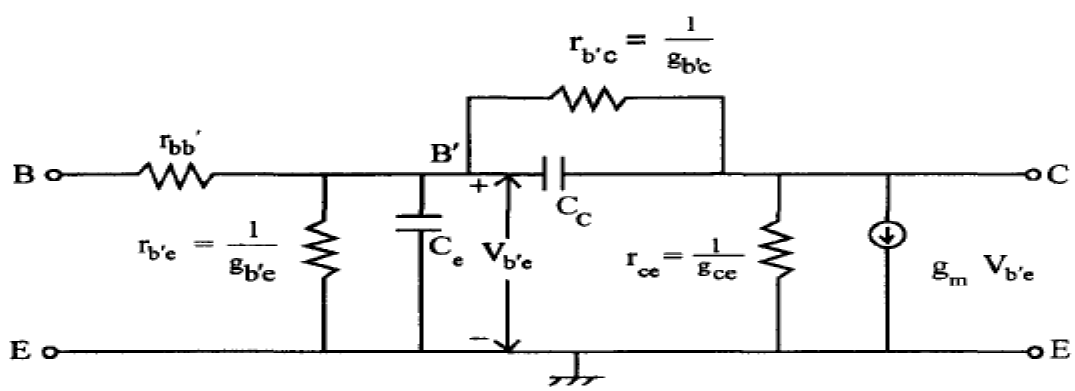
respond, then the Transistor amplifier will not respond instantaneously. Thus, the junction capacitances of the transistor, puts a limit to the highest frequency signal which the transistor can handle. Thus depending upon doping area of the junction etc, we have transistors which can respond in AF range and also RF range.

To study and analyze the behavior of the transistor to high frequency signals an equivalent model based upon transmission line equations will be accurate. But this model will be very complicated to analyze. So some approximations are made and the equivalent circuit is simplified. If the circuit is simplified to a great extent, it will be easy to analyze, but the results will not be accurate. If no approximations are made, the results will be accurate, but it will be difficult to analyze. The desirable features of an equivalent circuit for analysis are simplicity and accuracy. Such a circuit which is fairly simple and reasonably accurate is the Hybrid-pi or Hybrid-π model, so called because the circuit is in the form of π.

**Hybrid - π Common Emitter Transconductance Model**

For Transconductance amplifier circuits Common Emitter configuration is preferred. Why? Because for Common Collector (hrc< 1). For Common Collector Configuration, voltage gain Av < 1. So even by cascading you can't increase voltage gain. For Common Base, current gain is hib< 1. Overall voltage gain is less than 1. For Common Emitter, hre>>1. Therefore Voltage gain can be increased by cascading Common Emitter stage. So Common Emitter configuration is widely used. The Hybrid-x or Giacoletto Model for the Common Emitter amplifier circuit (single stage) is as shown below.




Analysis of this circuit gives satisfactory results at all frequencies not only at high frequencies but also at low frequencies. All the parameters are assumed to be independent of frequency.

Where B’ = internal node in base
$r_{bb'}$ = Base spreading resistance
$r_{b'e}$ = Internal base node to emitter resistance
$r_{ce}$ = collector to emitter resistance
$C_e$ = Diffusion capacitance of emitter base junction
$r_{b'c}$ = Feedback resistance from internal base node to collector node
$g_m$ = Transconductance
$C_C$= transition or space charge capacitance of base collector junction

**Circuit Components**

B' is the internal node of base of the Transconductance amplifier. It is not physically accessible. The base spreading resistance rbb is represented as a lumped parameter between base B and internal node B'. $g_{mVb'e}$ is a current generator. Vb'e is the input voltage across the emitter junction. If Vb'e increases, more carriers are injected into the base of the transistor. So the increase in the number of carriers is proportional to Vb'e. This results in small signal current since we are taking into account changes in Vb'e. This effect is represented by the current generator $g_m$Vb'e. This represents the current that results because of the changes in Vb'e' when C is shorted to E.

When the number of carriers injected into the base increase, base recombination also increases. So this effect is taken care of by gb'e. As recombination increases, base current increases. Minority carrier storage in the base is represented by $C_e$ the diffusion capacitance.

According to Early Effect, the change in voltage between Collector and Emitter changes the base width. Base width will be modulated according to the voltage variations between Collector and Emitter. When base width changes, the minority carrier concentration in base changes. Hence the current which is proportional to carrier concentration also changes. $I_E$ changes and $I_C$ changes. This feedback effect [IE on input side, $I_C$ on output side] is taken into account by connecting gb'e between B', and C. The conductance between Collector and Base is $g_{ce}$.$C_c$ represents the collector junction barrier capacitance.

**Hybrid - *n* Parameter Values**

Typical values of the hybrid-n parameter at $I_C$ = 1.3 rnA are as follows:

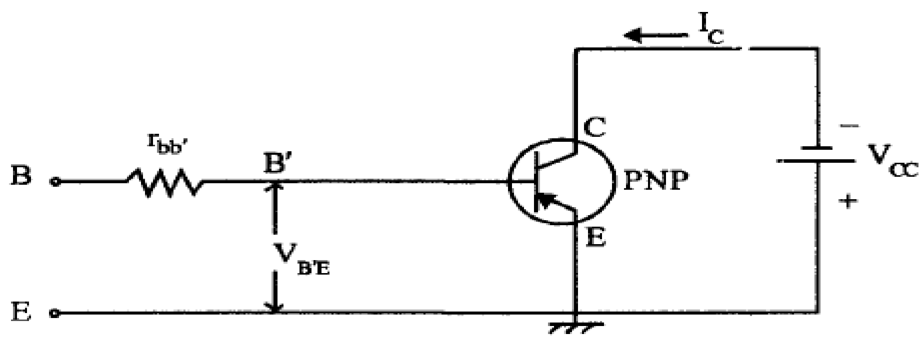
$$g_m = 50\ mA/v$$

$r_{bb'} = 100\ \Omega$
$r_{b'e} = 1\ k\Omega$
$r_{ee} = 80\ k\Omega$
$C_c = 3\ pf$
$C_e = 100\ pf$
$r_{b'c} = 4\ M\Omega$

These values depend upon:
1. Temperature 2. Value of $I_C$

**Determination of Hybrid-x Conductances**

**1. Trans conductance or Mutual Conductance ($g_m$)**




The above figure shows PNP transistor amplifier in Common Emitter configuration for AC purpose, Collector is shorted to Emitter.

$$I_C = I_{C0} - \alpha_0 \cdot I_E$$

$I_{CO}$ opposes $I_E$. $I_E$ is negative. Hence $I_C = I_{CO} - \alpha_0 IE$ $\alpha_0$ is the normal value of $\alpha$ at room temperature.

In the hybrid - $\pi$ equivalent circuit, the short circuit current = $g_m V_{b'e}$

Here only transistor is considered, and other circuit elements like resistors, capacitors etc are not considered.

$$g_m = \left. \frac{\partial I_C}{\partial V_{b'e}} \right|_{V_{CE} = K}$$

Differentiate (l) with respect to Vb'e partially. $I_{CO}$ is constant

$$g_m = 0 - \alpha_0 \frac{\partial I_E}{\partial V_{b'e}}$$

For a PNP transistor, Vb'e = $-V_E$ Since, for PNP transistor, base is n-type. So negative voltage is given

$$g_m = \alpha_0 \frac{\partial I_E}{\partial V_E}$$

If the emitter diode resistance is $r_e$ then

$$r_e = \frac{\partial V_E}{\partial I_E}$$

$$g_m = \frac{\alpha_0}{r_e}$$

$$r = \frac{\eta . V_T}{I} \quad \eta = 1, \qquad I = I_E \qquad r = \frac{V_T}{I_E}$$

$$g_m = \frac{\alpha_0 . I_E}{V_T} \qquad \alpha_0 \simeq 1, \qquad I_E \simeq I_C$$

$$I_E = I_{C0} - I_C$$

$$g_m = \frac{I_{C0} - I_C}{V_T}$$

Neglect $I_{C0}$

$$g_m = \frac{|I_C|}{V_T}$$

$g_m$ is directly proportional to $I_C$ is also inversely proportional to T. For PNP transistor, $I_C$ is negative

$$\boxed{g_m = \frac{-I_C}{V_T}}$$

At room temperature i.e. $T=300^0K$

$$g_m = \frac{|I_C|}{26}, \; I_C \text{ is in mA.}$$

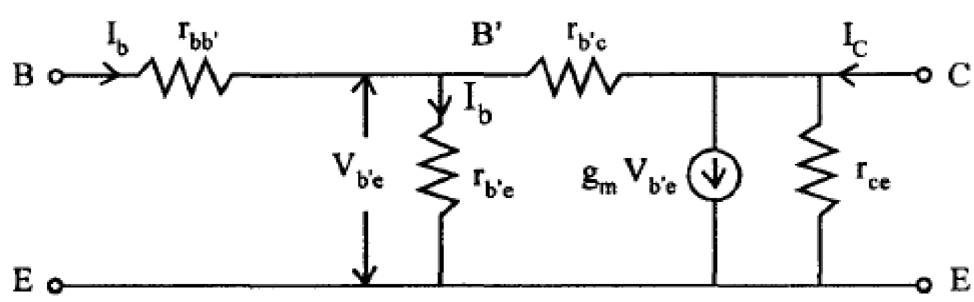
If $\quad I_C = 1.3$ mA, $g_m = 0.05$ A/V

If $\quad I_C = 10$ mA, $g_m = 400$ mA/V

**Input Conductance (gb'e):**

At low frequencies, capacitive reactance will be very large and can be considered as Open circuit. So in the hybrid-π equivalent circuit which is valid at low frequencies, all the capacitances can be neglected.

The equivalent circuit is as shown in Fig.



The value of rb'c» rb'e (Since Collector Base junction is Reverse Biased)So $I_b$ flows into rb'e only. [This is lb' (IE - $I_b$)will go to collector junction]

$$V_{b'e} \simeq I_b \cdot r_{b'e}$$

The short circuit collector current,

$$I_C = g_m \cdot V_{b'e}; \qquad V_{b'e} = I_b \cdot r_{b'e}$$

$$I_C = g_m \cdot I_b \cdot r_{b'e}$$

$$h_{fe} = \left.\frac{I_C}{I_B}\right|_{V_{CE}} = g_m \cdot r_{b'e}$$

$$\boxed{r_{b'e} = \frac{h_{fe}}{g_m}}$$

$$g_m = \frac{|I_C|}{V_T}$$

$$r_{b'e} = \frac{h_{fe} \cdot V_T}{|I_C|}$$

$$g_{b'e} = \boxed{\frac{|I_C|}{h_{fe} V_T}} \text{ or } \boxed{\frac{g_m}{h_{fe}}}$$

**Feedback Conductance ($g_{b'c}$)**

hre = reverse voltage gain, with input open or $I_b = 0$
hre =Vb'e/Vce = Input voltage/Output voltage

$$h_{re} = \frac{r_{b'e}}{r_{b'e} + r_{b'c}}$$

[With input open, i.e., $I_b - 0$, $V_{ce}$ is output. So it will get divided between $r_{b'e}$ and $r_{b'c}$ only]

or $$h_{re}(r_{b'e} + r_{b'c}) = r_{b'e}$$

$$r_{b'e}[1 - h_{re}] = h_{re} r_{b'c}$$

But $$h_{re} \ll 1$$

$\therefore$ $$r_{b'e} = h_{re} \; r_{b'c}; \quad r_{b'c} = \frac{r_{b'e}}{h_{re}}$$

or $$\boxed{g_{b'c} = h_{re} \; g_{b'e}} \quad \frac{1}{r_{b'c}} = g_{b'c} = \frac{h_{re}}{r_{b'e}}$$

$$h_{re} = 10^{-4}$$

$\therefore$ $$r_{b'c} \gg r_{b'e}$$

**Base Spreading Resistance (r bb')**

The input resistance with the output shorted is hie. If output is shorted, i.e., Collector and Emitter arejoined; $r_{b'e}$ is in parallel with $r_{b'c}$.

$$h_{ie} = r_{bb'} + r_{b'e}$$

$$\boxed{r_{bb'} = h_{ie} - r_{b'e}}$$

$$h_{ie} = r_{bb'} + r_{b'e}$$

$$r_{b'e} = \frac{h_{fe}.V_T}{|I_C|}$$

$$h_{ie} = r_{bb'} + \frac{h_{fe}.V_T}{|I_C|}$$

**Output Conductance ($g_{ce}$)**

This is the conductance with input open circuited. In h-parameters it is represented as hoe. For $I_b = 0$, we have,

$$I_C = \frac{V_{ce}}{r_{ce}} + \frac{V_{ce}}{r_{b'c} + r_{b'e}} + g_m \ V_{b'e}$$

$$h_{re} = \frac{V_{b'e}}{V_{ce}} \qquad \therefore \qquad V_{b'e} = h_{re}.\ V_{ce}$$

$$I_C = \frac{V_{ce}}{r_{ce}} + \frac{V_{ce}}{r_{b'c} + r_{b'e}} + g_m.\ h_{re}.V_{ce}$$

$$h_{oe} = \frac{1}{r_{ce}} + \frac{1}{r_{b'c}} + g_m \cdot h_{re}$$

$$= g_{ce} + g_{b'c} + g_m \, h_{re}$$

$$g_{b'e} = \frac{g_m}{h_{fe}}$$

$$g_m = g_{b'e} \cdot h_{fe}$$

$$h_{re} = \frac{r_{b'e}}{r_{b'e} + r_{b'c}} \approx \frac{r_{b'e}}{r_{b'c}} = \frac{g_{b'c}}{g_{b'e}}$$

$$h_{oe} = g_{ce} + g_{b'c} + g_{b'e} \, h_{fe} \cdot \frac{g_{b'c}}{g_{b'e}}$$

$$g_{ce} = h_{oe} - (1 + h_{fe}) \cdot g_{b'c}$$

$$h_{fe} >> 1, \; 1 + h_{fe} \approx h_{fe}$$

$$\boxed{\mathbf{g_{ce} = h_{oe} - h_{fe} \cdot g_{b'c}}}$$

$$g_{b'c} = h_{re} \cdot g_{b'e}$$

$$g_{ce} = h_{oe} - h_{fe} \cdot h_{re} \cdot g_{b'e}$$

**Hybrid - π Capacitances**

In the hybrid - **π** equivalent circuit, there are two capacitances, the capacitance between the Collector Base junction is the $C_c$ or $C_{b'e}$'. This is measured with input open i.e., $I_E = 0$, and is specified by the manufacturers as $C_{Ob}$. 0 indicates that input is open. Collector junction is reverse biased.

$$C_C \ \alpha \ \frac{1}{(V_{CE})^n}$$

$$n = \frac{1}{2} \quad \text{for abrupt junction}$$

$$= 1/3 \quad \text{for graded junction.}$$

$C_e$ = Emitter diffusion capacitance $C_{De}$ + Emitter junction capacitance $C_{Te}$

$C_T$ = Transition capacitance.

$C_D$ = Diffusion capacitance.

$C_{De} >> C_{Te}$

$C_e \simeq C_{De}$

$C_{De} \ \alpha \ I_E$ and is independent of Temperature T.

**Validity of hybrid-π model**

The high frequency hybrid Pi or Giacoletto model of BJT is valid for frequencies less than the unit gain frequency.

**High frequency model parameters of a BJT in terms of low frequency hybrid parameters**

The main advantage of high frequency model is that this model can be simplified to obtain low frequency model of BJT. This is done by eliminating capacitance's from the high frequency model so that the BJT responds without any significant delay (instantaneously) to the input signal. In practice there will be some delay between the input signal and output signal of BJT which will be very small compared to signal period (1/frequency of input signal) and hence can be neglected. The high frequency model of BJT is simplified at low frequencies and redrawn as shown in the figure below along with the small signal low frequency hybrid model of BJT.

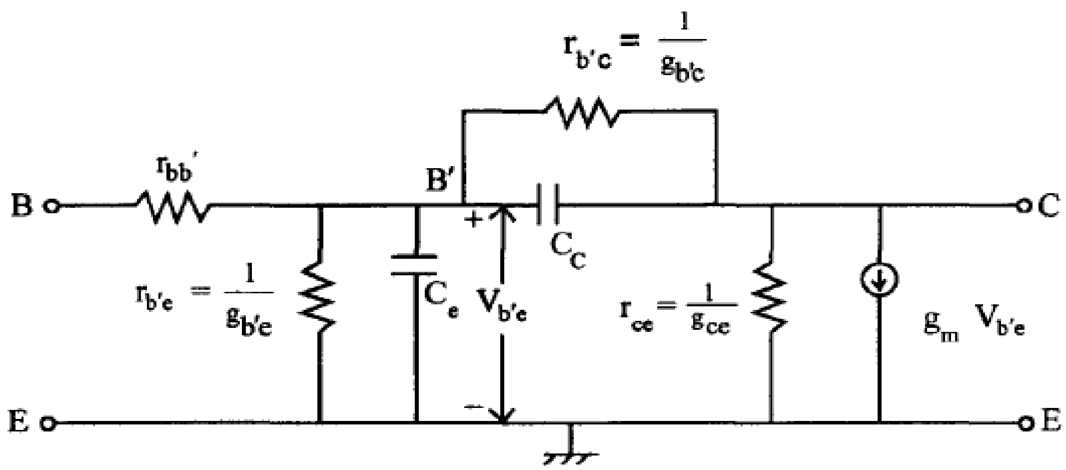



Fig. high frequency model of BJT at low frequencies

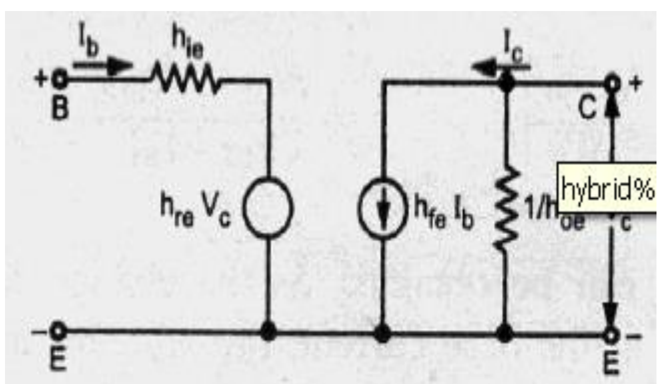



Fig hybrid model of BJT at low frequencies

The High frequency model parameters of a BJT in terms of low frequency hybrid parameters are given below:

Transconductance $g_m = I_c/V_t$
Internal Base node to emitter resistance $r_{b'e} = h_{fe}/ g_m = (h_{fe}* V_t)/ I_c$
Internal Base node to collector resistance $r_{b'e} = (h_{re}* r_{b'c}) / (1- h_{re})$ assuming $h_{re} << 1$ it reduces to $r_{b'e} = (h_{re}* r_{b'c})$
Base spreading resistance $r_{bb'} = h_{ie} – r_{b'e} = h_{ie} – (h_{fe}* V_t)/ I_c$
Collector to emitter resistance $r_{ce} = 1 / ( h_{oe} – (1+ h_{fe})/r_{b'c})$

**Collector Emitter Short Circuit Current Gain**

Consider a single stage Common Emitter transistor amplifier circuit. The hybrid-1t equivalent circuit is as shown:

$$I_L = -g_m V_{b'e}$$

$$V_{b'e} = \frac{I_i}{g_{b'e} + j\omega(C_e + C_c)}$$

$A_i$ under short circuit condition is,

$$A_i = \frac{I_L}{I_i} = \frac{-g_m}{g_{b'e} + j\omega(C_e + C_c)}$$

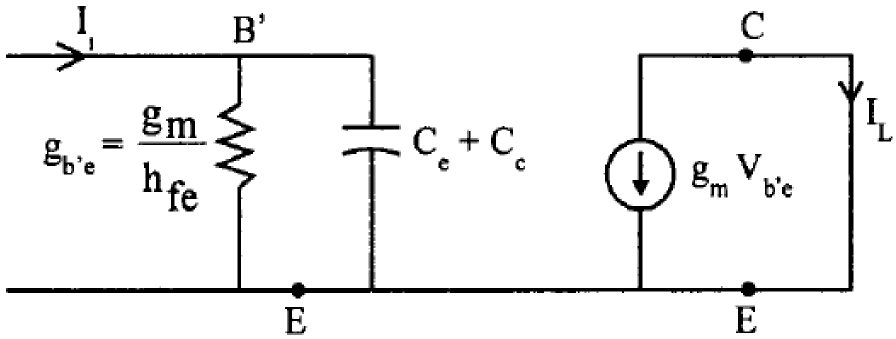
But $$g_{b'e} = \frac{g_m}{h_{fe}}, \quad C_e + C_c \simeq C_e$$

$$C_e = \frac{g_m}{2\pi f_T}$$

$$= \frac{-g_m}{\frac{g_m}{h_{fe}} + \frac{j\,2\pi.g_m.f}{2\pi f_T}}$$

$\therefore$ $$A_i = \frac{-1}{\frac{1}{h_{fe}} + j\left(\frac{f}{f_T}\right)}$$

If the output is shorted i.e. $R_L = 0$, what will be the flow response of this circuit? When$R_L = 0$, $V_o = 0$. Hence $A_v = 0$. So the gain that we consider here is the current gain $I_L/Ic$. The simplified equivalent circuit with output shorted is,




A current source gives sinusoidal current Ic. Output current or load current is IL· $g_{b'c}$ isneglected since $g_{b'c} \ll g_{b'e}$, $g_{ce}$ is in shunt with short circuit R = 0. Therefore $g_{ce}$ disappears. The current is delivered to the output directly through Ce and $g_{b'c}$ is also neglected since this will be very small.

$$I_L = -g_m V_{b'e}$$

$$V_{b'e} = \frac{I_i}{g_{b'e} + j\omega(C_e + C_c)}$$

$A_i$ under short circuit condition is,

$$A_i = \frac{I_L}{I_i} = \frac{-g_m}{g_{b'e} + j\omega(C_e + C_c)}$$

But $\quad g_{b'e} = \frac{g_m}{h_{fe}}, \quad C_e + C_c \simeq C_e$

$$C_e = \frac{g_m}{2\pi f_T}$$

$$= \frac{-g_m}{\frac{g_m}{h_{fe}} + \frac{j\, 2\pi . g_m . f}{2\pi f_T}}$$

$\therefore$

$$A_i = \frac{-1}{\frac{1}{h_{fe}} + j\left(\frac{f}{f_T}\right)}$$

$$= \frac{-h_{fe}}{1 + j\,h_{fe}\left(\frac{f}{f_T}\right)}$$

$$A_i = \frac{-h_{fe}}{1 + j\left(\frac{f}{f_\beta}\right)}$$

$$\frac{f_T}{h_{fe}} = f_\beta$$

$$|A_i| = \frac{h_{fe}}{\sqrt{1 + \left(\frac{f}{f_\beta}\right)^2}}$$

Where $f_\beta = \frac{g_{b'e}}{2\pi(C_e + C_C)}$

$$g_{b'e} = \frac{g_m}{h_{fe}}$$

∴ $f_\beta = \frac{g_m}{h_{fe}\, 2\pi(Ce + C_c)}$

At $f = f_\beta$, $A_i = \frac{1}{\sqrt{2}}$ $= 0.707$ of $h_{fe}$.

**Current Gain with Resistance Load:**

$$f_T = f_\beta \, . \, h_{fe} = \frac{g_m}{2\pi(C_e + C_c)}$$

Considering the load resistance $R_L$
$V_{b'e}$ is the input voltage and is equal to $V_1$
$V_{ce}$ is the output voltage and is equal to $V_2$

$$K_2 = \frac{V_{ce}}{V_{b'e}}$$

This circuit is still complicated for analysis. Because, there are two time constants associated with the input and the other associated with the output. The output time constant will be much smaller than the input time constant. So it can be neglected.

K = Voltage gain. It will be >> 1

$$g_{b'c}\left(\frac{K-1}{K}\right) \simeq g_{b'c}$$

$$g_{b'c} < g_{ce} \quad \because \quad r_{b'c} \simeq 4\ M\Omega, \qquad r_{ce} = 80\ K \ \text{(typical values)}$$

So $g_{b'c}$ can be neglected in the equivalent circuit. In a wide band amplifier RL will not exceed 2KΩ. If $R_L$ is small $f_H$ is large.

$$f_H = \frac{1}{2\pi C_S (R_C \| R_L)}$$

Therefore $g_{ce}$ can be neglected compared with $R_L$. Therefore the output circuit consists of current generator gm $V_{b'e}$ feeding the load $R_L$ so the Circuit simplifies as shown in Fig.