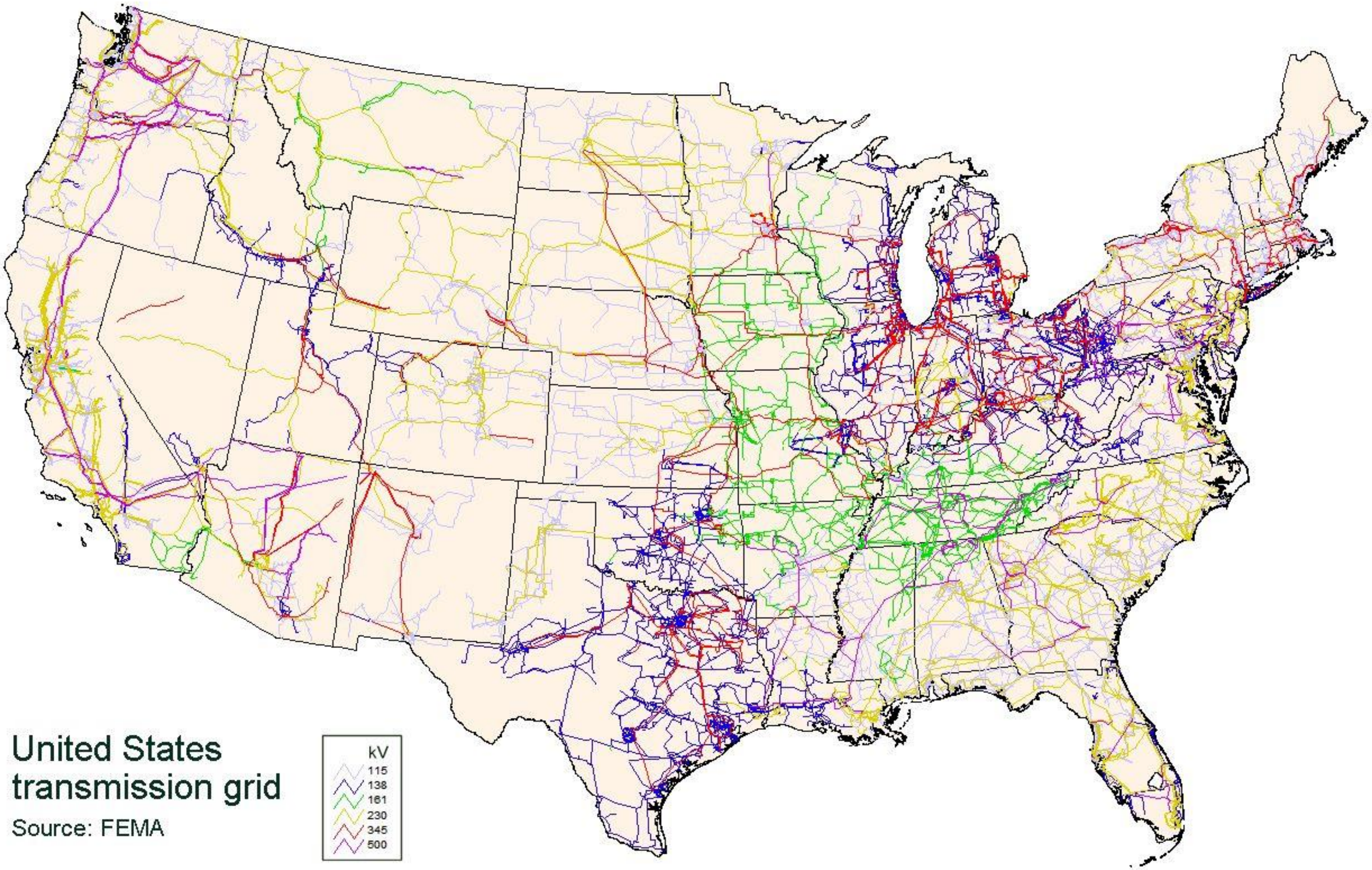
EE 740 – Transmission Lines

Spring 2013

US Power Transmission Grid



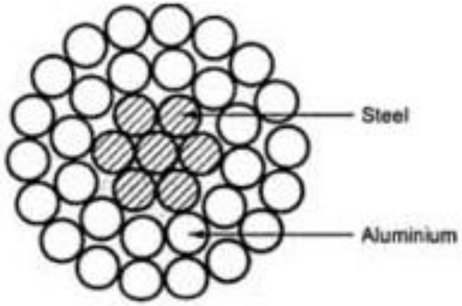
Physical Characteristics – underground cables

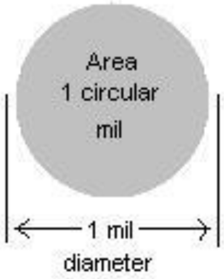
- Cable lines are designed to be placed underground or under water. The conductors are insulated from one another and surrounded by protective sheath.
- Cable lines are more expensive and harder to maintain. They also have capacitance problem – not suitable for long distance.

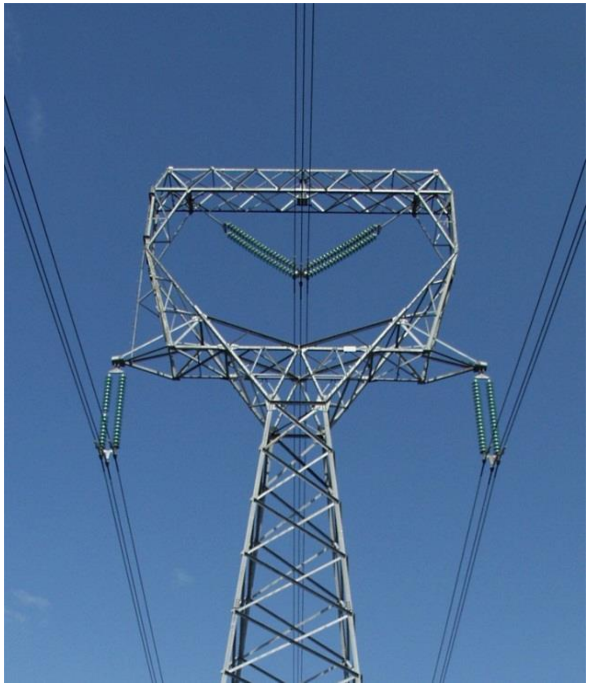


High Voltage Power Lines (overhead)

- Common voltages in north America: 138, 230, 345, 500, 765 kV
- Bundled conductors are used in extra-high voltage lines
- Stranded instead of solid conductors are used.





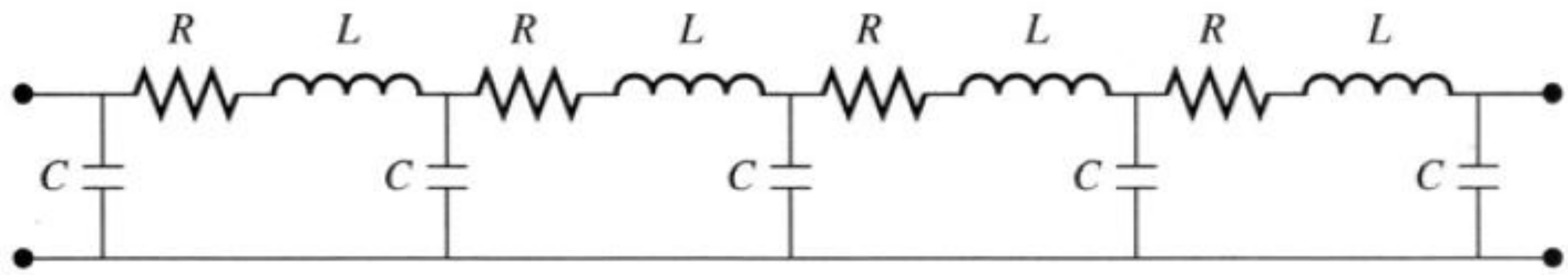


HVDC Transmission

- Because of the large fixed cost necessary to convert ac to dc and then back to ac, dc transmission is only practical in specialized applications
  - long distance overhead power transfer (> 400 miles)
  - long underwater cable power transfer
  - providing an asynchronous means of joining different power systems.

Electrical Characteristics

- Transmission lines are characterized by a **series resistance**, **inductance**, and **shunt capacitance** per unit length.
- These values determine the power-carrying capacity of the transmission line and the voltage drop across it at full load.



- The DC resistance of a conductor is expressed in terms of resistively, length and cross sectional area as follows:

$$R_{DC} = \frac{\rho l}{A}$$

Cable resistance

- The resistively increases linearly with temperature over normal range of temperatures.
- If the resistively at one temperature and material temperature constant are known, the resistively at another temperature can be found by

$$\rho_{T2} = \frac{M + T_2}{M + T_1} \rho_{T1}$$

Material	Resistivity at 20°C [Ω·m]	Temperature constant [°C]
Annealed copper	$1.72 \cdot 10^{-8}$	234.5
Hard-drawn copper	$1.77 \cdot 10^{-8}$	241.5
Aluminum	$2.83 \cdot 10^{-8}$	228.1
Iron	$10.00 \cdot 10^{-8}$	180.0
Silver	$1.59 \cdot 10^{-8}$	243.0

Cable Resistance

- AC resistance of a conductor is always higher than its DC resistance due to the skin effect forcing more current flow near the outer surface of the conductor. The higher the frequency of current, the more noticeable skin effect would be.
- Wire manufacturers usually supply tables of resistance per unit length at common frequencies (50 and 60 Hz). Therefore, the resistance can be determined from such tables.

Aluminum Conductor Steel Reinforced

Electrical Properties

	SIZE & STRANDING		RESISTANCE				60 HZ REACTANCE 1 FOOT EQUIVALENT SPACING		
			DC (Ohms/1000 Ft.)	AC-60-HZ(Ohms/1000 Ft.)			Capacitive (Megohms-1000 Ft.)	Inductive (Ohms/1000 Ft.)	
CODE WORD	AWG or kcmil	Aluminum/ Steel	@20°	@25° C	@50° C	@75° C		@25° C	
								Inductive (Ohms/1000 Ft.)	GMR (Ft.)
WAXWING	266.8	18/1	0.0644	0.0657	0.0723	0.0788	0.576	0.0934	0.0197
PARTRIDGE	266.8	26/7	0.0637	0.0652	0.0714	0.0778	0.565	0.0881	0.0217
MERLIN	336.4	18/1	0.0510	0.0523	0.0574	0.0625	0.560	0.0877	0.0221
LINNET	336.4	26/7	0.0506	0.0517	0.0568	0.0619	0.549	0.0854	0.0244
ORIOLE	336.4	30/7	0.0502	0.0513	0.0563	0.0614	0.544	0.0843	0.0255
CHICKADEE	397.5	18/1	0.0432	0.0443	0.0487	0.0528	0.544	0.0856	0.0240
IBIS	397.5	26/7	0.0428	0.0438	0.0481	0.0525	0.539	0.0835	0.0265
LARK	397.5	30/7	0.0425	0.0434	0.0477	0.0519	0.533	0.0824	0.0277
PELICAN	477.0	18/1.	0.0360	0.0369	0.0405	0.0441	0.528	0.0835	0.0263
FLICKER	477.0	24/7	0.0358	0.0367	0.0403	0.0439	0.524	0.0818	0.0283
HAWK	477.0	26/7	0.0357	0.0366	0.0402	0.0438	0.522	0.0814	0.0290
HEN	477.0	30/7	0.0354	0.0362	0.0389	0.0434	0.517	0.0803	0.0304
OSPREY	556.5	18/1	0.0309	0.0318	0.0348	0.0379	0.518	0.0818	0.0284
PARAKEET	556.5	24/7	0.0307	0.0314	0.0347	0.0377	0.512	0.0801	0.0306

ACSR Conductor Table Data

TABLE A8.1. BARE ALUMINUM CONDUCTORS, STEEL REINFORCED (ACSR) ELECTRICAL PROPERTIES OF MULTILAYER SIZES (*Cont'd*)

Code Word	Size (kcmil)	Stranding Al./St.	Number of Aluminum Layers	Resistance dc 20°C (Ohms/Mile)	Resistance ac-60 Hz 25°C (Ohms/Mile)	Resistance ac-60 Hz 50°C (Ohms/Mile)	Resistance ac-60 Hz 75°C (Ohms/Mile)	GMR (ft)	Phase-to-Neutral, 60 Hz Reactance at One ft Spacing: Inductive Ohms/Mile X_a	Phase-to-Neutral, 60 Hz Reactance at One ft Spacing: Capacitive Megohm-Miles X'_a
Flicker	477	24/7	2	0.1889	0.194	0.213	0.232	0.0283	0.432	0.0992
Hawk	477	26/7	2	0.1883	0.193	0.212	0.231	0.0290	0.430	0.0988
Hen	477	30/7	2	0.1869	0.191	0.210	0.229	0.0304	0.424	0.0980
Osprey	556.5	18/1	2	0.1629	0.168	0.184	0.200	0.0284	0.432	0.0981
Parakeet	556.5	24/7	2	0.1620	0.166	0.183	0.199	0.0306	0.423	0.0969
Dove	556.5	26/7	2	0.1613	0.166	0.182	0.198	0.0313	0.420	0.0965
Eagle	556.5	30/7	2	0.1602	0.164	0.180	0.196	0.0328	0.415	0.0957
Peacock	605	24/7	2	0.1490	0.153	0.168	0.183	0.0319	0.418	0.0957
Squab	605	26/7	2	0.1485	0.153	0.167	0.182	0.0327	0.415	0.0953

Geometric Mean Radius

Inductive and Capacitive Reactance for 1-foot Spacing

Line inductance

- The series inductance of a transmission line consists of two components: internal and external inductances, which are due the magnetic flux inside and outside the conductor respectively.
- The inductance of a transmission line is defined as the number of flux linkages [Wb-turns] produced per ampere of current flowing through the line:

$$L = \frac{\lambda}{I}$$

- The inductance of a single-phase transmission line is given by (see derivation in the book): (*r* : conductor radius - assumed solid, *D*: distance between cables, $\mu = 4\pi \times 10^{-7}$ H/m, $r' = r\, e^{-1/4} = .7788\, r$)

$$l = \frac{\mu}{\pi}\left(\frac{1}{4} + \ln\frac{D}{r}\right) \quad [H/m]$$

$$L = 4x10^{-7}\ln\left(\frac{D}{r'}\right) \text{ H/m}$$

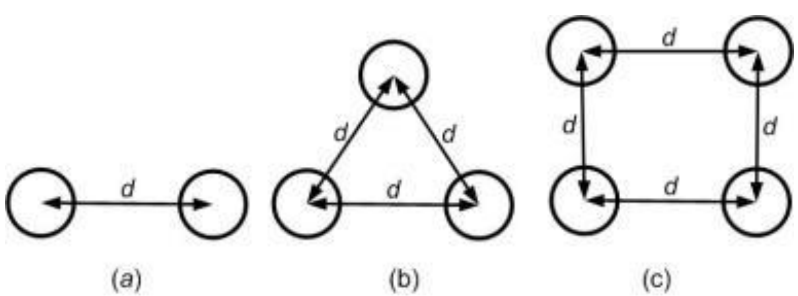
Remarks on line inductance

- **The greater the spacing between the phases of a transmission line, the greater the inductance of the line.**
  - Since the phases of a high-voltage overhead transmission line must be spaced further apart to ensure proper insulation, a high-voltage line will have a higher inductance than a low-voltage line.
  - Since the spacing between lines in buried cables is very small, series inductance of cables is much smaller than the inductance of overhead lines
- **The greater the radius of the conductors in a transmission line, the lower the inductance of the line.** In practical transmission lines, instead of using heavy and inflexible conductors of large radii, two and more conductors are bundled together to approximate a large diameter conductor, and reduce corona loss.

$$GMR_2 = \sqrt{GMR.d}$$

$$GMR_3 = \sqrt[3]{GMR.d^2}$$

$$GMR_4 = 1.09\sqrt[4]{GMR.d^3}$$



Inductance of 3-phase transmission line

$$L = 2x10^{-7}\ln(\frac{GMD}{GMR})\ \text{H/m}$$

where the Geometric Mean Distance (GMD) is defined by

$$GMD = \sqrt[3]{D_1 D_2 D_3}$$

where D1, D2, and D3 are the distances between the 3 conductors. The Geometric Mean Radius (GMR) is supplied by the manufacturer (takes into account the cable strands). For a solid conductor, GMR = 0.7788 r.

For a 60 Hz system, the reactance of the line is

$$X_L = 0.754\text{x}10^{-4}\ln\left(\frac{GMD}{GMR}\right)\ \frac{\text{Ohms}}{\text{m}}$$

$$X_L = 0.1213\ln\left(\frac{GMD}{GMR}\right)\ \frac{\text{Ohms}}{\text{mi}}$$

Shunt capacitance

- Since a voltage *V* is applied to a pair of conductors separated by a dielectric (air), charges *q* of equal magnitude but opposite sign will accumulate on the conductors. Capacitance C between the two conductors is defined by

$$C = \frac{q}{V}$$

- The capacitance of a single-phase transmission line is given by (see derivation in the book): (ε = 8.85 x 10^{-12} F/m)

$$C = \frac{2\pi\epsilon}{\ln(\frac{D}{r})} \text{ F/m}$$

Capacitance of 3-phase transmission line

- The capacitance per phase is computed by

$$C = \frac{2\pi\epsilon}{\ln(\frac{GMD}{r})} \text{ F/m}$$

- The shunt admittance per phase at 60 Hz is given by

$$y = 2\pi f C = \frac{0.35x10^{-8}}{\ln(\frac{GMD}{r})} \text{ S.m}$$

- The shunt capacitive reactance per phase at 60 Hz is given by

$$X_c = 47.7 \text{ x}10^6 \ln\left(\frac{GMD}{r}\right) \text{ Ohm.m}$$

$$X_C = 0.02965 \ln\frac{GMD}{r} \; (M\Omega.mi)$$

Remarks on line capacitance

1. **The greater the spacing between the phases of a transmission line, the lower the capacitance of the line**.
   - Since the phases of a high-voltage overhead transmission line must be spaced further apart to ensure proper insulation, a high-voltage line will have a lower capacitance than a low-voltage line.
   - Since the spacing between lines in buried cables is very small, shunt capacitance of cables is much larger than the capacitance of overhead lines.
2. **The greater the radius of the conductors in a transmission line, the higher the capacitance of the line**. Therefore, bundling increases the capacitance.

Use of Tables

- Inductive reactance (in Ω/mi):

$$X_L = 0.1213 \ln \frac{GMD}{GMR} = 0.1213 \ln \frac{1}{GMR} + 0.1213 \ln GMD$$

  - The first term is defined as X_a: the inductive reactance at 1-foot spacing
  - The second term is defined as X_d : the inductive reactance spacing factor
  - Both of the above components are already calculated in Table A.3 & A.4

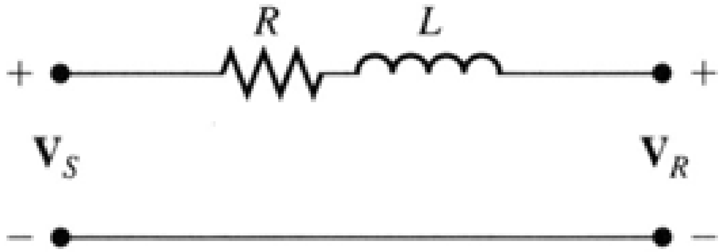
- Capacitive reactance (in MΩ.mi):

$$X_C = 0.02965 \ln \frac{GMD}{r} = 0.02965 \ln \frac{1}{r} + 0.02965 \ln GMD$$

  - The first term is defined as X'_a: the capacitive reactance at 1-foot spacing
  - The second term is defined as X'_d : the capacitive reactance spacing factor
  - Both of the above components are already calculated in Table A.3 & A.5

Short line model

- Overhead transmission lines shorter than 50 miles can be modeled as a series resistance and inductance, since the shunt capacitance can be neglected over short distances.



- The total series resistance and series reactance can be calculated as

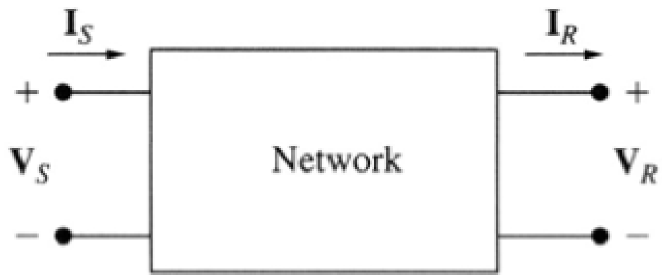
$$R = rd$$

$$X = xd$$

- where *r*, *x* are resistance and reactance per unit length and *d* is the length of the transmission line.

Short line model

- Two-port network model:



$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

$$A = 1$$
$$B = Z$$
$$C = 0$$
$$D = 1$$

$$I_S = I_R$$

$$V_R = V_S - RI - jX_L I$$

- The equation is similar to that of a synchronous generator and transformer (w/o shunt impedance)

Short line

Voltage Regulation: $$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \cdot 100\%$$

1. If lagging (inductive) loads are added at the end of a line, the voltage at the end of the transmission line **decreases significantly** – large positive VR.
2. If unity-PF (resistive) loads are added at the end of a line, the voltage at the end of the transmission line **decreases slightly** – small positive VR.
3. If leading (capacitive) loads are added at the end of a line, the voltage at the end of the transmission line **increases** – negative VR.

Short line – simplified

- If the resistance of the line is ignored, then

$$I\cos\theta = \frac{V_S \sin\delta}{X_L} \qquad \boxed{P = \frac{3V_S V_R \sin\delta}{X_L}}$$

- Therefore, the power flow through a transmission line depends on the angle between the input and output voltages.
- Maximum power flow occurs when δ = 90º.

$$\boxed{P_{\max} = \frac{3V_S V_R}{X_L}}$$

- Notes:
  - The maximum power handling capability of a transmission line is a function of the **square of its voltage.**
  - The maximum power handling capability of a transmission line is inversely proportional to its series reactance (some very long lines include series capacitors to reduce the total series reactance).
  - The angle δ controls the power flow through the line. Hence, it is possible to control power flow by placing a phase-shifting transformer.

Line Characteristics

- To prevents excessive voltage variations in a power system, the ratio of the magnitude of the receiving end voltage to the magnitude of the ending end voltage is generally within

$$0.95 \leq V_S/V_R \leq 1.05$$

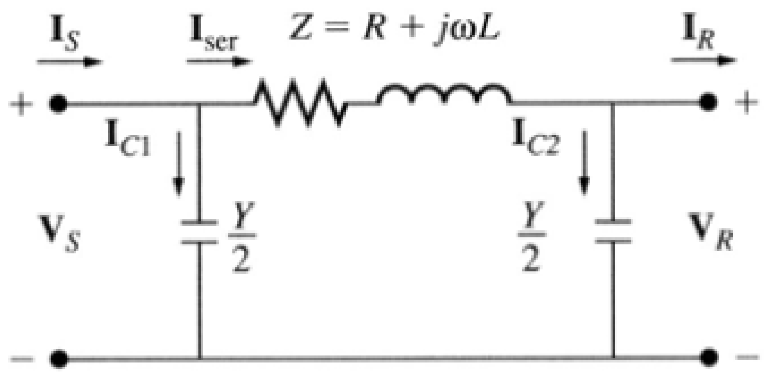
- The angle δ in a transmission line should typically be ≤ 30º to ensure that the power flow in the transmission line is well below the static stability limit.
- Any of these limits can be more or less important in different circumstances.
  - In short lines, where series reactance *X* is relatively small, the **resistive heating** usually limits the power that the line can supply.
  - In longer lines operating at lagging power factors, the **voltage drop** across the line is usually the limiting factor.
  - In longer lines operating at leading power factors, the **maximum angle δ** can be the limiting f actor.

Example

- A line with reactance X and negligible resistance supplies a pure resistive load from a fixed source V_S. Determine the maximum power transfer, and the load voltage V_R at which this occurs. *(Hint: recall the maximum power transfer theorem from your basic circuits course)*

Medium Line (50-150 mi)

- the shunt admittance must be included in calculations. However, the total admittance is usually modeled (π model) as two capacitors of equal values (each corresponding to a half of total admittance) placed at the sending and receiving ends.



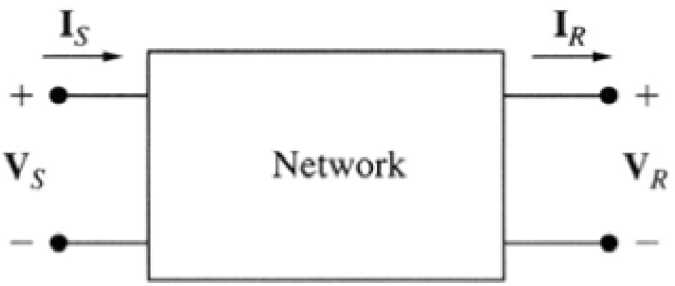
- The total series resistance and series reactance are calculated as before. Similarly, the total shunt admittance is given by

$$Y = yd$$

- where *y* is the shunt admittance per unit length and *d* is the length of the transmission line.

Medium Line

- Two-port network:



$$V_S = AV_R + BI_R$$
$$I_S = CV_R + DI_R$$

$$A = \frac{ZY}{2} + 1$$

$$B = Z$$

$$C = Y\left(\frac{ZY}{4} + 1\right)$$

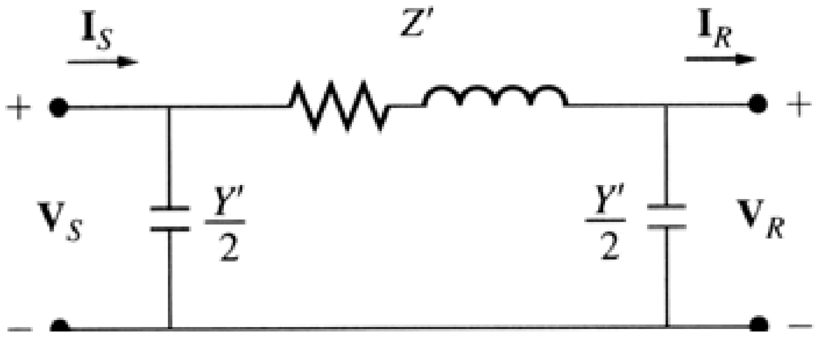
$$D = \frac{ZY}{2} + 1$$

Long Lines (> 150 mi)

- For long lines, both the shunt capacitance and the series impedance must be treated as distributed quantities. The voltages and currents on the line are found by solving differential equations of the line.
- However, it is possible to model a long transmission line as a π model with a *modified* series impedance *Z'* and a *modified* shunt admittance *Y'* and to perform calculations on that model using ABCD constants. These modified values are

$$Z' = Z\frac{\sinh \gamma d}{\gamma d}$$

$$Y' = Y\frac{\tanh(\gamma d/2)}{\gamma d/2}$$



where the propagation constant is defined by $\gamma = \sqrt{yz}$

Surge Impedance Loading

- The surge impedance of a line is defined as

$$Z_C = \sqrt{z / y} \approx \sqrt{L / C}$$

- Surge Impedance Loading (SIL) is the power delivered by a line to a pure resistive load that is equal to its surge impedance:

$$SIL = 3\frac{V_\phi^2}{\sqrt{L / C}} = \frac{V_L^2}{\sqrt{L / C}} MW$$

- Under such loading, the line consumes as much reactive power as it generates and the terminal voltages are equal to each other.
- Power system engineers sometime find it convenient to express the power transmitted by a line in terms of per-unit of SIL.

Reactive Power Generation/Consumption

- Note that a transmission line both absorbs and generates reactive power:
  - Under light load, the line generates more reactive power than it consumes.
  - Under “surge impedance loading”, the line generates and consumes the same amount of reactive power.
  - Under heavy load, the line absorbs more reactive power than it generates.

Input/Output Power and efficiency

- Input powers

$$P_{in} = 3V_S I_S \cos\theta_S = \sqrt{3} V_{LL,S} I_S \cos\theta_S$$

$$Q_{in} = 3V_S I_S \sin\theta_S = \sqrt{3} V_{LL,S} I_S \sin\theta_S$$

$$S_{in} = 3V_S I_S = \sqrt{3} V_{LL,S} I_S$$

- Output powers

$$P_{out} = 3V_R I_R \cos\theta_R = \sqrt{3} V_{LL,R} I_R \cos\theta_R$$

$$Q_{out} = 3V_R I_R \sin\theta_R = \sqrt{3} V_{LL,R} I_R \sin\theta_R$$

$$S_{out} = 3V_R I_R = \sqrt{3} V_{LL,R} I_R$$

- Efficiency

$$\eta = \frac{P_{out}}{P_{in}} \cdot 100\%$$

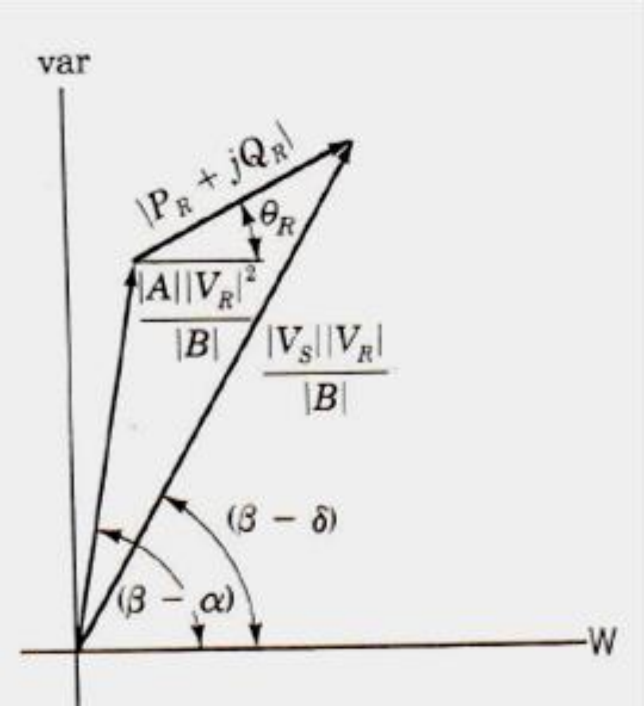
Power Flow Through a Transmission Line

- Let

$$A = |A|\angle\alpha,\ B = |B|\angle\beta, V_S = |V_S|\angle\delta, V_R = |V_R|\angle 0^o$$

- Then the complex power at the receiving end is given by

$$P_R + jQ_R = V_R I_R^* = \frac{|V_S||V_R|}{|B|}\angle(\beta-\delta) - \frac{|A||V_R|^2}{|B|}\angle(\beta-\alpha)$$

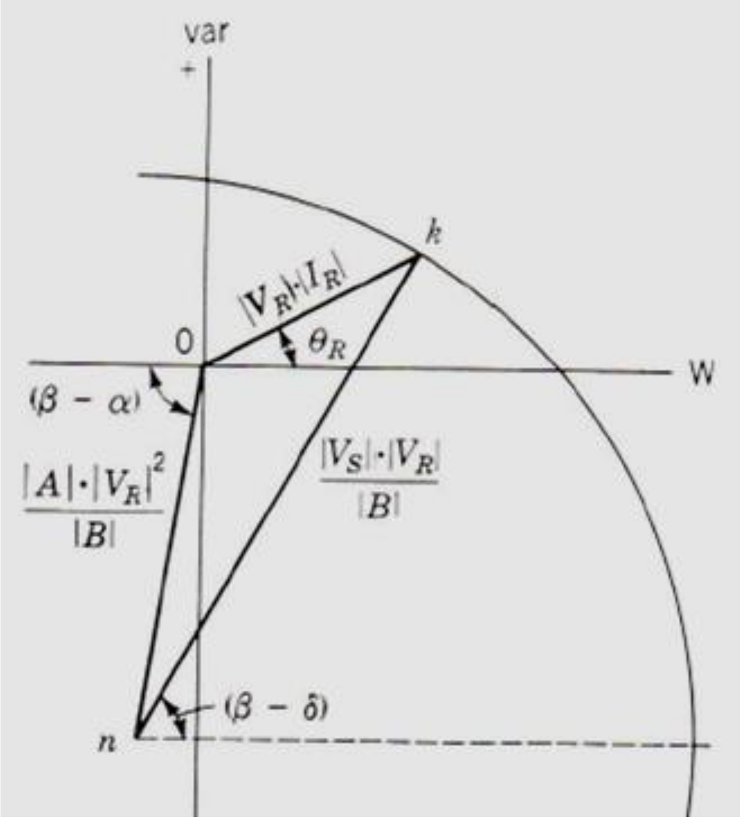


Power Diagram (by shifting origin of coordinate axes)

- For fixed values of both voltage and as the load changes, point k moves on a circle of center n.
  - Any change in P_R will require a change in Q_R.
  - The limit of the power that can be transmitted occurs whenβ = δ.
  - The maximum power transfer is

$$P_{R.\max} = \frac{|V_S||V_R|}{|B|} - \frac{|A||V_R|^2}{|B|}\cos(\beta - \alpha)$$

  - This above requires a large leading current.
  - Normally,
    - $\delta \leq 35^o$
    - $0.95 \leq \frac{|V_S|}{|V_R|} \leq 1.05$



Long line series and shunt compensation

- **Shunt reactors** are used to compensate the line shunt capacitance under light load or no load to regulate voltage.
- **Series capacitors** are often used to compensate the line inductive reactance in order to transfer more power.

Problems (Chap 6)

- 8, 16, 20, 23, 25