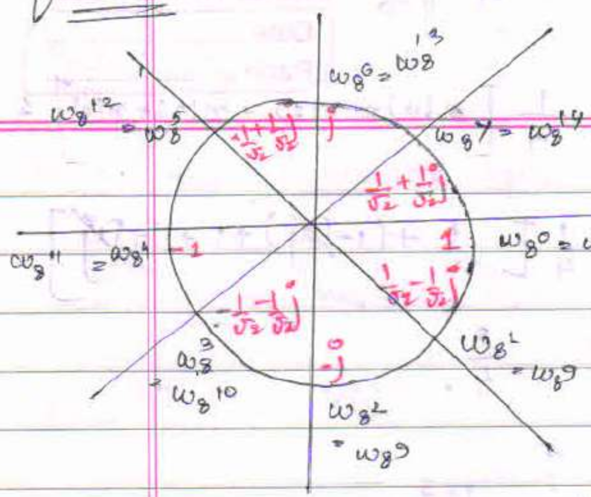
Q. for N = 8



→ DFT and IDFT as Linear Transformation

Since, DFT ⇒

$$X(K) = \sum_{n=0}^{N-1} x(n) \cdot w_N^{nK}$$

$K = 0, 1, \ldots, N-1$

$$X(0) = \sum_{n=0}^{N-1} x(n) = x(0) + x(1) + x(2) + \ldots$$

$$X(1) = x(0) + x(1) \cdot w_N^1 + x(2) W_N^2 + \ldots + x(N-1) \cdot w_N^{N-1}$$

$$\therefore X(N-1) = \sum_{n=0}^{N-1} x(n) \cdot w_N^{(N-1)n} =$$

$$x(0) + x(1) \cdot w_N^{(N-1)} + \ldots + x(N-1) \cdot w_N^{(N-1)(N-1)}$$

This can be expressed as matrix.

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ \vdots \\ X(N-1) \end{bmatrix}_{N\times 1} = \begin{bmatrix} 1 & 1 & 1 & \ldots & 1 \\ 1 & w_N^1 & \ldots & & w_N^{N-1} \\ 1 & w_N^2 & \ldots & & w_N^{2(N-1)} \\ \vdots & \vdots & \ldots & & \vdots \\ 1 & \ldots & & & w_N^{N(N-1)} \end{bmatrix}_{N\times N} \cdot \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ \vdots \\ x(N-1) \end{bmatrix}$$

Ex :-

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ \cancel{X(N-1)} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & w_4^1 & w_4^2 & w_4^3 \\ 1 & w_4^2 & w_4^4 & w_4^6 \\ 1 & w_4^3 & w_4^6 & w_4^9 \end{bmatrix} \cdot \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} x \\ x \\ x \\ x \end{bmatrix}$$

Q. Determine DFT of the sequence -

$$x(n) = \begin{cases} 1/4 & 0 \le n \le 2 \\ 0 & \text{otherwise} \end{cases}$$

The N point DFT of $x(n)$

$$X(K) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N} nK}$$

$K = 0, 1, \ldots, (N-1)$

$x(n) = (1/4, 1/4, 1/4)$

$\therefore X(K) = \frac{1}{4}\left[1 + e^{-j\omega} + e^{-j2\omega}\right]\Big|_{\omega = \frac{2\pi K}{N}}$

$= \frac{1}{4} e^{-j2\pi K/3}\left[1 + 2\cos\left(\frac{2\pi K}{3}\right)\right]$

Hence,

$X(K) = \frac{1}{4} \cdot e^{-j\frac{2\pi K}{3}}\left[1 + 2\cos\left(\frac{2\pi K}{3}\right)\right]$

where $K = 0, 1, \ldots N-1$

Qo. Determine DFT of sequence

$x(n) = \begin{cases} 1/5 & -1 \le n \le 1 \\ 0 & \text{otherwise} \end{cases}$

$x(n) = \left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$

$X(K) = \sum_{n=0}^{\infty} x(n)\, e^{-j\omega n}$, at $\omega = \frac{2\pi k}{N}$.

$X(K) = \frac{1}{5}\left[e^{j\omega} + 1 + e^{-j\omega}\right]\Big|_{\omega = \frac{2\pi k}{N}}$

$= \frac{1}{5}\left[1 + 2\cos\left(\frac{2\pi K}{3}\right)\right]$

$X(K) = \frac{1}{5}\left[1 + 2\cos\left(\frac{2\pi K}{3}\right)\right]$ → ans

Qo Find N-point DFT for

$x(n) = a^n \qquad 0 < a < 1$

$X(K) = \sum_{n=0}^{N-1} x(n)\, e^{-j\frac{2\pi}{N}\cdot nK}, \quad K = 0, 1, \ldots (N-1)$

$X(K) = \sum_{n=0}^{N-1} a^n e^{-j\frac{2\pi}{N}nk}$

$= \frac{1 - \left(ae^{-j\frac{2\pi k}{N}}\right)^N}{1 - ae^{-j2\pi k/N}}$

$X(K) = \frac{1 - a^N}{1 - ae^{-j2\pi nk/N}}$ → ans

Qo. Determine IDFT of

$X(K) = \{3, (2+j), 1, (2-j)\}$

$x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X(k)\, e^{j\frac{2\pi}{N}nk}$. $\quad 0 \le n \le N-1$

$N = 4, \quad x(n) = \frac{1}{4}\sum_{K=0}^{3} X(K)\, e^{j\frac{\pi nk}{2}}$

when $n = 0$

$x(0) = \frac{1}{4}\sum_{k=0}^{3} X(k)e^{0}$

$= \frac{1}{4}\left[3 + (2+j) + 1 + (2-j)\right] = 2$

When $n = 1$.

$x(1) = \frac{1}{4}\left[3 + (2+j)\, e^{j\pi/2} + e^{j\pi} + (2-j)\, e^{j3\pi/2}\right]$.

$= \frac{1}{4}\left[3 + (2+j)j - 1 + (2-j)(-j)\right]$

$= 0$.

when $n = 2$.

$x(2) = \frac{1}{4}\left[3 + (2+j)(-1) + 1 + (2-j)(-1)\right] = 0$

Q. when $n=3$.

$$x(3) = \frac{1}{4}\sum_{k=0}^{3} X(k)\, e^{j3\pi k/2}.$$

$$x(3) = \frac{1}{4}\left[3 + (2+j)(-j) - 1 + (2-j)(j)\right] = 1$$

$\therefore x(n) = \{2, 0, 0, 1\}$ → ans.

Imp.

- Properties of DFT :-

1. Linearity
2. Periodicity
3. Circular shift of a Sequence
4. Time Reversal
5. Circular time shift.
6. Circular freq. shift.
7. Circular Convolution.
8. Complex Conjugation.
9. Multiplication of two Sequences
10. Parseval's theorem.
11. Symmetric property.

1. Linearity :-

If $x_1(n) \xleftrightarrow[DFT]{N} X(K) = \sum_{n=0}^{N-1} x_1(n)\, W_N^{nK}$,

$K = 0, 1, \ldots, N-1$

$x_2(n) \xleftrightarrow[DFT]{N} X_2(K) = \sum_{n=0}^{N-1} x_2(n)\, W_N^{nK}$

$K = 0, 1, \ldots, (N-1)$

then,

$x_3(K) = a_1 X_1(K) + a_2 X_2(K)$

$K = 0, 1, \ldots, (N-1)$

Proof :-

$$DFT[x_3(n)] = \sum_{n=0}^{(N-1)} x_3(n)\, W_N^{nK}$$

$K = 0, 1, \ldots, (N-1)$

$$= \sum_{n=0}^{N-1} \left[a_1 x_1(n) + a_2 x_2(n)\right] W_N^{nK},$$

$K = 0, 1, \ldots, (N-1)$

$$= a_1 X_1(K) + a_2 X_2(K).$$

proved.

2. Periodicity :-

If $x(n) \xleftrightarrow[DFT]{N} X(K)$

then, $X(K+N) = X(K)$

proof :- $X(K) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{nK}$.

$$X(K+N) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{n(K+N)}$$

$$= \sum_{n=0}^{N-1} x(n) \cdot W_N^{nK} \cdot W_N^{Nn}$$

Here $W_N^{Nn} = \left(e^{-j\frac{2\pi}{N}N}\right)^n$

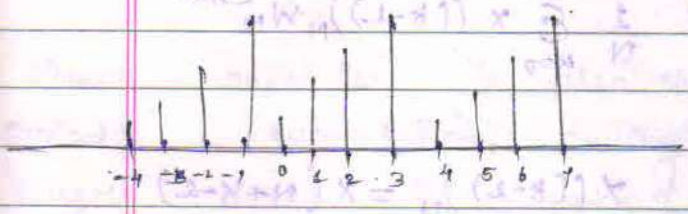
$= (e^{-j2\pi})^n = (1)^n = 1$

$$\therefore X(K+N) = \sum_{n=0}^{N-1} x(n) \cdot W_N^{nK} = X(K)$$

proved.

3. Circular shift of a Sequence :-

In DFT we cannot combine linear shifted seq. with the original one due to loss of signal in the period. So we do periodic shifting.



In periodic shifting - sequences can be (1 2 3 4) (4 1 2 3) (3 4 1 2) (2 3 4 1) (1 2 3 4)

$$\boxed{x_c(n) = x_p(n-K)}$$

circular shift ↓ periodic shifting ↓

K = no. of positions (shifted / shifting periodic)

ie
$x_c(0) = x_p(0-1) = 4$
$x_c(1) = x_p(1-1) = 1$
$x_c(2) = x_p(2-1) = 2$
$x_c(3) = x_p(3-1) = 3$.

$x_c(n) = x_p(n-K)$
$= x((n-K))_N$

another way of representing periodic shifting.

N = periodicity.

$$\boxed{\begin{aligned} x_c(n) &= x(N+n-K) \\ &= x_p((n-K))_N \end{aligned}}$$

Eg. $K=1$ $N=4$

then.

$x_c(n) = x((N+n-K))$
$x_c(0) = x(4+0-1) = x(3)$
$x_c(1) = x(4+1-1) = x(4)$
$x_c(2) = x(4+2-1) = x(1)$
$x_c(3) = x(4+3-1) = x(2)$.

4. Time Reversal:-

If $x(n) \xleftrightarrow[DFT]{N} X(K)$.

then,

$x((-n))_N \xleftrightarrow[DFT]{N} X((-K))_N$

$x(N-n) \xleftrightarrow[DFT]{N} X(N-K)$.

Proof :- $DFT[x((-n))] = \sum_{n=0}^{N-1} x((-n))_N W_N^{nK}$.

$K = 0, 1, \ldots, (N-1)$

$x((-n))_N = x(N-n)$

$= \sum_{n=0}^{N-1} x(N-n)\, W_N^{nK}$.

$K = 0, 1, \ldots, (N-1)$

let $N-n = t$.

$DFT[x((-n))_N] = \sum_{t=N}^{1} x(t) \cdot W_N^{(N-t)K}$

$= \sum_{t=N}^{1} x(t) \cdot W_N^{-tK}$

$DFT[x((-n))_N] = \sum_{t=N}^{1} x(t)\, W_N^{-tK}$

Q. (2)

$$DFT\left[x((-n))_N^{*}\right] = \sum_{t=N}^{1} x(t) W_N^{-tk}$$

$$= \sum_{t=N}^{1} x(t) W_N^{-tk}$$

$$W_N^{Nt} = \sum_{t=N}^{1} x(t) \cdot W_N^{(N-K)t}$$

$$= X(N-K) = X((-K))_N$$

5. Circular time shift :-

$$x(n) \xleftrightarrow[DFT]{N} X(K)$$

then,

$$x((n-l)) \xleftrightarrow[DFT]{N} W_N^{lk} X(K)$$

proof :-

$$DFT\left[x((n-l))_N\right] = DFT\left[x(N+n-l)\right]$$

$$= \sum_{n=0}^{N-1} x(N+n-l) W_N^{nk},$$

$K = 0, 1, \ldots (N-1)$,

put $N+n-l = t$

then

$$DFT\left[x((n-l))_N\right] = \sum_{t=N-l}^{2N-l-1} x(t) \cdot W_N^{(t+l-N)k}$$

$$= \sum_{t=N-l}^{2N-l-1} x(t) W_N^{(tk)} \cdot W_N^{lk} \cdot W_N^{-NK}$$

$$= \sum_{t=N-l}^{2N-l-1} x(t) W_N^{tk} * W_N^{lk}$$

$$= W_N^{lk} X(K)$$

proved.

6. Circular freq. shift

If $x(n) \longleftrightarrow X(K)$

then,

$$x(n) W_N^{-ln} \xleftrightarrow[N]{DFT} X((K-l))$$

proof :-

$$IDFT\left[X((K-l))_N\right] =$$

$$\frac{1}{N}\sum_{K=0}^{N-1} X((K-l))_N W_N^{-nK}$$

$$X((K-l))_N = X(N+K-l)$$

$$IDFT\left[X((K-l))_N\right] = \frac{1}{N}\sum_{K=0}^{N-1} X(N+K-l) W_N$$

put $N+K-l = t$

$$IDFT\left[X((K-l))_N\right] = \frac{1}{N}\sum_{N-l}^{2N-l-1} X(t) \cdot W_N$$

$$= \frac{1}{N}\sum_{N-l}^{2N-l-1} x(t) \cdot W_N^{-tn} \cdot W_N^{-ln} \cdot W_N$$

$$IDFT\left[X((K-l))_N\right] = W_N^{-ln} \frac{1}{N}\sum_{t=N-l}^{2N-l-1} X(t)$$

$$= (W_N^{-ln} \cdot x(n))$$

now taking DFT on both sides we get -

$$X((K-l))_N = DFT\left[W_N^{-ln} x(n)\right]$$

$$\therefore X((K-l)) = x(n) W_N^{-ln}.$$

proved

Circular Convolution :-

If $x_1(n) \xleftrightarrow[DFT]{N} X_1(K)$

$x_2(n) \xleftrightarrow[DFT]{N} X_2(K)$

and $X_3(K) = X_1(K) \cdot X_2(K)$

then,

$x_3(n) = x_1(n) \circledast x_2(n)$

circular convolution is also cld periodic convolution. having length equal to length of convolved sequences.

proof :- $x_3(n) = IDFT[X_3(K)]$

$$x_3(n) = \frac{1}{N} \sum_{K=0}^{N-1} X_3(K) W_N^{-nK}$$

$n = 0, 1, \ldots (N-1)$

Since,

$X_3(K) = X_1(K) \cdot X_2(K)$

$$\therefore x_3(n) = \frac{1}{N} \sum_{K=0}^{(N-1)} X_1(K) \cdot X_2(K) \cdot W_N^{-nK} \quad \text{---(I)}$$

let

$$X_1(K) = \sum_{m=0}^{(N-1)} x_1(m) \cdot W_N^{mK} \quad \text{---(II)}$$

$K = 0, 1, 2, \ldots (N-1)$

$$X_2(K) = \sum_{l=0}^{(N-1)} x_2(l) W_N^{lk} \quad \text{---(III)}$$

$K = 0, 1, \ldots, (N-1)$

now substituting the values from eq. (II) & (III) into eq. (I),

$$\therefore x_3(n) = \frac{1}{N} \sum_{K=0}^{(N-1)} \left[\sum_{m=0}^{(N-1)} x_1(m) \cdot W_N^{+mk}\right] \cdot \left[\sum_{l=0}^{(N-1)} x_2(l) W_N^{lk}\right] \cdot W_N^{-nK} \quad \text{---(IV)}$$

$$= \frac{1}{N} \sum_{m=0}^{N-1} x_1(m) \cdot \sum_{l=0}^{N-1} x_2(l) \sum_{K=0}^{N-1} W_N^{(m+l-n)k}$$

for $\sum_{K=0}^{(N-1)} W_N^{(m+l-n)k}$

suppose $W_N^{(m+l-n)} = a$.

$$\therefore \sum_{k=0}^{(N-1)} a^k = \begin{cases} \frac{1-a^N}{1-a} & \text{when } a \neq 1 \\ N & a = 1 \end{cases}$$

Here we see that on considering $\sum_{k=0}^{(N-1)} a^k = \frac{1-a^N}{1-a}$,

then eq. (IV) becomes zero.

$\therefore$ we will consider the next case. ie $\sum_{k=0}^{N-1} a^k = N$, $(a=1)$

$$\frac{1-W_N^{(m+l-n)N}}{1-W_N^{(m+l-n)}}$$

for existance of circular conv.

$a = 1$.

ie $W_N^{(m+l-n)} = 1$,

when,

$m+l-n = qN$

$$x_3(n) = \frac{1}{N} \sum_{m=0}^{N-1} x_1(m) \sum_{l=0}^{N-1} x_2(l) \cdot N$$

$m+l-n = q$

Q. $x_3(n) = \sum_{m=0}^{N-1} x_1(m) \sum_{l=0}^{N-1} x_2(l)$

$m+l-n = qn.$

$m+l-n = q_N$

$l = q_N + n - m$

(2) $l = ((n-m))_N.$

$x_3(n) = \sum_{m=0}^{N-1} x_1(m) \cdot x_2((n-m))_N.$

(circular conv.)

$n = 0, 1, \ldots, (N-1)$

$x_3(n) = \sum_{K=-\infty}^{\infty} x_1(K) \cdot x_2(n-K).$

$= x_1(n) \circledast x_2(n)$

(linear conv).

$x_3(n) = x_1(n) \odot x_2(n).$

proved.

Methods of Circular Convolution :-

- Tabular
- Using Circular array.
- using DFT & IDFT (Stockham's Method)
- Matrix

Q. Determine $\odot$ of $x_1(n) = \{1, 2, 2, 1\}$

$x_2(n) = \{2, 1, 1, 2\}$

By Tabular Method –

range $-(N-1) \le n \le (N-1)$

	-3	-2	-1	0	1	2	3
$x_1(n)$				$\boxed{1}$	$\boxed{2}$	$\boxed{2}$	$\boxed{1}$
$x_2(n)$				2	1	1	[illegible]
$x_{2p}(n)$	1	1	2	2	1	1	[illegible]
$x_{2p}(-n)$	2	1	1	$\boxed{2}$	$\boxed{2}$	$\boxed{1}$	[illegible]
$x_2(1-n)$		2	1	1	2	2	[illegible]
$x_2(2-n)$			2	1	1	2	[illegible]
$x_2(3-n)$				2	1	1	[illegible]

(no -ve value)

$x_3(n) = \sum_{m=0}^{N-1} x_1(m)\, x_2(n-m)_N$

$n = 0, 1, \ldots (N-1).$

$n=0$

$x_3(n) = \sum_{m=0}^{3} x_1(m) \cdot x_2(0-m)_N.$

$x_3(0) = 1\times2 + 2\times2 + 2\times1 \times 1\times1 = 9$

$x_3(1) = 1\times1 + 2\times2 + 2\times2 + 1\times1 = 10$

$x_3(2) = 1\times1 + 2\times1 + 2\times2 + 1\times2 = 9$

$x_3(n) = 1\times2 + 2\times1 + 2\times1 + 1\times2 = 8.$

$\therefore x_3(n) = \{9, 10, 9, 8\}$

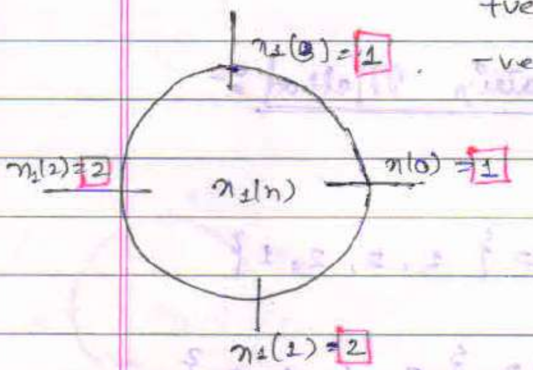
By Using Circular Array :-

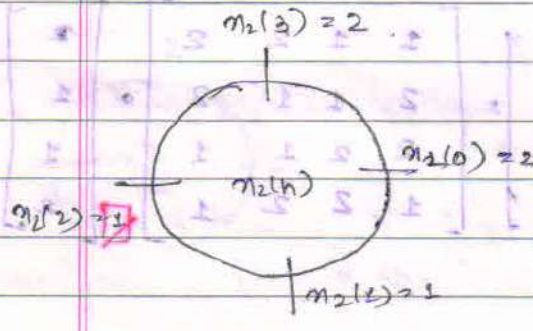
$x_1(n) = \{1, 2, 2, 1\}$

$x_2(n) = \{2, 1, 1, 2\}$

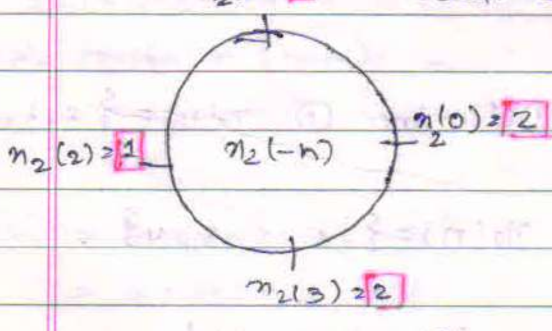
+ve → clockwise

−ve → anticlockwise



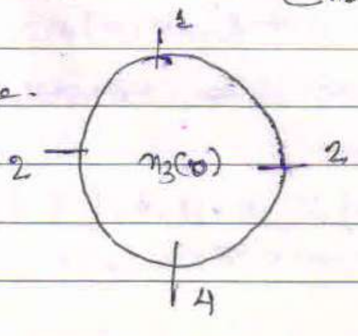


$x_2(1) = 1$ (anticlockwise)

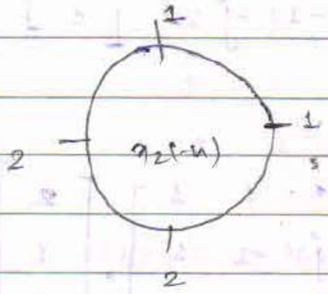


$\Sigma x_1(n) \cdot x_2(-n) = 9.$

product sequence.



$x_3(1) = x_1(n) +$ clockwise shift of $x_2(-n)$.



$\Sigma x_1(n) \cdot x_2(-n)$

$x_3(1) = 1\times1 + 1\times1 + 2\times2 + 2\times2 = 10$

$x_3(2)$ $x_1(n)$ + 2 times clockwise shift of $x_2(-n)$

$x_3(2) = 2\times1 + 1\times1 + 1\times2 + 2\times2 = 9.$

$x_3(3) = x_1(n)$ + 3 times clockwise shift of $x_2(-n)$

$x_3(n) = 8.$

$\therefore x_3(n) = \{9, 10, 9, 8\}$

Imp. By Using DFT & IDFT Method :-

If $X_3(K) = X_1(K) \cdot X_2(K)$

$x_3(n) = x_1(n) \circledast x_2(n)$

$x_1(n) = \{1, 2, 2, 1\}$

$x_2(n) = \{2, 1, 1, 2\}$

$$\begin{bmatrix} x_1(0) \\ x_1(1) \\ x_1(2) \\ x_1(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x_2(0) \\ x_2(1) \\ x_2(2) \\ x_2(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & 1 \\ 1 & -j & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$x_1(0) = 1+2+2+1 = 6$

$X_1(1) = 1 - 2j - 2 + j = -1-j$

$X_1(2) = \cancel{1} - \cancel{2} + \cancel{2} - j = \cancel{1} - \cancel{j}\ 0$

$X_1(3) = 2 + j - 1 - 2j = 1-j$

$X_2(0) = 6 \qquad X_2(1) = 1+j$

$X_2(3) = 0 \qquad X_2(3) = 1-j$

$X_1(K) = \{6, (-1-j), 0, (-1+j)\}$

$X_2(K) = \{6, (1+j) \cdot 0, (1-j)\}$

$\therefore\ X_3(K) = \{36, -2j, 0, 2j\}$.

$$\begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ x_3(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & W_4^{-1} & W_4^{-2} & W_4^{-3} \\ 1 & W_4^{-2} & W_4^{-4} & W_4^{-6} \\ 1 & W_4^{-3} & W_4^{-6} & W_4^{-9} \end{bmatrix} \cdot \begin{bmatrix} 36 \\ -2j \\ 0 \\ 2j \end{bmatrix}$$

$$\begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ x_3(3) \end{bmatrix} = \frac{1}{4} \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -j & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix} \begin{bmatrix} 36 \\ -2j \\ 0 \\ 2j \end{bmatrix}.$$

$x_3(n) = \{9, 10, 9, 8\}$

<u>By Matrix Method :-</u>

$x_1(n) = \{1, 2, 2, 1\}$

$x_2(n) = \{2, 1, 1, 2\}$

$$\begin{bmatrix} x_3(0) \\ x_3(1) \\ x_3(2) \\ x_3(3) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 1 & 1 & 2 \\ 2 & 2 & 1 & 1 \\ 1 & 2 & 2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

$x_3(n) = \{9, 10, 9, 8\}$.

Q. Determine (i) $x_1(n) = \{2, 1, 2, \ldots$

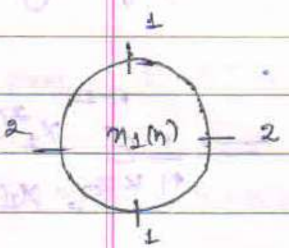
$x_2(n) = \{1, 2, 3, 4\}$.

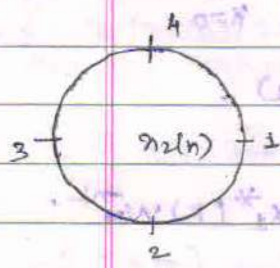
ans - $\{14, 16, 14, 16\}$

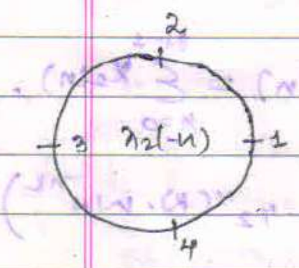
Circular array.

$x_1(n) = \{2, 1, 2, 1\}$

$x_2(n) = \{1, 2, 3, 4\}$







$x_3(0) = \sum x_1(m)\, x_2(-m)$

$= 2+6+4+2 = 14$

$x_3(1) = \sum x_1(m)\, x_2(1-m)$

$= 1+8+3+4 = 16.$

$x_3(2) = \sum x_1(m)\, x_2(2-m)$

$= 6+4+2+2 = 14$

$x_3(3) = \sum x_1(m)\, x_2(3-m)$

$= 4+3+8+1 = 16.$

$\therefore x_3(n) = \{14, 16, 14, 16\}$

Matrix Method.

$$x_3(n) = \begin{bmatrix} 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 \\ 1 & 2 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix} =$$

$x_3(n) = \{14, 16, 14, 16\}$

8. Complex Conjugate Property :-

If $x(n) \xleftrightarrow[DFT]{N} X(K)$

then $x^*(n) \xleftrightarrow[DFT]{N} X^*((-K))_N$

$= X^*(N-K)$

proof :- $DFT[x(n)] = \sum_{n=0}^{(N-1)} x(n) W_N^{nk}$

$K = 0, 1, \dots, (N-1).$

$DFT[x^*(n)] = \sum_{n=0}^{N-1} x^*(n) W_N^{nk}$

$K = 0, 1, \dots, (N-1)$

$DFT[x^*(n)] = \sum_{n=0}^{N-1} x^*(n) W_N^{nk} W_N^{-Nn}$

$= \sum_{n=0}^{N-1} x^*(n) W_N^{-(N-K)n}.$

(multiplying $W_N^{-Nn} = 1$)

$DFT[x^*(n)] = \sum_{n=0}^{(N-1)} x^*(n) W_N^{nk} W_N^{-Nn}$

$= \sum_{n=0}^{N-1} [x(n)\, W_N^{(N-K)n}]^*$

$= X^*(N-K)$