the fundamental component of the input ac line current is equal to  $\alpha > 90^\circ$ . Therefore, power in the ac side flows from the converter to the source.

It is observed from Fig. 13.3(b) that an outgoing thyristor (thyristor  $T_6$  in Fig. 13.3(b)) after commutation is impressed with a negative voltage of duration  $\beta = \pi - \alpha$ . For successful commutation of the outgoing thyristor it is essential that this interval is larger than the turn off time of the thyristor i.e,

$$\beta \geq \omega t_q, \text{ } t_q \text{ is the thyristor turn off time}$$

Therefore  $\pi - \alpha \geq \omega t_q$  or  $\alpha \leq \pi - \omega t_q$ .

Which imposes an upper limit on the value of  $\alpha$ . In practice this upper value of  $\alpha$  is further reduced due to commutation overlap.

## Exercise 13.2

1. A three phase fully controlled bridge converter operating from a 3 phase 220 V, 50 Hz supply is used to charge a battery bank with nominal voltage of 240 V. The battery bank has an internal resistance of 0.01  $\Omega$  and the battery bank voltage varies by  $\pm 10\%$  around its nominal value between fully charged and uncharged condition. Assuming continuous conduction find out.
  - (i) The range of firing angle of the converter.
  - (ii) The range of ac input power factor.
  - (iii) The range of charging efficiency.

When the battery bank is charged with a constant average charging current of 100 Amps through a 250 mH lossless inductor.

**Answer:** The maximum and minimum battery voltages are,  $V_{B \text{ Min}} = 0.9 \times V_{B \text{ Nom}} = 216$  volts and  $V_{B \text{ Max}} = 1.1 \times V_{B \text{ Nom}} = 264$  volts respectively.

Since the average charging current is constant at 100 A.

$$V_{0 \text{ Max}} = V_{B \text{ Max}} + 100 \times R_B = 264 + 100 \times 0.01 = 265 \text{ volts}$$

$$V_{0 \text{ Min}} = V_{B \text{ Min}} + 100 \times R_B = 216 + 100 \times 0.01 = 217 \text{ volts.}$$

$$(i) \quad \text{But} \quad V_{0 \text{ Max}} = \frac{3\sqrt{2}}{\pi} V_L \cos \alpha_{\text{Min}} \quad \therefore \quad \alpha_{\text{Min}} = 26.88^\circ$$

$$V_{0 \text{ Min}} = \frac{3\sqrt{2}}{\pi} V_L \cos \alpha_{\text{Max}} \quad \therefore \quad \alpha_{\text{Max}} = 43.08^\circ$$

- (ii) Input power factor is maximum at minimum  $\alpha$  and vice versa

$$\therefore \text{p.f. max} = \text{Distortion factor} \times \text{Displacement factor} \Big|_{\text{Max}} = \frac{3}{\pi} \times \cos \alpha_{\text{min}} = 0.85$$

$$\text{p.f. Min} = \frac{3}{\pi} \times \cos \alpha_{\text{Max}} = 0.697$$

(iii) Power loss during charging =  $I_{\text{ORMs}}^2 R_B$

But  $I_{\text{ORMs}}^2 = I_0^2 + I_1^2 + I_2^2 + \dots$  and  $I_K \approx \frac{V_K}{6K\omega L} = \frac{\sqrt{V_{AK}^2 + V_{BK}^2}}{6\sqrt{2}K\omega L}$

For  $\alpha = \alpha_{\text{Min}}$

$$V_{A1} = 0.439 \text{ V}, \quad V_{B1} = 48.48 \text{ V}, \quad I_1 = 0.073 \text{ Amps}$$

$$V_{A2} = 10.76, \quad V_{B2} = 20.15 \text{ V}, \quad I_2 = 0.017 \text{ Amps.}$$

$$\therefore J_{\text{ORMs}}^2 \approx 100^2 + (0.073)^2 + (0.017)^2 = 10000.00562$$

$$\therefore P_{\text{loss}} = 100 \text{ watts.}$$

At  $\alpha_{\text{Min}}$ ,  $P_0 = I_0 \times V_{B \text{ Max}} = 26400 \text{ watts.}$

$$\therefore \text{Charging efficiency} = \frac{26400}{26400 + 100} = 99.6\%$$

Similarly for  $\alpha_{\text{Max}}$ ,  $I_{\text{ORMs}}^2 \approx I_0^2$

$$\therefore P_{\text{loss}} = 100 \text{ watts}$$

$$P_0 = I_0 \times V_{B \text{ Min}} = 21600 \text{ watts.}$$

$$\therefore \text{Charging efficiency} = \frac{21600}{21600 + 100} = 99.54\%$$

2. A three phase fully controlled converter operates from a 3 phase 230 V, 50 Hz supply through a Y/ $\Delta$  transformer to supply a 220 V, 600 rpm, 500 A separately excited dc motor. The motor has an armature resistance of 0.02  $\Omega$ . What should be the transformer turns ratio such that the converter produces rated motor terminal voltage at 0° firing angle. Assume continuous conduction. The same converter is now used to brake the motor regeneratively in the reverse direction. If the thyristors are to be provided with a minimum turn off time of 100  $\mu\text{s}$ , what is the maximum reverse speed at which rated braking torque can be produced.

**Answer:** From the given question

$$\frac{3\sqrt{2}}{\pi} V_L = 220 \quad \therefore V_L = 162.9 \text{ V}$$

Where  $V_L$  is the secondary side line and also the phase voltage since the secondary side is  $\Delta$  connected.

$$\text{Primary side phase voltage} = \frac{230}{\sqrt{3}} \text{ V} = 132.79 \text{ V}$$

$$\therefore \text{ Turns ratio} = \frac{132.79}{162.9} = 1 : 1.2267 .$$

During regenerative braking in the reverse direction the converter operates in the inverting mode.

$$\begin{aligned} \text{tq}|_{\text{Min}} &= 100\mu\text{S} & \therefore \quad \beta_{\text{Min}} &= \omega\text{tq}|_{\text{Min}} = 1.8^\circ \\ \therefore \alpha_{\text{Max}} &= 180 - \beta_{\text{Min}} = 178.2^\circ \end{aligned}$$

$\therefore$  Maximum negative voltage that can be generated by the converter is

$$\frac{3\sqrt{2}}{\pi} V_L \cos 178.2^\circ = -219.89 \text{ V}$$

For rated braking torque  $I_a = 500 \text{ A}$

$$\therefore E_b = V_a - I_a r_a = -229.89 \text{ V} .$$

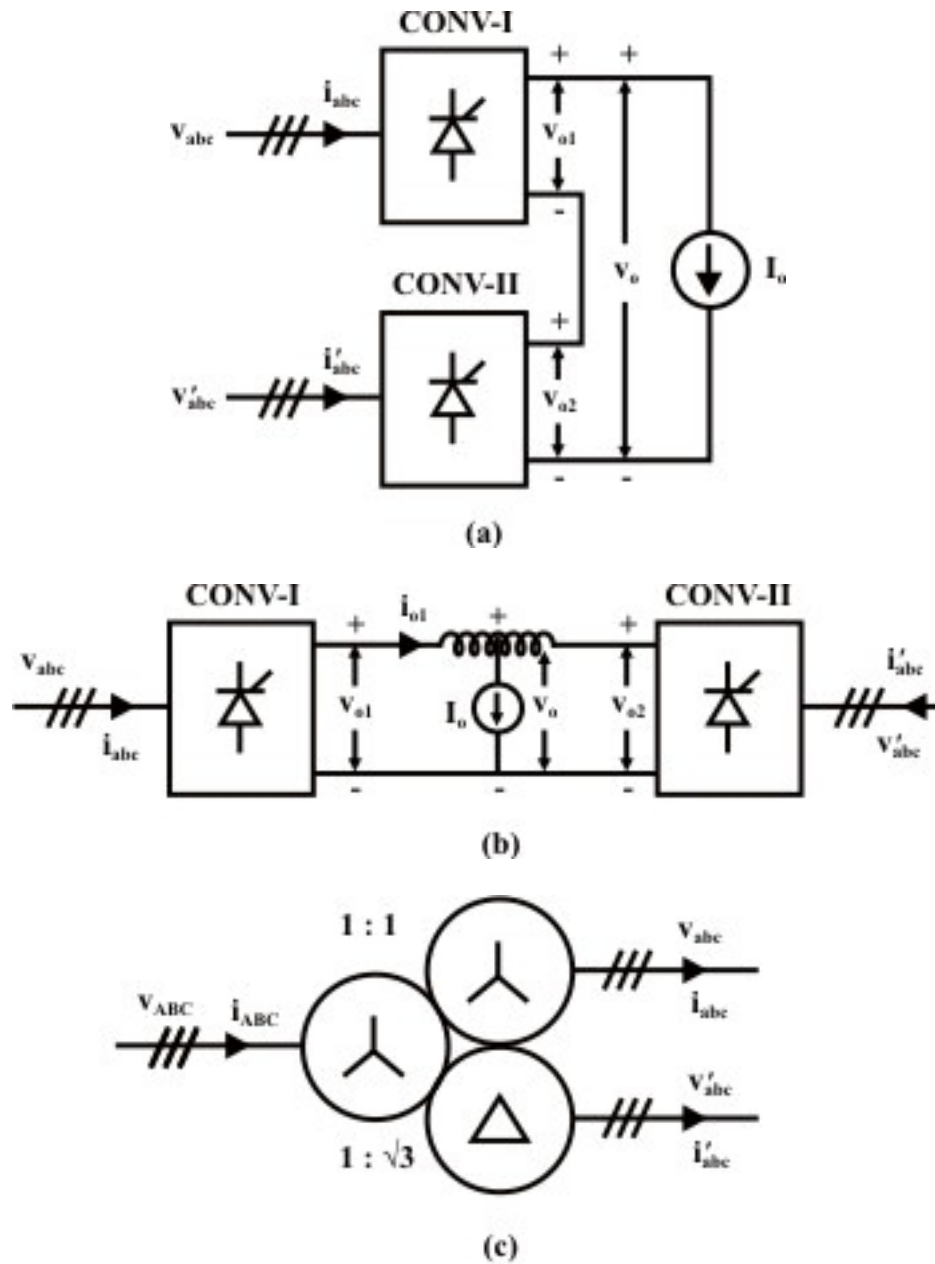
At 600 RPM  $E_b = 220 - 500 \times 0.02 = 210 \text{ V} .$

$$\therefore \text{ Max reverse speed is } \frac{229.89}{210} \times 600 = 656.83 \text{ RPM} .$$

### 13.3 Higher pulse number converters and dual converter

The three phase fully controlled converter is widely used in the medium to moderately high power applications. However in very large power applications (such as HV DC transmission systems) the device ratings become impractically large. Also the relatively low frequency (6<sup>th</sup> in the dc side, 5<sup>th</sup> and 7<sup>th</sup> in the ac side) harmonic voltages and currents produced by this converter become unacceptable. Therefore several such converters are connected in series parallel combination in order to increase the voltage / current rating of the resulting converter. Furthermore if the component converters are controlled properly some lower order harmonics can be eliminated both from the input and output resulting in a higher pulse converter.





**Fig. 13.4: Series and parallel connection of 6 phase converters**  
**(a) Series connection,**  
**(b) parallel connection,**  
**(c) Transformer connection.**

Fig. 13.4(a) schematically represents series connection of two six pulse converters where as Fig. 13.4(b) can be considered to be a parallel connection. The inductance in between the converters has been included to limit circulating harmonic current. In both these figures CONV – I and CONV – II have identical construction and are also fired at the same firing angle  $\alpha$ . Their input supplies also have same magnitude but displaced in phase by an angle  $\phi$ . Then one can write

$$v_{o1} = \frac{3\sqrt{2}}{\pi} V_L \cos\alpha + \sum_{K=1}^{\infty} V_{AK} \cos 6 K\omega t + \sum_{K=1}^{\alpha} V_{BK} \sin 6 K\omega t \quad (13.26)$$

$$v_{02} = \frac{3\sqrt{2}}{\pi} V_L \cos\alpha + \sum_{K=1}^{\alpha} V_{AK} \cos 6K(\omega t - \phi) + \sum_{K=1}^{\alpha} V_{BK} \sin 6K(\omega t - \phi) \quad (13.27)$$

Therefore for Fig 13.4(a)

$$v_0 = v_{01} + v_{02} = \frac{6\sqrt{2}}{\pi} V_L \cos\alpha + 2 \sum_{K=1}^{\alpha} \cos 3K\phi [V_{AK} \cos 3K(2\omega t - \phi) + V_{BK} \sin 3K(2\omega t - \phi)] \quad (13.28)$$

Now if  $\cos 3K\phi = 0$  for some K then the corresponding harmonic disappear from the fourier series expression of  $v_0$ .

In particular if  $\phi = 30^\circ$  then  $\cos 3K\phi = 0$  for  $K = 1, 2, 3, 5, \dots$

This phase difference can be obtained by the arrangement shown in Fig. 13.4(c).

Then

$$v_0 = \frac{6\sqrt{2}}{\pi} V_L \cos\alpha + 2 \sum_{m=1}^{\alpha} [V_{Am} \cos 12m\omega t + V_{Bm} \sin 12m\omega t] \quad (13.29)$$

It can be seen that the frequency of the harmonics present in the output voltage has the form  $12\omega, 24\omega, 36\omega, \dots$

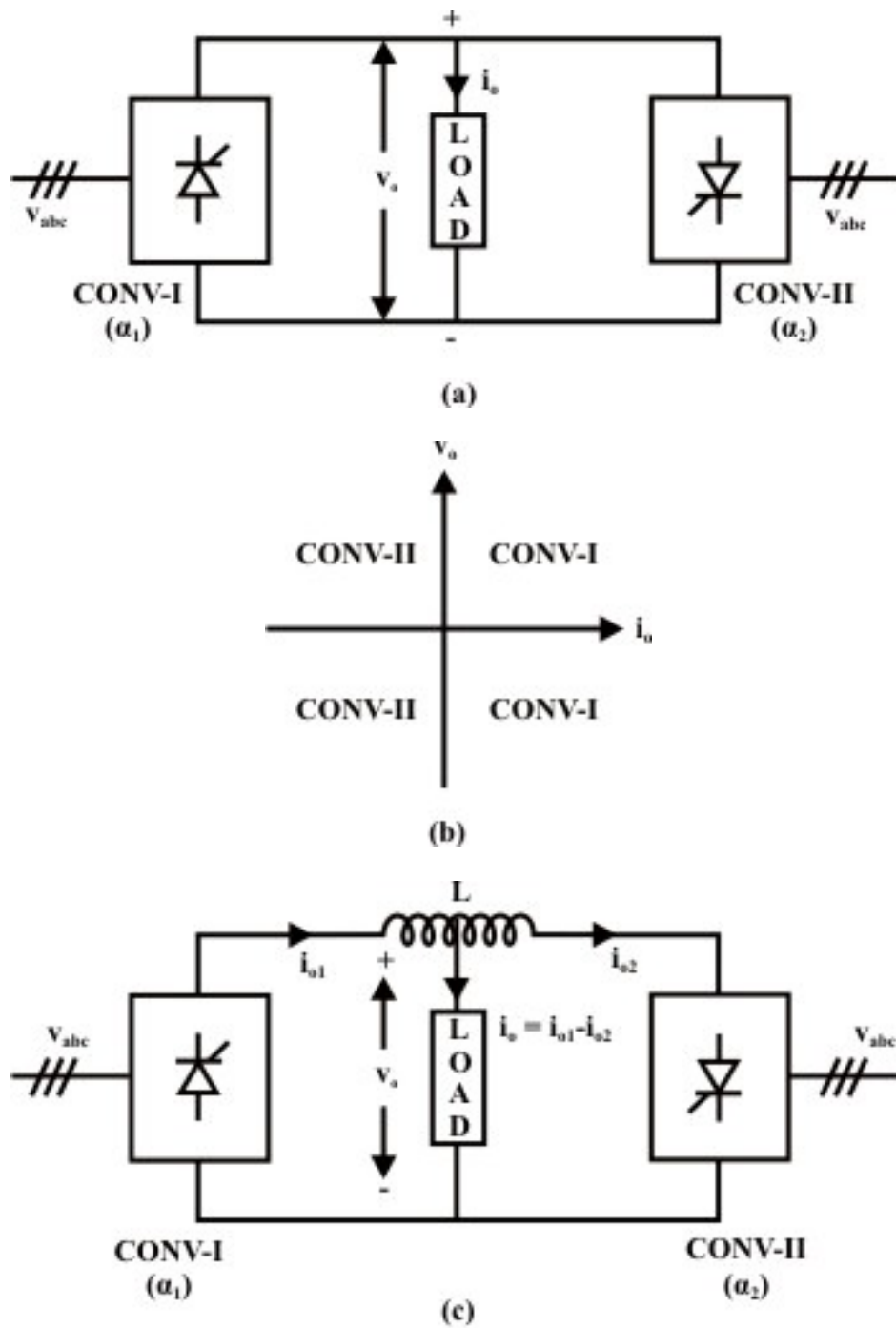
Similarly it can be shown that the input side line current  $i_{ABC}$  have harmonic frequency of the form

$$11\omega, 13\omega, 23\omega, 25\omega, 35\omega, 37\omega, \dots$$

Which is the characteristic of a 12 pulse converter.

In a similar manner more number of 3 phase 6 pulse converters can be connected in series / parallel and the  $\phi$  angle can be adjusted to obtain 18 and 24 pulse converters.

One of the shortcomings of a three phase fully controlled converter is that although it can produce both positive and negative voltage it can not supply current in both directions. However, some applications such as a four quadrant dc motor drive require this capability from the dc source. This problem is easily mitigated by connecting another three phase fully controlled converter in anti parallel as shown in Fig. 13.5 (a). In this figure converter-I supplies positive load current while converter-II supplies negative load current. In other words converter-I operates in the first and fourth quadrant of the output  $v - i$  plane whereas converter-II operates in the third and fourth quadrant. Thus the two converters taken together can operate in all four quadrants and is capable of supplying a four quadrant dc motor drive. The combined converter is called the Dual converter.



**Fig. 13.5: Dual converter circuits**  
 (a) non circulating type  
 (b) output V-I plane  
 (c) circulating current type

Obviously since converter-I and converter-II are connected in antiparallel they must produce the same dc voltage. This requires that the firing angles of these two converters be related as

$$\alpha_2 = \pi - \alpha_1 \quad (13.30)$$

Although Equation 13.30 ensures that the dc voltages produced by these converters are equal the output voltages do not match on an instantaneous basis. Therefore to avoid a direct short circuit between two different supply lines the two converters must never be gated simultaneously. Converter-I receives gate pulses when the load current is positive. Gate pulses to converter-II are blocked at that time. For negative load current converter-II thyristors are fired while converter-I gate pulses are blocked. Thus there is no circulating current flowing through the converters and therefore it is called the non-circulating current type dual converter. It requires precise sensing of the zero crossing of the output current which may pose a problem particularly at light load due to possible discontinuous conduction. To overcome this problem an interphase reactor may be incorporated between the two converters. With the interphase reactor in place both the converters can be gated simultaneously with  $\alpha_2 = \pi - \alpha_1$ . The resulting converter is called the circulating current type dual converter.

### 13.4 Gate Drive circuit for three phase fully controlled converter

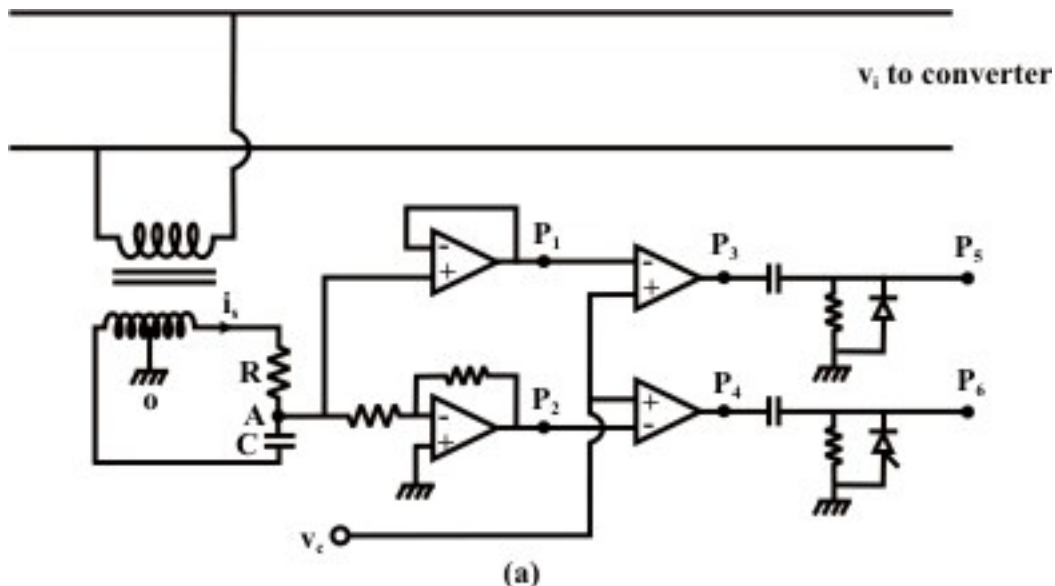
Several schemes exist to generate gate drive pulses for single phase or three phase converters. In many applications it is required that the output of the converter be proportional to a control voltage. This can be achieved as follows.

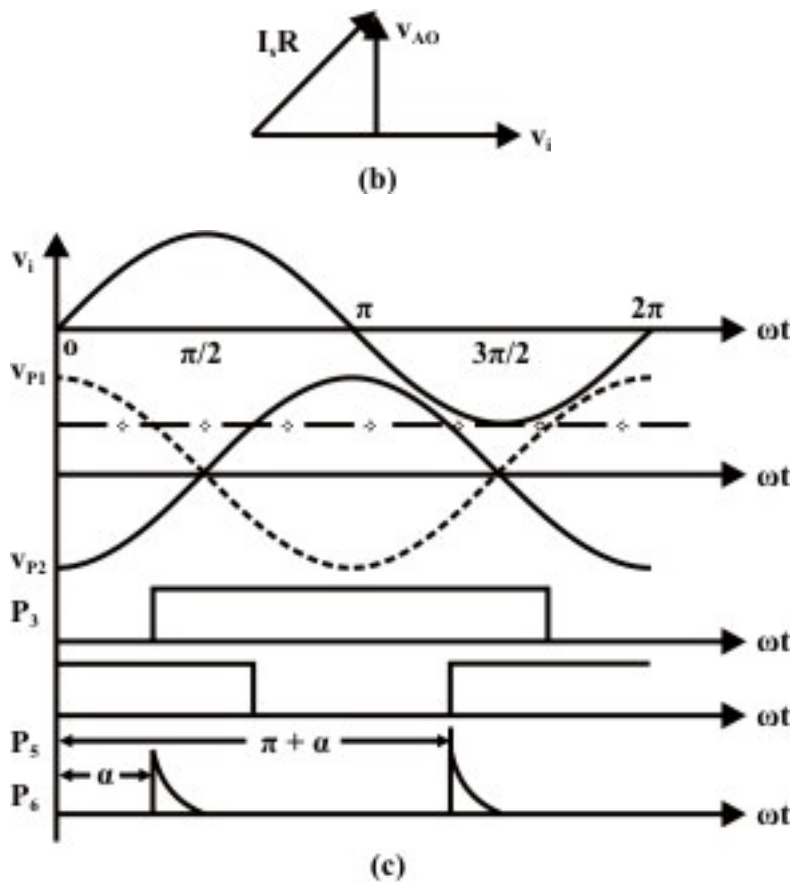
In either single or three phase converters

$$V_0 \propto \cos \alpha \quad \text{or} \quad \alpha = \cos^{-1} \frac{V_0}{K_1} \quad (13.31)$$

To get  $V_0 \propto v_c \quad \alpha = \cos^{-1} \frac{v_c}{K}$  (13.32)

The following circuit can be used to generate “ $\alpha$ ” according to equation 13.32.





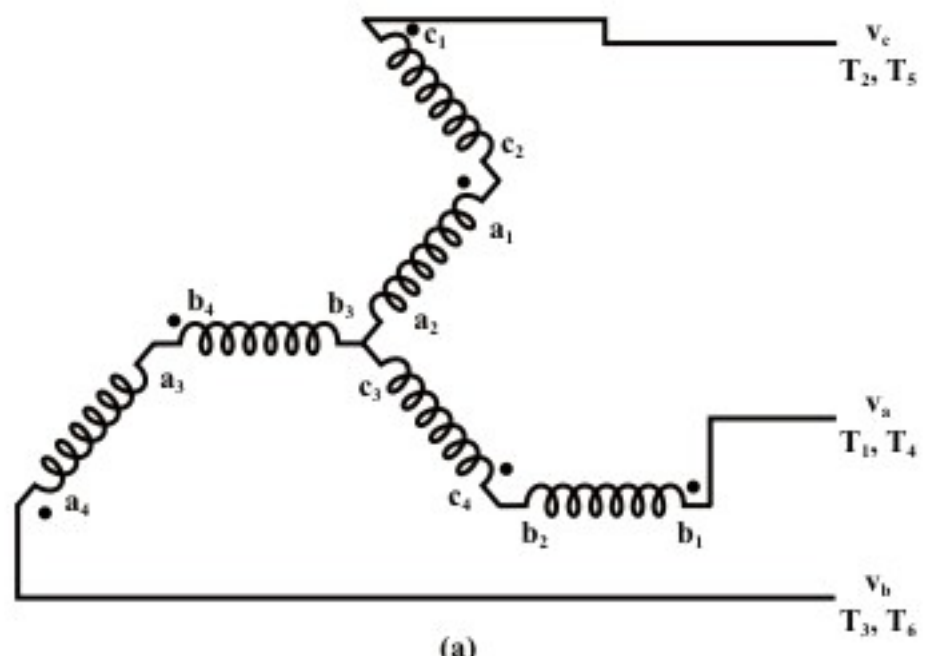
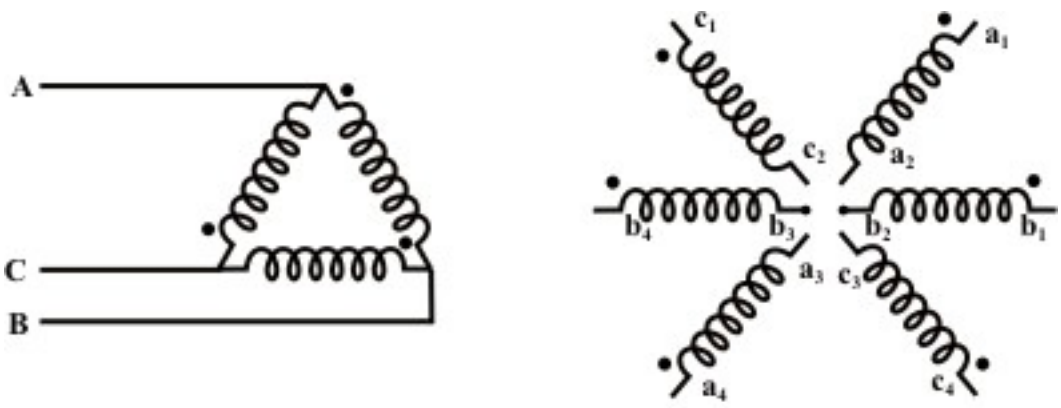
**Fig. 13.6: Triggering circuit for single phase converter**  
**(a) circuit diagram**  
**(b) phasor diagram**  
**(c) waveforms**

In the circuit of Fig. 13.6(a) a phase shift network is used to obtain a waveform leading  $v_i$  by  $90^\circ$ . The phasor diagram of the phase shift circuit is shown in Fig. 13.6(b). The output of the phase shift waveform (and its inverse) is compared with  $v_c$ . The firing pulse is generated at the point when these two waveforms are equal. Obviously at this instant

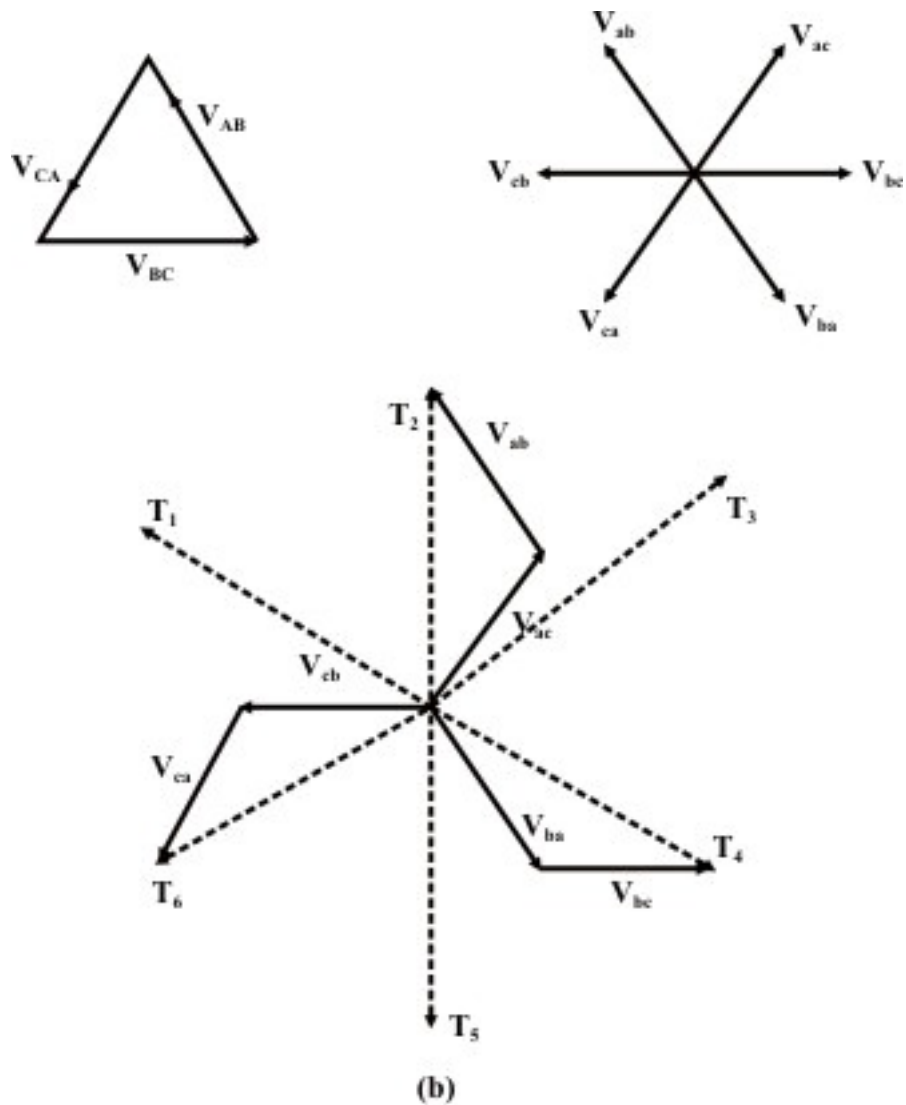
$$v_c \propto V_s \cos \alpha \quad \text{or} \quad \alpha = \cos^{-1} \frac{v_c}{V_s} \quad (13.33)$$

Therefore this method of generation of converter firing pulses is called “inverse cosine” control. The output of the phase shift network is called carrier waveform.

Similar technique can be used for three phase converters. However the phase shift network here consists of a three phase signal transformer with special connections as shown in Fig. 13.7.



(a)



**Fig. 13.7: Carrier wave generation three phase converters**  
**(a) Transformer connection**  
**(b) phasor diagram.**

The signal transformer uses three single phase transformer each of which has two secondary windings. The primary windings are connected in delta while the secondary windings are connected in zigzag. From Fig. 13.1 (c)  $T_2$  is fired  $\alpha$  angle after the positive going zero crossing of  $v_{bc}$ . Therefore, to implement inverse cosine the carrier wave for  $T_2$  must lead  $v_{bc}$  by  $90^\circ$ . This waveform is obtained from zigzag connection of the winding segments  $a_1a_2$  and  $c_1c_2$  as shown in Fig. 13.7(a). The same figure also shows the zigzag connection for other phase. The voltage across each zigzag phase can be used to fire two thyristors belonging to the same phase leg using a circuit similar to Fig. 13.6 (a). The phase shift network will not be required in this case.

### Exercise 13.3

1. Fill in the blank(s) with the appropriate word(s)
  - i) Higher pulse number converters can be realized by \_\_\_\_\_ and \_\_\_\_\_ connection of six pulse converters.
  - ii) Constituent six pulse converters of a 12 pulse converter have \_\_\_\_\_ firing angles.
  - iii) The input supply voltages to the converters of a 12 pulse converter have \_\_\_\_\_ magnitudes and are phase shifted from one another by \_\_\_\_\_ degrees.
  - iv) The input supply to a 12 pulse converter can be obtained through a \_\_\_\_\_ connected transformer.
  - v) Dual converters are used for supplying \_\_\_\_\_ quadrant dc motor drives.
  - vi) In a dual converter if one converter is fired at an angle ' $\alpha$ ' the other has to be fired at \_\_\_\_\_.
  - vii) In \_\_\_\_\_ current dual converter only one converter conducts at any time.
  - viii) In a circulating current type dual converter an \_\_\_\_\_ is used between the converters to limit the circulating current.
  - ix) To obtain a linear control relation between the control voltage and the output dc voltage of a converter \_\_\_\_\_ control logic is used.
  - x) In a three phase fully controlled converter the carrier waves for firing pulse generation are obtained using three \_\_\_\_\_ connected single phase transformers.

**Answers:** (i) Series, parallel; (ii) same, (iii) equal, 30, (iv) star – star – delta; (v) four; (vi)  $\pi - \alpha$ , (vii) non-circulating ; (viii) inductor, (ix) inverse-cosine; (x) delta-zigzag.

2. A 220V, 750 RPM, 200A separately excited dc motor has an armature resistance of 0.05  $\Omega$ . The armature is fed from a three phase non circulating current dual converter. If the forward converter operates at a firing angle of 70°
  - i) At what speed will the motor deliver rated torque.
  - ii) What should be the firing angle in the regenerative braking mode when the motor delivers half the rated torque at 600 rpm.  
Assume continuous conduction. Supply voltage is 400 V.

**Answer:**

i) The output voltage =  $\frac{3\sqrt{2}}{\pi} \times 400 \cos 70^\circ = 184.7 \text{ V}$



$$\therefore E_b = V_a - I_a r_a = 184.7 - 200 \times 0.05 = 174.75 \text{ V.}$$

$$\therefore \text{Operating speed} = \frac{174.75}{220 - 0.05 \times 200} \times 750 = 624 \text{ RPM.}$$

$$\text{ii) } E_b \Big|_{600\text{RPM}} = -\frac{600}{750} \times 210 = -168 \text{ V}_1 \quad I_a = 100 \text{ A}$$

$$\therefore V_a = -E_b + I_a r_a = -173 \text{ r.}$$

$$V_a = -173 = \frac{3\sqrt{2}}{\pi} 400 \cos \alpha \quad \therefore \alpha = 108.67^\circ$$

3. What will happen if the signal transformers generating the carrier wave have delta – double star connection instead of delta-zigzag connection.

**Answer:** With delta-double star connection of the signal transformers the carrier wave forms will be in phase with the line voltage waveforms. Therefore, without a phase shift network it will not be possible to generate carrier waveforms which are in quadrature with the line voltages. Hence inverse casine control law cannot be implemented.

## References

1. “Power Electronics”; P.C. Sen; Tata-McGrawhill publishing company limited; 1995.
2. “Power Electronics, Converters, Applications and Design”, Mohan, Undeland, Robbins; John Willey and Sons Inc; Third Edition, 2003.

## Lesson Summary

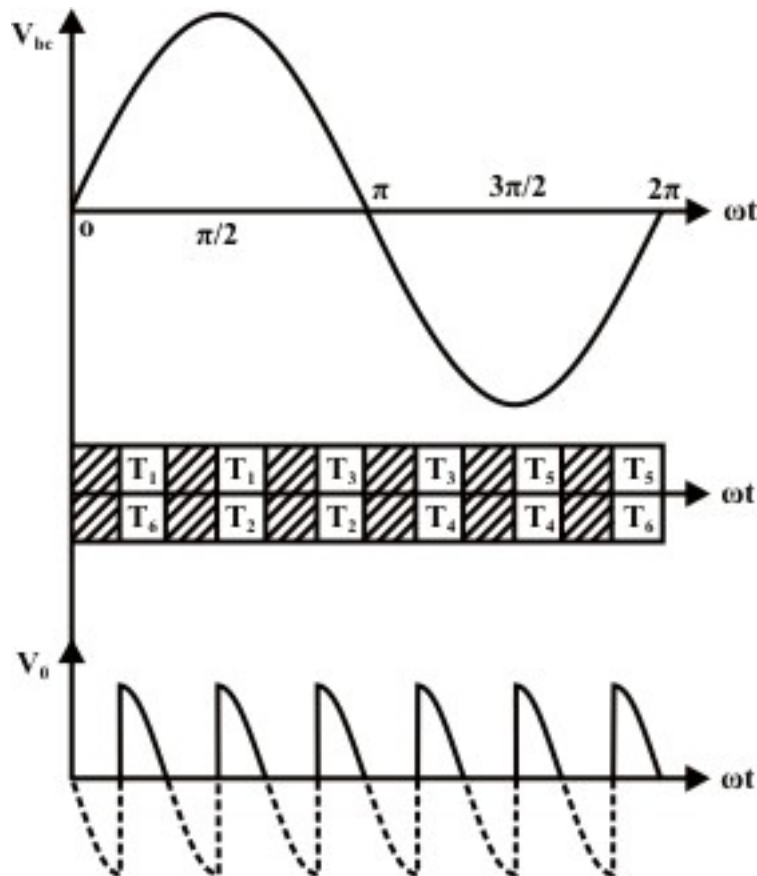
- A three phase fully controlled converter is realized by replacing the diodes of an uncontrolled converter with thyristors.
- A three phase fully controlled converter can operate either as a rectifier or as an inverter.
- The output voltage of a three phase fully controlled converter contains multiple of sixth harmonic of the input frequency in addition to the dc component.
- The input current of a three phase fully controlled converter contains only odd harmonics other than tripler harmonics.
- The input current displacement factor of a three phase fully controlled converter is  $\cos \alpha$ .  $\alpha$  being the firing angle.
- In the continuous conduction mode a three phase fully controlled converter may operate in the inverting mode by increasing  $\alpha$  beyond  $90^\circ$ .
- In the inverting mode the firing angle should be less than  $180^\circ$  for safe commutation of the thyristors.
- Several units of three phase fully controlled converters can be connected in series parallel to form higher pulse number (12, 18, 24 etc) converters.
- In higher pulse number converters all component converters are fired at the same firing angle while their input supplies are phase shifted from one another by a predetermined angle.
- Two three phase fully controlled converter can be connected in anti parallel to form a dual converter which can operate in all four quadrants of the V-I plane.
- Dual converters can be of circulating and non circulating current type.
- Fully controlled converters employ “inverse cosine control” strategy for generating firing pulses which gives linear relationship between the output voltage and the control voltage. In a three phase fully controlled converter, a three phase delta/zig-zag connected signal transformer is used to generate the required carrier waves for this purpose.

## Practice problems and answers

1. A three phase fully controlled converter operates from a 3 phase 230 V, 50 Hz supply and supplies a resistive load of  $25 \Omega$  at a firing angle  $90^\circ$ . Find out the power supplied to the load.
2. A three phase fully controlled converter supplies a 220 V 1500 RPM, 50 A separately excited dc motor from a 230 V, 3 phase, 50 Hz supply. The motor holds an overhauling load at 1000 RPM while producing full load torque. The motor has an armature resistance of  $0.2 \Omega$ . What should be the firing angle? Assume continuous conduction.
3. What precaution should be taken in the gate drive circuit so that a three phase fully controlled converter can continue to operate even when the load current becomes discontinuous.

### Answers

1.



The figure above shows the output voltage with  $\alpha = 90^\circ$  and a resistive load. Since the load is resistive the load current becomes zero when the voltage becomes zero. Both the voltage and amount remains zero thereafter till the next thyristor is fired.

Therefore for  $\frac{5\pi}{6} \leq \omega t \leq \pi$

$$\begin{aligned}
v_0 &= V_{bc} = \sqrt{2}V_L \sin \omega t \\
\pi &\leq \omega t \leq \frac{7\pi}{6} \\
v_0 &= 0 \\
\therefore V_0 |_{\text{RMS}} &= \sqrt{\frac{3}{\pi} \int_{\frac{5\pi}{6}}^{\pi} 2V_L^2 \sin^2 \omega t \, d\omega t} \\
&= V_L \sqrt{\frac{3}{\pi} \int_{\frac{5\pi}{6}}^{\pi} (1 - \cos 2\omega t) \, d\omega t} \\
\therefore P_0 &= \frac{V_0 |_{\text{RMS}}^2}{R} = V_L \sqrt{\frac{1}{2} - \frac{3}{\pi} \int_{\frac{5\pi}{6}}^{\pi} \cos 2\omega t \, d\omega t} \\
&= 183 \text{ Watts} \\
&= V_L \sqrt{\frac{1}{2} - \frac{3\sqrt{3}}{4\pi}} \\
&= 67.65 \text{ V}
\end{aligned}$$

2. To hold the overhauling load the motor must operate in the regenerative braking mode.

$$\text{At 1000 RPM } E_b = \frac{220 - 50 \times 0.2}{1500} \times 1000 = 140 \text{ volts}$$

To supply full load torque, the motor armature current = 50 A.

$\therefore$  Supply voltage =  $V_a = E_b + I_a r_a = 140 + 50 \times 0.2 = 150 \text{ V}$  in the reverse direction.

$$\therefore \frac{3\sqrt{2}}{\pi} V_L \cos \alpha = -150 \text{V} \quad \therefore \alpha = 118.9^\circ.$$

3. With reference to the conduction diagram of problem – 1 it can be seen that the load current becomes zero  $30^\circ$  after a new thyristor is fired (for example,  $T_2$ ). Therefore, both the conducting thyristor ( $T_1$  and  $T_2$  in this case) turns off. However, when  $T_3$  is fired the converter will be unable to resume operation from  $T_2T_3$  mode unless  $T_2$  is fired simultaneously. Similar explanation holds for all other thyristor firing. Therefore, to ensure that the converter operates properly even under discontinuous load current condition the final gate pulse for a particular thyristors must be generated by logically “ANDing” the outputs of its own firing circuit with the output of the firing circuit of the thyristor in the commutation sequence as shown in the table next below

To generate the

gate pulse of                    :     $T_1$          $T_2$          $T_3$          $T_4$          $T_5$          $T_6$

AND the outputs of    :     $T_1 \& T_2$      $T_2 \& T_3$      $T_3 \& T_4$      $T_4 \& T_5$      $T_5 \& T_6$      $T_6 \& T_1$

