CHAPTER

Conduction–Steady-State One Dimension

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2.1. INTRODUCTION

In this chapter an attempt will be made to derive general heat conduction equation and examine the applications of Fourier's law of heat conduction to the calculation of heat flow in some simple one-dimensional systems. Under the category of one-dimensional systems several different physical shapes may fall; *when the temperature of the body is a function only of radial distance and is independent of azimuth angle or axial distance cylindrical and spherical systems are treated as one-dimensional.* In case of problems of two-dimensional nature the effect of a second-space coordinate may be so small that it may be neglected and the heat-flow problems of multi-dimensional type may be approximated with a one-dimensional analysis; in such cases the differential equations are simplified and as a consequence of this simplification much easier solution in available.

2.2. GENERAL HEAT CONDUCTION EQUATION IN CARTESIAN COORDINATES

Consider an infinitesimal rectangular parallelopiped (volume element) of sides dx, dy and dz parallel, respectively, to the three axes (X, Y, Z) in a medium in which temperature is varying with location and time as shown in Fig. 2.1.

Let, t = Temperature at the left face *ABCD*; this temperature may be assumed uniform over the entire surface, since the area of this face can be made arbitrarily *small*, and

$\frac{dt}{dx}$ = Temperature changes and rate of change along *X*-direction.

Then, $\left(\frac{\partial t}{\partial x}\right)dx$ = Change of temperature through distance dx, and

$t + \left(\frac{\partial t}{\partial x}\right)dx$ = temperature on the right face *EFGH* (at a distance dx from the left face *ABCD*).

Further, let, k_x, k_y, k_z = Thermal conductivities (direction characteristics of the material) along *X*, *Y* and *Z* axes.

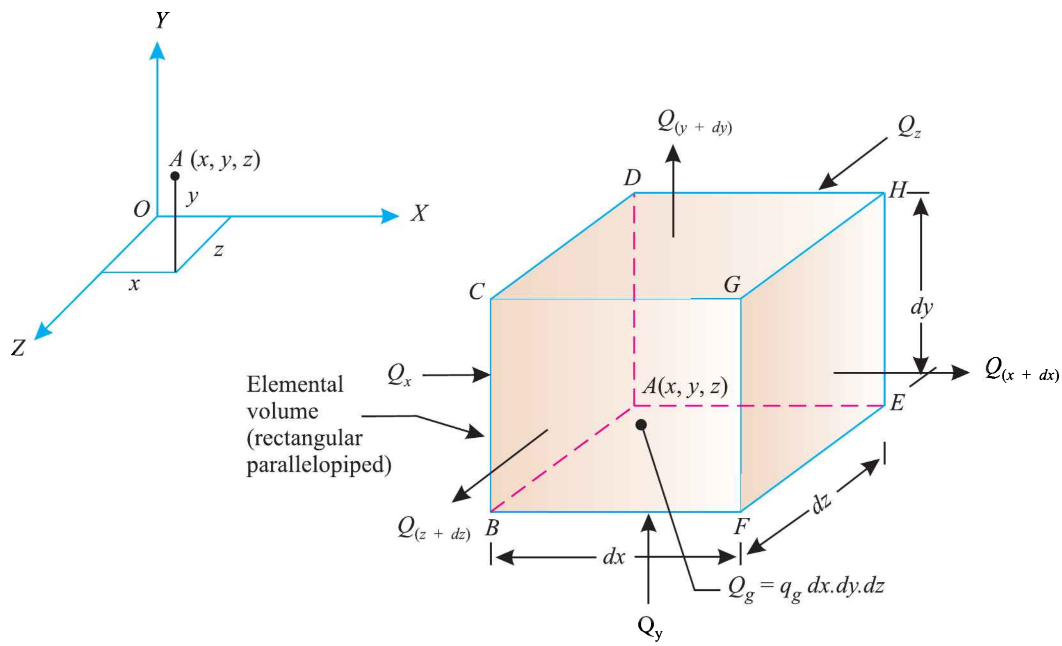


Fig. 2.1. Elemental volume for three-dimensional heat conduction analysis - Cartesian coordinates.

[If the directional characterisitics of a material are equal/same, it is called an "*Isotropic material*" and if unequal/different "*Anisotropic material*".]

q_g = Heat generated per unit volume per unit time.

[Inside the control volume there may be heat sources due to flow of electric current in electric motors and generators, nuclear fission etc.]

(**Note :** q_g may be function of position or time, or both).

ρ = Mass density of material, and

c = Specific heat of the material.

Energy balance/equation for volume element :

Net heat accumulated in the element due to conduction of heat from all the coordinate directions considered (*A*) + heat generated within the element (*B*) = Energy stored in the element (*C*). ...(1)

Let, Q = Rate of heat flow in a direction, and

$Q' = (Q.d\tau)$ = Total heat flow (flux) in that direction (in time $d\tau$).

A. *Net heat accumulated in the element due to conduction of heat from all the directions considered*:

Quantity of heat flowing into the element from the left face *ABCD* during the time interval $d\tau$ in *X*-direction is given by :

Heat influx, $$Q'_x = -\, k_x\, (dy.dz)\, \frac{\partial t}{\partial x} \cdot d\tau \qquad ...(i)$$

During the same time interval $d\tau$ the heat flowing out of the right face of control volume (*EFGH*) will be :

Heat efflux, $$Q'_{(x+dx)} = Q'_x + \frac{\partial}{\partial x}(Q'_x)\, dx \qquad ...(ii)$$

∴ Heat accumulation in the element due to heat flow in *X*-direction,

$$dQ'_x = Q'_x - \left[Q'_x + \frac{\partial}{\partial x}(Q'_x)\, dx\right] \qquad \text{[Subtracting } (ii) \text{ from } (i)]$$

$$= -\frac{\partial}{\partial x}(Q'_x)\, dx$$

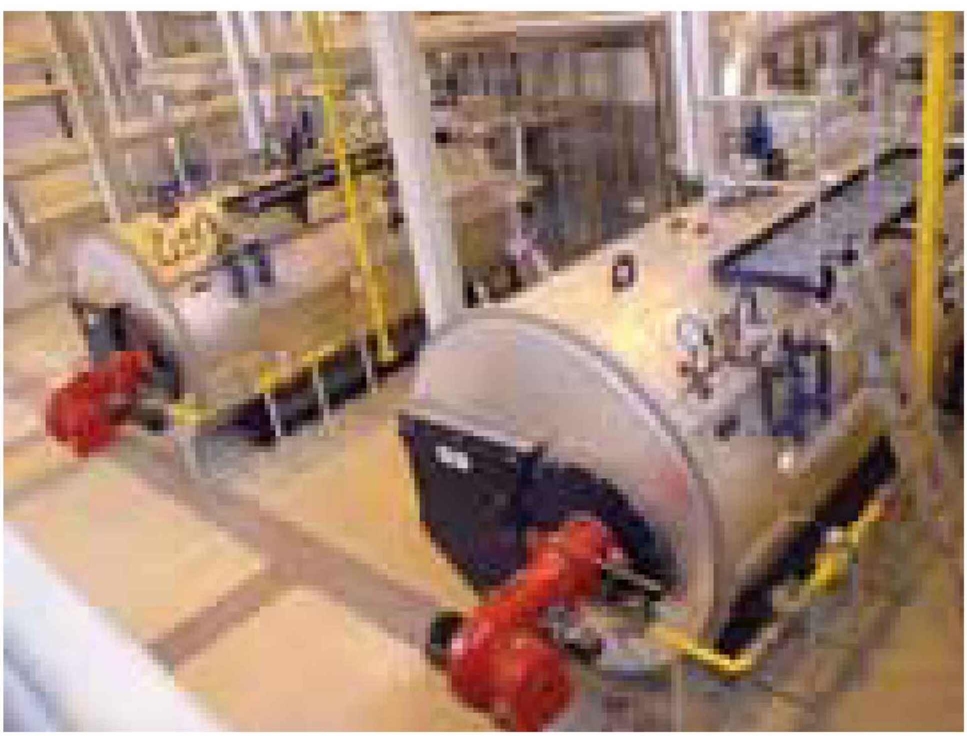
$$= -\frac{\partial}{\partial x}\left[-k_x\, (dy.dz)\, \frac{\partial t}{\partial x} \,.\, d\tau\right] dx$$

$$= \frac{\partial}{\partial x}\left[k_x \frac{\partial t}{\partial x}\right] dx.dy.dz.d\tau \qquad ...(2.1)$$

Similarly the heat accumulated due to heat flow by conduction along *Y* and *Z* directions in time $d\tau$ will be :

$$dQ'_y = \frac{\partial}{\partial y}\left[k_y \frac{\partial t}{\partial y}\right] dx.dy.dz.d\tau \qquad ...(2.2)$$

$$dQ'_z = \frac{\partial}{\partial z}\left[k_z \frac{\partial t}{\partial z}\right] dx.dy.dz.d\tau \qquad ...(2.3)$$



Boilers in a plant. A good boiler should efficiently take the heat from the fuel and minimise loss of heat from inside to outside.

$\therefore$ *Net heat accumulated* in the element due to conduction of heat from all the coordinate directions considered

$$= \frac{\partial}{\partial x}\left[k_x \frac{\partial t}{\partial x}\right] dx.dy.dz.d\tau + \frac{\partial}{\partial y}\left[k_y \frac{\partial t}{\partial y}\right] dx.dy.dz.d\tau + \frac{\partial}{\partial z}\left[k_z \frac{\partial t}{\partial z}\right] dx.dy.dz.d\tau$$

$$= \left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dx.dy.dz.d\tau \qquad ...(2.4)$$

B. *Total heat generated within the element* (Q_g') :

The total heat generated in the element is given by

$$Q_g' = q_g\,(dx.dy.dz)\,d\tau \qquad ...(2.5)$$

C. *Energy stored in the element* :

The total heat accumulated in the element due to heat flow along coordinate axes (Eqn. 2.4) and the heat generated within the element (Eqn. 2.5) together serve to increase the thermal energy of the element/lattice. This increase in thermal energy is given by

$$\rho\,(dx.dy.dz)\,c.\frac{\partial t}{\partial \tau}\,.\,d\tau \qquad ...(2.6)$$

[$\because$ Heat stored in the body = Mass of the body × specific heat of the body material × rise in the temperature of body].

Now, substituting eqns. (2.4), (2.5), (2.6), in the eqn. (1), we have

$$\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right)\right] dx.dy.dz.d\tau + q_g\ (dx.dy.dz.)d\tau = \rho\,(dx.dy.dz)\ c.\frac{\partial t}{\partial \tau}\,.\,d\tau$$

Dividing both sides by $dx.dy.dz.d\tau$, we have

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial t}{\partial x}\right)+\frac{\partial}{\partial y}\left(k_y \frac{\partial t}{\partial y}\right)+\frac{\partial}{\partial z}\left(k_z \frac{\partial t}{\partial z}\right) + q_g = \rho.c.\frac{\partial t}{\partial \tau} \qquad ...(2.7)$$

or, using the vector operator ∇, we get

$$\nabla\,.(k\nabla t) + q_g = \rho.c.\frac{\partial t}{\partial \tau} \qquad ...[2.7\ (a)]$$

This is known as the **general heat conduction equation for 'non-homogeneous material'**, **'self heat generating'** and **'unsteady three-dimensional heat flow'**. This equation establishes in *differential form the relationship between the time and space variation of temperature at any point of solid through which heat flow by conduction takes place.*

General heat conduction equation for constant thermal conductivity :

In case of homogeneous (in which properties *e.g.*, specific heat, density, thermal conductivity etc. are same everywhere in the material) and isotropic (in which properties are independent of surface orientation) material, $k_x = k_y = k_z = k$ and diffusion equation Eqn. (2.7) becomes

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = \frac{\rho.c}{k}\cdot\frac{\partial t}{\partial \tau} = \frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau} \qquad ...(2.8)$$

where, $\alpha = \frac{k}{\rho.c} = \frac{\text{Thermal conductivity}}{\text{Thermal capacity}}$

The quantity, $\alpha = \frac{k}{\rho.c}$ is known as **thermal diffusivity**.

- The larger the value of α, the faster will the heat diffuse through the material and its temperature will change with time. This will result either due to a high value of thermal conductivity k or a low value of heat capacity $\rho.c$. A low value of heat capacity means the less amount of heat entering the element, would be absorbed and used to raise its temperature and more would be available for onward transmission. Metals and gases have relatively high value of α and their response to temperature changes is quite rapid. The non-metallic solids and liquids respond slowly to temperature changes because of their relatively small value of thermal diffusivity.

– Thermal diffusivity is an important characteristic quantity for *unsteady conduction situations*.

Eqn. (2.8) by using Laplacian ∇^2, may be written as :

$$\nabla^2 t + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \qquad \text{...[2.8 }(a)]$$

Eqn. (2.8), governs the temperature distribution under unsteady heat flow through a material which is homogeneous and isotropic.

Other simplified forms of heat conduction equation in cartesian coordinates :

(*i*) For the case when *no internal source of heat generation is present*, Eqn. (2.8) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$ [Unsteady state $\left(\frac{\partial t}{\partial \tau} \neq 0\right)$ heat flow with no internal heat generation]

or,
$$\nabla^2 t = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \qquad \text{(Fourier's equation)} \qquad \text{...(2.9)}$$

(*ii*) Under the situations when temperature does not depend on time, the conduction then takes place in the steady state $\left(i.e., \frac{\partial t}{\partial \tau} = 0\right)$ and the eqn. (2.8) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} + \frac{q_g}{k} = 0$$

or,
$$\nabla^2 t + \frac{q_g}{k} = 0 \qquad \text{(Poisson's equation)} \qquad \text{...(2.10)}$$

In the absence of internal heat generation, Eqn. (2.10) reduces to

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} = 0$$

or,
$$\nabla^2 t = 0 \qquad \text{(Laplace equation)} \qquad \text{...(2.11)}$$

(*iii*) *Steady state and one-dimensional heat transfer*:

$$\frac{\partial^2 t}{\partial x^2} + \frac{q_g}{k} = 0 \qquad \text{...(2.12)}$$

(*iv*) *Steady state, one-dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} = 0 \qquad \text{...(2.13)}$$

(*v*) *Steady state, two dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = 0 \qquad \text{...(2.14)}$$

(*vi*) *Unsteady state, one dimensional, without internal heat generation*

$$\frac{\partial^2 t}{\partial x^2} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau} \qquad \text{...(2.15)}$$

2.3. GENERAL HEAT CONDUCTION EQUATION IN CYLINDRICAL COORDINATES

While dealing with problems of conduction of heat through systems having cylindrical geometries (*e.g.*, rods and pipes) it is convenient to use cylindrical coordinates.

Consider an elemental volume having the coordinates (r, ϕ, z), for three-dimensional heat conduction analysis, as shown in Fig. 2.2.

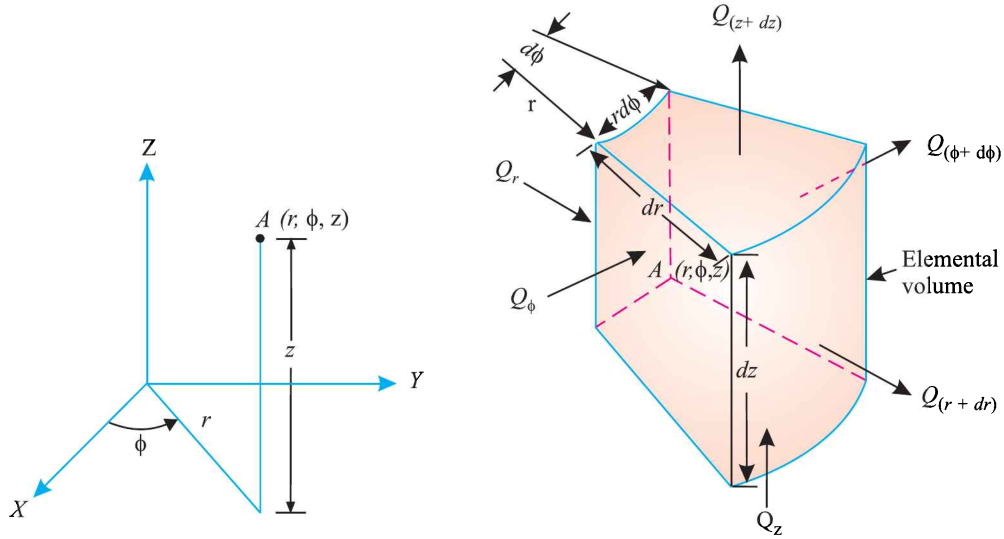


Fig. 2.2. Elemental volume for three-dimensional heat conduction analysis - Cylindrical coordinates.

The volume of the element = $rd\phi.dr.dz$

Let, q_g = Heat generation (uniform) per unit volume per unit time.

Further, let us assume that k (thermal conductivity), ρ (density), c (specific heat) do not alter with position.

A. *Net heat accumulated in the element due to conduction of heat from all the coordinate directions considered* :

Heat flow in radial direction (x–ϕ) plane :

Heat influx, $$Q'_r = -k\ (rd\phi.dz)\ \frac{\partial t}{\partial r}\ .\ d\tau \qquad ...(i)$$

Heat efflux, $$Q'_{(r+dr)} = Q'_r + \frac{\partial}{\partial r}\ (Q_r)\ dr \qquad ...(ii)$$

∴ Heat accumulation in the element due to heat flow in *radial direction*,

$$dQ'_r = Q'_r - Q'_{(r+dr)} \qquad \text{[subtracting } (ii) \text{ from } (i)]$$

$$= -\frac{\partial}{\partial r}\ (Q'_r)\ dr$$

$$= -\frac{\partial}{\partial r}\left[-k\ (rd\phi.dz)\ \frac{\partial t}{\partial r}\ .\ d\tau\right] dr$$

$$= k\ (dr.d\phi.dz)\ \frac{\partial}{\partial r}\left(r \cdot \frac{\partial t}{\partial r}\right) d\tau$$

$$= k\ (dr.d\phi.dz)\left(r\ \frac{\partial^2 t}{\partial r^2} + \frac{\partial t}{\partial r}\right) d\tau$$

$$= k\,(dr.rd\phi.dz)\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\frac{\partial t}{\partial r}\right] d\tau \qquad ...(2.16)$$

Heat flow in *tangential direction* (*r–z*) *plane* :

Heat influx, $$Q'_\phi = -\,k\,(dr.dz)\,\frac{\partial t}{r.\partial\phi}\,d\tau \qquad ...(iii)$$

Heat efflux, $$Q'_{(\phi + d\phi)} = Q'_\phi + \frac{\partial}{r.\partial\phi}(Q'_\phi)\,rd\phi \qquad ...(iv)$$

Heat accumulated in the element due to heat flow in *tangential direction*,

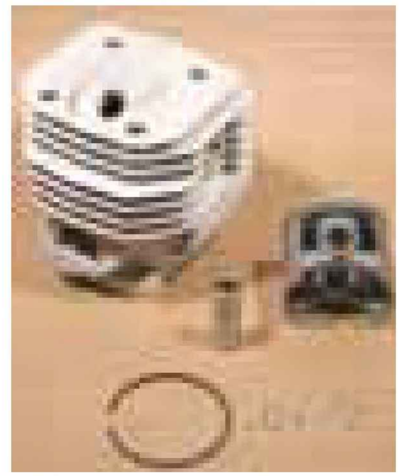
$$dQ'_\phi = Q'_\phi - Q'_{(\phi + d\phi)} \qquad \text{[subtracting } (iv) \text{ from } (iii)]$$

$$= -\frac{\partial}{r.\partial\phi}(Q_\phi)\,r.d\phi$$

$$= -\frac{\partial}{r.\partial\phi}\left[-\,k\,(dr.dz)\,\frac{\partial t}{r.\partial\phi}\,.\,d\tau\right] r.d\phi$$

$$= k\,(dr.d\phi.dz)\,\frac{\partial}{\partial\phi}\left(\frac{1}{r}\cdot\frac{\partial t}{\partial\phi}\right) d\tau$$

$$= k\,(dr.rd\phi.dz)\,\frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial\phi^2}\,.\,d\tau$$



Piston assembly. The fins around the cylinder are meant to spread the heat and speed-up cooling.

Heat flow in *axial direction* (*r-φ plane*) :

Heat influx, $$Q'_z = -\,k\,(r.d\phi.dr)\,\frac{\partial t}{\partial z}\,d\tau \qquad ...(v)$$

Heat efflux, $$Q'_{(z + dz)} = Q'_z + \frac{\partial}{\partial z}(Q'_z)\,dz \qquad ...(vi)$$

Heat accumulated in the element due to heat flow in *axial direction*,

$$dQ'_z = Q'_z - Q'_{(z + dz)} \qquad \text{[subtracting } (vi) \text{ from } (v)]$$

$$= -\frac{\partial}{\partial z}\left[-\,k\,(r.d\phi.dr)\,\frac{\partial t}{\partial z}\,.\,d\tau\right] dz$$

$$= k\,(dr.rd\phi.dz)\,\frac{\partial^2 t}{\partial z^2}\,.\,d\tau \qquad ...(2.18)$$

Net heat accumulated in the element

$$= k.dr.rd\phi.dz\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}\right] d\tau \qquad ...(2.19)$$

B. *Heat generated within the element* (Q'_g) :

The total heat generated within the element is given by

$$Q'_g = q_g\,(dr.rd\phi.dz).d\tau \qquad ...(2.20)$$

C. *Energy stored in the element* :

The increase in thermal energy in the element is equal to

$$= \rho\,(dr.rd\phi.dz).c.\frac{\partial t}{\partial\tau}\,.\,d\tau \qquad ...(2.21)$$

Now, $(A) + (B) = (C)$ ... Energy balance/equation

$$\therefore \quad k.dr.rd\phi.dz\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r}\cdot\frac{\partial t}{\partial r} + \frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial\phi^2} + \frac{\partial^2 t}{\partial z^2}\right] d\tau + q_g\,(dr.rd\phi.dz).d\tau$$

$$= \rho\,(dr.rd\phi.dz).c.\frac{\partial t}{\partial\tau}\,.\,d\tau$$

Dividing both sides by $dr.rd\phi.dz.d\tau$, we have

$$k\left[\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial t}{\partial r}+\frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial \phi^2}+\frac{\partial^2 t}{\partial z^2}\right]+q_g=\rho.c.\frac{\partial t}{\partial \tau}$$

or,
$$\left[\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial t}{\partial r}+\frac{1}{r^2}\cdot\frac{\partial^2 t}{\partial \phi^2}+\frac{\partial^2 t}{\partial z^2}\right]+\frac{q_g}{k}=\frac{\rho c}{k}\cdot\frac{\partial t}{\partial \tau}=\frac{1}{\alpha}\cdot\frac{\partial t}{\partial \tau} \quad ...(2.22)$$

Equation (2.22) is the **general heat conduction equation in cylindrical coordinates**.

In case there are no *heat sources present* and the heat flow is *steady* and *one-dimensional*, then eqn. (2.22) reduces to

$$\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\cdot\frac{\partial t}{\partial r}=0 \quad ...(2.23)$$

or,
$$\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\cdot\frac{dt}{dr}=0$$

or,
$$\frac{1}{r}\cdot\frac{d}{dr}\left(r\cdot\frac{dt}{dr}\right)=0$$

Since $\frac{1}{r}\neq 0$, therefore,

$$\frac{d}{dr}\left(r\cdot\frac{dt}{dr}\right) \text{ or } r\cdot\frac{dt}{dr}=\text{constant} \quad ...(2.24)$$

Equation (2.22) can also be derived by transformation of coordinates, as follows :

$$x = r\cos\phi,\ y = r\sin\phi \text{ and } z = z$$

Now, by chain rule :

$$\frac{\partial t}{\partial r}=\frac{\partial t}{\partial x}\cdot\frac{\partial x}{\partial r}+\frac{\partial t}{\partial y}\cdot\frac{\partial y}{\partial r}=\frac{\partial t}{\partial x}\cos\phi+\frac{\partial t}{\partial y}\sin\phi$$

or,
$$\cos\phi\frac{\partial t}{\partial r}=\cos^2\phi\cdot\frac{\partial t}{\partial x}+\sin\phi\,.\cos\phi\,.\frac{\partial t}{\partial y} \quad ...(i)$$

(Multiplying both sides by $\cos\phi$)

Also,
$$\frac{\partial t}{\partial \phi}=\frac{\partial t}{\partial x}\cdot\frac{\partial x}{\partial \phi}+\frac{\partial t}{\partial y}\cdot\frac{\partial y}{\partial \phi}=\frac{\partial t}{\partial x}(-r\sin\phi)+\frac{\partial t}{\partial y}(r\cos\phi)$$

or,
$$\frac{\sin\phi}{r}\cdot\frac{\partial t}{\partial \phi}=-\sin^2\phi\frac{\partial t}{\partial x}+\sin\phi\,.\cos\phi\,.\frac{\partial t}{\partial y} \quad ...(ii)$$

(Multiplying both sides by $\frac{\sin\phi}{r}$)

From Eqns. (*i*) and (*ii*), we have

$$\frac{\sin\phi}{r}\cdot\frac{\partial t}{\partial \phi}=-\sin^2\phi\frac{\partial t}{\partial x}+\left[\cos\phi\,.\frac{\partial t}{\partial r}-\cos^2\phi\frac{\partial t}{\partial x}\right]$$

$$=-\frac{\partial t}{\partial x}+\cos\phi\frac{\partial t}{\partial r}$$

$\therefore$
$$\frac{\partial t}{\partial x}=\cos\phi\frac{\partial t}{\partial r}-\frac{\sin\phi}{r}\cdot\frac{\partial t}{\partial \phi} \quad ...(iii)$$

Differentiating both sides with respect to x, we have

$$\frac{\partial}{\partial x}\left(\frac{\partial t}{\partial x}\right)=\frac{\partial}{\partial x}\left[\cos\phi\cdot\frac{\partial t}{\partial r}-\frac{\sin\phi}{r}\cdot\frac{\partial t}{\partial \phi}\right]$$

or, $$\frac{\partial^2 t}{\partial x^2} = \cos\phi . \frac{\partial}{\partial r}\left(\frac{\partial t}{\partial x}\right) - \frac{\sin\phi}{r} \cdot \frac{\partial}{\partial \phi}\left(\frac{\partial t}{\partial x}\right)$$

$$= \cos\phi . \frac{\partial}{\partial r}\left(\cos\phi . \frac{\partial t}{\partial r} - \frac{\sin\phi}{r} \cdot \frac{\partial t}{\partial \phi}\right) - \frac{\sin\phi}{r} \cdot \frac{\partial}{\partial \phi}\left(\cos\phi \cdot \frac{\partial t}{\partial r} - \frac{\sin\phi}{r} \cdot \frac{\partial t}{\partial \phi}\right)$$

[Substituting the value of $\frac{\partial t}{\partial x}$ from (*iii*)]

$$= \cos^2\phi . \frac{\partial^2 t}{\partial r^2} - \frac{\cos\phi . \sin\phi}{r^2} \cdot \frac{\partial t}{\partial \phi} + \frac{\sin^2\phi}{r} \cdot \frac{\partial t}{\partial r} + \frac{\sin^2\phi}{r^2} \cdot \frac{\partial^2 t}{d\phi^2} + \frac{\sin\phi . \cos\phi}{r^2} \cdot \frac{\partial t}{\partial \phi} \quad ...(iv)$$

Similarly, $$\frac{\partial^2 t}{\partial y^2} = \sin^2\phi . \frac{\partial^2 t}{\partial r^2} + \frac{\cos^2\phi}{r} \cdot \frac{\partial t}{\partial r} - \frac{\cos\phi . \sin\phi}{r^2} \cdot \frac{\partial t}{\partial \phi} + \frac{\cos^2\phi}{r^2} \cdot \frac{\partial^2 t}{\partial \phi^2} - \frac{\cos\phi . \sin\phi}{r^2} \cdot \frac{\partial t}{\partial \phi} \quad ...(v)$$

By adding (*iii*) and (*iv*), we get

$$\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} = \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial t}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 t}{\partial \phi^2}$$

Substituting it in eqn (2.8), we get,

$$\left[\frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \cdot \frac{\partial t}{\partial r} + \frac{1}{r^2} \cdot \frac{\partial^2 t}{\partial \phi^2} + \frac{\partial^2 t}{\partial z^2}\right] + \frac{q_g}{k} = \frac{1}{\alpha} \cdot \frac{\partial t}{\partial \tau}$$

which is the same as eqn. (2.22)

2.4. GENERAL HEAT CONDUCTION EQUATION IN SPHERICAL COORDINATES

Consider an elemental volume having the coordinates (r, ϕ, θ), for three dimensional heat conduction analysis, as shown in Fig. 2.3.

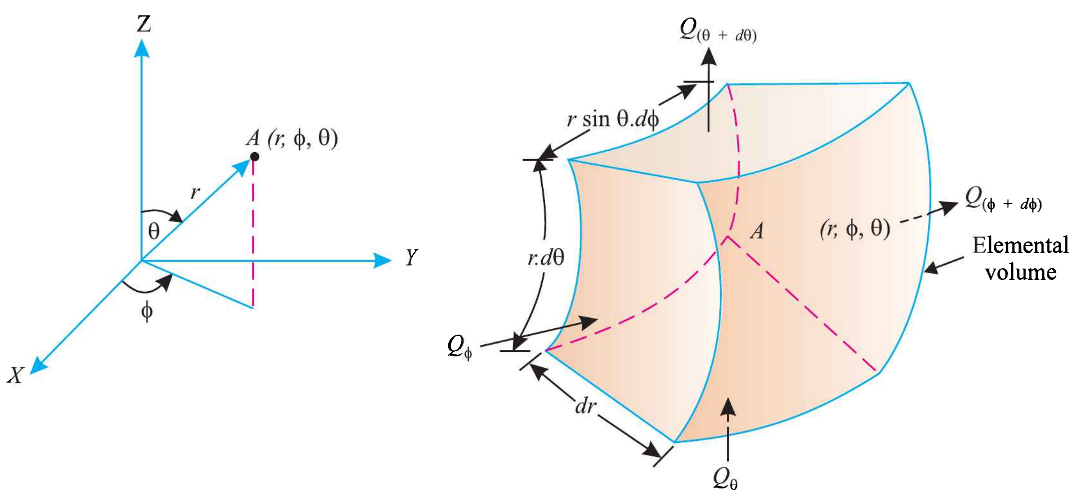


Fig. 2.3. Elemental volume for three-dimensional heat conduction analysis - Spherical coordinates.

The volume of the element = $dr.rd\theta.r\sin\theta\, d\phi$

Let, q_g = Heat generation (uniform) per unit volume per unit time.

Further let us assume that k (thermal conductivity), ρ (density), c (specific heat) do not alter with position.