## C H A P T E R

8

## Joints

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Joints consist of separate structural elements joined with fasteners or welds. Useful formulas and tables for the analysis and design of joints are provided in this chapter. Most commonly used in engineering structures and machines are riveted, bolted, and welded connections. Figure 8-1 illustrates these three types of joints.

### 8.1 NOTATION

The units for each definition are given in parentheses, using $L$ for length and $F$ for force.

A Cross-sectional area $\left(L^{2}\right)$
$A_{b}$ Nominal bearing area, bolt cross-sectional area $\left(L^{2}\right)$
$A_{e}$ Effective net area ( $L^{2}$ )
$A_{g}$ Cross-sectional gross area $\left(L^{2}\right)$
$A_{n}$ Net sectional area ( $L^{2}$ )

(a)

(b)

(c)

Figure 8-1: Common joints: $(a)$ bolted; $(b)$ riveted; $(c)$ welded.
$A_{w}$ Weld area ( $L^{2}$ )
$d$ Nominal or major diameter of fastener (rivet or bolt) ( $L$ )
$E$ Elastic modulus (Young's modulus) $\left(F / L^{2}\right)$
$f_{r}$ Nominal resultant stress in weld $\left(F / L^{2}\right)$
$F_{i}$ Initial tensile force ( $F$ )
$g$ Transverse spacing (gage) ( $L$ )
$k$ Stiffness constant ( $F / L$ )
$\ell$ Distance ( $L$ )
$M$ Moment ( $F L$ )
$P$ Applied load ( $F$ )
$P_{e}$ External tensile load ( $F$ )
$P_{T}$ Total load-carrying capacity ( $F$ )
$s$ Longitudinal spacing, pitch ( $L$ )
$t$ Plate thickness ( $L$ )
$T$ Torque or twisting moment ( $F L$ )
$w$ Leg size of fillet weld ( $L$ )
$\sigma_{p}$ Allowable bearing stress $\left(F / L^{2}\right)$
$\sigma_{t w}$ Allowable tensile stress $\left(F / L^{2}\right)$
$\sigma_{u}$ Ultimate tensile strength $\left(F / L^{2}\right)$
$\sigma_{w a}$ Allowable strength of particular type of weld $(F / L)$
$\sigma_{y s}$ Yield strength $\left(F / L^{2}\right)$
$\tau$ Shear stress $\left(F / L^{2}\right)$
$\tau_{w}$ Allowable shear stress $\left(F / L^{2}\right)$

### 8.2 RIVETED AND BOLTED JOINTS

When joints are used for connecting members of structures such as building frames, trusses, or cranes, they are generally referred to as connections. Here we will not distinguish between these two terms.

Much of the material in this section deals with steel and conforms to the specifications of the American Institute of Steel Construction (AISC) as provided in Ref. [8.1], using "Allowable Steel Design." However, the steel construction community is replacing the design methodology of Ref. [8.1] with the "Load and Resistance Factor Design" (LRFD) design approach of Ref. [8.2]. Allowable stress design is the traditional procedure which begins with the identification of an allowable stress, which is obtained by dividing either the yield stress $\sigma_{y s}$ or the ultimate stress $\sigma_{u}$ by a factor of safety. Thus, a typical allowable stress would be $0.6 \sigma_{u}$. Then, a structural member is designed by choosing cross-sectional properties such that the maximum stress, as determined using elastic strength of materials relations, does not exceed the allowable stress. Load and resistance factor design is quite different. In this procedure, the structural member is selected so that its resistance, multiplied by a resistance factor, is not less than the service load combination, multiplied by load factors. This procedure permits differing reliabilities in the prediction of the load and member resistance to be taken into account.

Rivets in connections are usually made from a soft grade of steel that does not become brittle when heated and hammered with a pneumatic riveting gun. They are manufactured with one formed head and are installed in holes that are $\frac{1}{16}$ in. larger in diameter than the nominal diameter of the rivet. When installing a rivet, its head end is held tightly against the pieces being joined while the opposite end is hammered until another, similar head is formed. Steel rivets, as used in most connections, are usually heated to a cherry-red color (approximately $1800^{\circ} \mathrm{F}$ ) and can then be more easily driven. As the rivets cool, they shrink and squeeze the joined parts together. Copper and aluminum rivets used in aircraft engineering are generally driven cold.

The AISC mandates that rivets must conform to the American Society for Testing and Materials (ASTM) provisions "Standard Specifications for Structural Rivets" [8.3]. The size of rivets used in general steel construction ranges in diameter from $\frac{5}{8}$ to $1 \frac{1}{2}$ in. in $\frac{1}{8}$-in. increments.

Riveted joints are either lap or butt joints (Fig. 8-2). The lap joint has two plates that are lapped over each other and fastened together by one or more rows of rivets or fasteners. In the case of a butt joint, the edges of two plates are butted together and the plates are connected by cover plates.

A bolt is a threaded fastener with a head and a nut that screws on to the end without the head (Fig. 8-1a). The bolts most commonly used in steel construction are unfinished bolts (also called ordinary or common bolts) and high-strength bolts. Unfinished bolts are used primarily in light structures subjected to static loads. Unfinished bolts must conform to the specifications for low-carbon steel externally and internally threaded fasteners, ASTM A307 [8.3].

High-strength bolts are made from medium-carbon, heat-treated, or alloy steel and have tensile strengths several times greater than those of unfinished bolts. Spec-


Figure 8-2: Rivet joints: (a) lap; (b) butt.
ifications of the AISC state that high-strength bolts must conform to "Specifications for Structural Joints Using ASTM A325 or A490 Bolts" [8.3].

Owing to better performance and economy compared to riveted joints, highstrength bolting has become the leading technique used for connecting structures in the field. In general, three types of connections are made of high-strength bolts:

1. The friction $(F)$ connection, in which slip between the connected parts cannot be tolerated and must be resisted by a high clamping force
2. The bearing $(N)$ connection, with threads included in the shear plane (Fig. 8-3a)
3. The bearing $(X)$ connection, with threads excluded from the shear plane (Fig. 8-3b)

The allowable stresses recommended by the AISC Manual of Steel Construction [8.1] are given in Table 8-1. Generally, these stresses are based on the results of a large number of laboratory tests in which the ultimate strength of the rivets is determined. Division of the ultimate strength by a suitable factor of safety gives the allowable stresses.


Figure 8-3: Bearing connections of high-strength bolts: (a) type $N$ bolt, with threads included in the shear plane; $(b)$ type $X$ bolt, with threads excluded from the shear plane.

## Joint Failure Mode under Shear Loading

Four modes of failure for joints are normally considered.
(a) Shearing of the fastener (bolt or rivet) in either single (one-sided) or double (two-sided) shear, depending on the type of joint (Fig. 8-4a): To prevent shear failure of the fastener, the number of fasteners should be determined to limit the maximum shear stress in the critical fastener to the allowable stress listed in Table 8-1. The average shearing stress in a fastener (Fig. 8-4a) is

$$
\begin{equation*}
\tau=P_{s} / A=4 P_{s} / \pi d^{2} \tag{8.1}
\end{equation*}
$$

where $P_{S}$ is the load acting on the fastener's cross section subject to shear and $A$ and $d$ are the area and diameter, respectively, of the bolt or rivet cross section. For single shear $P_{s}=P$ and for double shear joints (Fig. 8-4a)

$$
P_{S}=\frac{1}{2} P
$$

(b) Compression or bearing, that is, the crushing of either the fastener or the plate in front of it (Fig. 8-4b): To assure that no compression or bearing occurs due to the crushing force of the fasteners on the material, the minimum number of fasteners is determined.

The bearing is assumed to be uniformly distributed over an area $A=t d$ so that the bearing stress $\sigma_{\mathrm{br}}$ is

$$
\begin{equation*}
\sigma_{\mathrm{br}}=P / t d \tag{8.2}
\end{equation*}
$$

where $P$ is the load, $t$ is the thickness, and $d$ is the diameter of the shaft of the fastener, as shown in Fig. 8-4b.

The specifications of AISC recommend the allowable bearing stress $\sigma_{p}$, using a factor of safety of 2, to be [8.4]

$$
\begin{equation*}
\sigma_{p}=\frac{1}{2} \sigma_{u}(s / d-0.50) \leq 1.50 \sigma_{u} \tag{8.3}
\end{equation*}
$$

where $s$ is the distance between the centers of two fasteners in the direction of the stress and $\sigma_{u}$ is the ultimate strength of the material.
(c) Tension or tearing when a plate tears apart along some line of least resistance (Fig. 8-4c): To prevent this failure in steel, the connected parts should be designed so that the tensile stress is less than $0.6 \sigma_{y s}$ of the gross area $A_{g}$ and less than $0.5 \sigma_{u}$ on the effective net area (specifications of AISC), where the gross area $A_{g}$ of a member is defined as the product of the thickness and the gross width of the member as measured normal to the tensile force $\sigma$ (Fig. 8-5). The net area $A_{n}$ of the plate is the product of the net width and the member thickness, and the net width is determined by deducting from the gross width the sum of all hole widths in the section cut. To compute the net width, the width of a fastener (rivet or bolt) hole is taken as $\frac{1}{16} \mathrm{in}$. larger than the actual width of the hole. Moreover, the actual hole diameter is $\frac{1}{16}$ in. larger than the nominal fastener size $d$. Thus, for each hole, a value of fastener

(c)

(d)


End tearing
(e)

Figure 8-4: Failure modes of joints.


Figure 8-5: Gross area $A_{g}$ and net width $W_{n}$.
diameter $\frac{1}{16}+\frac{1}{16}=\frac{1}{8} \mathrm{in}$. should be used to calculate the net width and net area. For section 1-1 in Fig. 8-5 with two holes, for example, the net width $W_{n}$ is

$$
W_{n}=b-N\left(d+\frac{1}{8}\right)=b-2\left(d+\frac{1}{8}\right)
$$

where $N$ is the number of holes and

$$
A_{n}=t W_{n}=t\left[b-2\left(d+\frac{1}{8}\right)\right], \quad A_{g}=t b
$$

Holes are sometimes arranged in a zigzag pattern, as in Fig. 8-6. Then the net width is taken as the gross width minus the diameter of all the holes in a chain (line of failure), plus for each "out-of-line" space (i.e., each diagonal) in the chain, the value

$$
\begin{equation*}
s^{2} / 4 g \tag{8.4a}
\end{equation*}
$$

where $s$ is the pitch (longitudinal spacing) in inches (Fig. 8-6) and $g$ is the gage (transverse spacing) in inches. For example, for the chain (possible line of failure) of $A B G H I$ (or the section of $A B G H I$ ) shown in Fig. 8-6a, where only one diagonal is in the chain, the net width is

$$
\begin{equation*}
W_{n}=b-3\left(d+\frac{1}{8}\right)+s^{2} / 4 g \tag{8.4b}
\end{equation*}
$$



Figure 8-6: Zigzag pattern of holes: (a) chain ABGHI; (b) chain $A B G C D$.
and the net width for section $A B G C D$ (Fig. 8-6b), where two diagonals are in the chain, is

$$
\begin{equation*}
W_{n}=b-3\left(d+\frac{1}{8}\right)+s^{2} / 4 g+s^{2} / 4 g \tag{8.4c}
\end{equation*}
$$

For all possible chains, the critical net section is the chain that has the least net width.
If the tensile force is transmitted by fasteners (rivets or bolts) through some, but not all, of the segments of the cross section, an effective net area $A_{e}$ must be computed [8.2]. This is to account for the effect of shear stress concentration in the vicinity of connections or for the shear lag for the portion of the section distant from the connections. The effective net area is defined by

$$
\begin{equation*}
A_{e}=U A_{n} \tag{8.5}
\end{equation*}
$$

where $U$ is a reduction factor assumed to be 1.0 unless otherwise determined. Values of $U$ recommended by AISC are listed in Table 8-2. The tensile stress can be obtained from

$$
\begin{align*}
\sigma_{g} & =P_{T} / A_{g}  \tag{8.6a}\\
\sigma_{n} & =P_{T} / A_{n} \tag{8.6b}
\end{align*}
$$

or

$$
\begin{equation*}
\sigma_{e}=P_{T} / A_{e} \tag{8.6c}
\end{equation*}
$$

where $\sigma_{g}, \sigma_{n}$, and $\sigma_{e}$ are the tensile stresses based on the gross, net, and effective areas, respectively, and $P_{T}$ is the total load on the connection member.
(d) End failure, including end shearing and end tearing (Figs. 8-4d and $e$ ): In the case of failure including shearing on the area $x t$ (Fig. 8-4d), the shear stresses are

$$
\begin{equation*}
\tau=P / 2 x t \tag{8.7}
\end{equation*}
$$

where $P$ is the load acting at the hole, $t$ is the thickness of the plate, and $x$ is as shown in Fig. 8-4d.

In actuality, the stress is probably more complicated. The AISC [8.1] recommends an experimentally determined formula. To prevent the failure mode of end tearing or shearing, the minimum edge distance $e$ from the center of a fastener hole to the edge (Fig. 8-4e) in the direction of the force shall be greater than $2 P / t \sigma_{u}$, that is,

$$
\begin{equation*}
e \geq 2 P / t \sigma_{u} \tag{8.8}
\end{equation*}
$$

where $\sigma_{u}$ is the ultimate tensile strength. Table 8-3 gives some recommended $e$ values. These edge distances depend on the joint type, the plate thickness, and the type of fastener.

The analyses above are based on the assumption that the stresses in fasteners or connecting members are uniform. This is not always true. When the stress is below
the elastic limit, the true stresses are not necessarily equal to the average stress. Stress concentration may occur. Before the ultimate strength is reached, however, the material yields and stresses are redistributed so that they tend to approach uniform values. Because of plastic yielding of the material and because allowable stresses are obtained from tests on specimens similar to the actual structure, it is possible that this assumption corresponds to an acceptable approximation.

The four modes of failure analysis are suitable for riveted joints and $N, X$-type bolted joints. The only difference is in their corresponding allowable stresses for shearing and bearing (Table 8-1). For $F$-type connections, the design is based on the assumption that if the connection fails, the bolts will fail in shear alone, and the bearing stress of the fasteners on the connected parts need not be considered. Nevertheless, the bearing must be considered in the event the friction bolts slip and must resist bearing [8.1]. Formulas for the four modes of failure are summarized in Table 8-4.

## Boiler Joints

When a riveted joint is to remain airtight under pressure, it is sometimes called a boiler joint. Special consideration is given to the analysis and design of this type of joint. The Boiler Code of the American Society of Mechanical Engineers gives the ultimate strength of boiler steel to be used for boilers and tanks and also recommends a factor of safety of 3.5 . The efficiency of a riveted boiler joint is the ratio of the strength of the joint to the strength in tension of the unpunched plate.

## Bolted Joints in Machine Design

In machine design, friction-type high-strength bolting is most commonly utilized. A bolt is tightened to develop a minimum initial tension in the bolt shank equal to about $70 \%$ of the tensile strength of the bolt. In this case, no interface slip occurs at allowable loads so that the bolts are not actually stressed in shear and are not in bearing. A discussion follows of the analysis of friction-type high-strength bolting used in machine design in which the tensile load of the bolt must be considered.

The tightening load is created in a bolt by exerting an initial torque on the nut or on the head of the bolt. For a torqued-up bolt, the tensile force in the bolt due to the torque can be approximated as [8.5]

$$
\begin{equation*}
T=c d F_{i} \tag{8.9}
\end{equation*}
$$

where $T$ is the tightening torque, $c$ is a constant depending on the lubrication present, $d$ is a nominal outside diameter of the shank of the bolt, and $F_{i}$ is the initial tightening load in the bolt. When the threads of the bolt are well cleaned and dried, choose $c=0.20$.

It is important to understand that when a load in the bolt shank direction is applied to a bolted connection over and above the tightening load, special consideration must be given to the behavior of the connection, which changes the allowable external load


Figure 8-7: Inner force for a bolted joint under tension.
significantly. In the absence of an external load, the tensile force in the bolt is equal to the compressive force on the connected members. The external load will act to stretch the bolt beyond its initial length (Fig. 8-7). Thus, the resulting effect, which depends on the relative stiffness of the bolt and the connected members, is that only a part of the applied external load is carried by the bolt. The final tensile force $F_{b}$ in the bolt and compressive force $F_{c}$ in connected members can be obtained using [8.6]

$$
\begin{align*}
F_{b} & =F_{i}+k_{b} P_{e} /\left(k_{b}+k_{c}\right)  \tag{8.10}\\
F_{c} & =F_{i}-k_{c} P_{e} /\left(k_{b}+k_{c}\right) \tag{8.11}
\end{align*}
$$

where $F_{i}$ is the initial tensile force in the bolt, $k_{b}$ and $k_{c}$ are the stiffnesses of the bolt and the connected members, respectively (Fig. 8-7), and $P_{e}$ is the external load.

Since the external loading $\left(P_{e}\right)$ is shared by the bolt and the connected members according to their relative stiffness $\left(k_{c} / k_{b}\right)$, stiffness is usually given in the form of the ratio $k_{r}=k_{c} / k_{b}$, where $k_{r}$ is the stiffness ratio. For simple extension or compression, the stiffnesses are

$$
\begin{align*}
k_{b} & =A_{b} E_{b} / L_{b}  \tag{8.12}\\
k_{c} & =A_{c} E_{c} / L_{c} \tag{8.13}
\end{align*}
$$

where $A_{b}, E_{b}$, and $L_{b}$ are the cross-sectional area, modulus of elasticity, and length of the bolt, respectively. The quantities $A_{c}, E_{c}$, and $L_{c}$ are for the connected members.


Figure 8-8: Concentrically loaded connections.

### 8.3 LOAD ANALYSIS OF FASTENER GROUPS

A riveted or bolted joint may be subjected to a variety of forces. When the line of action of the resultant force that is to be resisted by the joint passes through the centroid $c$ of the fastener group (Fig. 8-8), the joint is said to be concentrically loaded. Otherwise, the joint is said to be eccentrically loaded (Fig. 8-9a). For a concentrically loaded connection, the load is assumed to be uniformly distributed among the fasteners. For an eccentrically loaded connection, the force may be replaced by an equal force at the centroid and a moment equal in magnitude to the force times its eccentricity. In this case, each fastener in the group is assumed to resist the force at the centroid uniformly and to resist the moment in proportion to its respective distance to the centroid of the fastener group.

Normally, the conditions of equilibrium, along with the assumptions above, permit the most significant forces in each fastener to be computed. In riveted and bolted connections the centroid of the fastener, which is sometimes referred to as the center of resistance, is of importance in the analysis and design. The location of the centroid can be found by using the method described in Chapter 2. But in most cases all fasteners of a group have the same cross-sectional area and are arranged in a symmetrical pattern; consequently, the location of the centroid can be readily determined by simple observation, as should be evident in Figs. 8-8 and 8-9.

Example 8.1 Eccentrically Loaded Connection To illustrate the determination of fastener forces, consider the simple rivet group of the four symmetrically located rivets of Fig. 8-9a. It is assumed that each rivet takes $\frac{1}{4} P$ of the load (Fig. 8-9b). Also, a moment of magnitude $P L$ is generated at the centroid of the rivet areas. This moment is resisted by the moment due to the rivet forces (see Fig. 8-9b). This force in each rivet is $F$. Because the moments are in equilibrium,

$$
\begin{equation*}
P L-4 F \ell=0 \tag{1}
\end{equation*}
$$



Figure 8-9: Eccentric loading. Positive moment $M$ and coordinate system are shown in (c).

Hence

$$
\begin{equation*}
F=P L / 4 \ell \tag{2}
\end{equation*}
$$

The total force on each rivet is the vector sum of $\frac{1}{4} P$ and $F$. In Fig. 8-9, it can be seen that the maximum resultant force occurs in the rivet closest to the eccentric load $P$, that is,

$$
\begin{equation*}
F_{\max }=F+\frac{1}{4} P=(P / 4 \ell)(\ell+L) \tag{3}
\end{equation*}
$$

There is another general, related method to find the moment-resisting forces $F$ on each fastener. If the center of rotation of the eccentric moment $P L$ is assumed to be the centroid of the fastener group, each fastener force $F_{j}$ will be perpendicular to a line joining the fastener and the centroid $c$, and the magnitude of the force will be proportional to its distance $\ell_{j}$ from $c$. Therefore,

$$
\begin{equation*}
F_{j}=k \ell_{j} \tag{4}
\end{equation*}
$$

where $F_{j}$ is the force on fastener $j$ due to eccentric loading and $k$ is a constant for the fastener group.

From the equilibrium condition, $\sum M_{c}=0$. For the connections shown in Fig. 8-9a,

$$
\begin{equation*}
P L=F_{1} \ell_{1}+F_{2} \ell_{2}+F_{3} \ell_{3}+F_{4} \ell_{4} \tag{5}
\end{equation*}
$$

where $F_{1}, F_{2}, F_{3}, F_{4}$ are the moment-resisting forces on rivet $1,2,3,4$, respectively, and $\ell_{1}, \ell_{2}, \ell_{3}, \ell_{4}$ are the distances from each rivet to the centroid $c$.

From (4) and (5),

$$
\begin{equation*}
P L=k\left(\ell_{1}^{2}+\ell_{2}^{2}+\ell_{3}^{2}+\ell_{4}^{2}\right)=k \sum_{j=1}^{4} \ell_{j}^{2} \tag{6}
\end{equation*}
$$

so that

$$
\begin{equation*}
k=\frac{P L}{\sum_{j=1}^{4} \ell_{j}^{2}} \tag{7}
\end{equation*}
$$

Thus, $F_{j}$ in (4) can be obtained by use of $k$ of (7). From Fig. 8-9a,

$$
\begin{equation*}
k=P L / 4 \ell^{2} \tag{8}
\end{equation*}
$$

which can be substituted into (4) to give

$$
\begin{equation*}
F_{1}=F_{2}=F_{3}=F_{4}=F=P L / 4 \ell \tag{9}
\end{equation*}
$$

This is, of course, the same as (2).

In more general terms Eq. (7) of Example 8.1 would be written

$$
\begin{equation*}
k=\frac{P L}{\sum_{j=1}^{n} \ell_{j}^{2}} \tag{8.14}
\end{equation*}
$$

where $L$ is the eccentric distance of load $P, n$ is the number of fasteners in a connection, and $k$ is the proportional constant of the fastener group under load $P$. It is often convenient to use the components $F_{x}$ and $F_{z}$ of force $F$ in vectorial summation with the direct force $P / n$. The formulas are

$$
\begin{equation*}
k=\frac{M}{\sum_{j=1}^{n}\left(\ell_{x j}^{2}+\ell_{z j}^{2}\right)} \tag{8.15}
\end{equation*}
$$

where $M=P L, L=\sqrt{L_{x}^{2}+L_{z}^{2}}$, and $\ell_{j}=\sqrt{\ell_{x j}^{2}+\ell_{z j}^{2}}$ and for coordinate system $x z$, as shown in Fig. 8-9c,

$$
\begin{align*}
F_{x j} & =\frac{-M \ell_{z j}}{\sum_{j=1}^{n}\left(\ell_{x j}^{2}+\ell_{z j}^{2}\right)}  \tag{8.16a}\\
F_{z j} & =\frac{M \ell_{x j}}{\sum_{j=1}^{n}\left(\ell_{x j}^{2}+\ell_{z j}^{2}\right)} \tag{8.16b}
\end{align*}
$$

Therefore, the components $F_{R x j}$ and $F_{R z j}$ of the resultant force $F_{R j}$ on the fastener $j$ are

$$
\begin{align*}
& F_{R x j}=\frac{P_{x}}{n}+\frac{-M \ell_{z j}}{\sum_{j=1}^{n} \ell_{x j}^{2}+\sum_{j=1}^{n} \ell_{z j}^{2}}  \tag{8.17a}\\
& F_{R z j}=\frac{P_{z}}{n}+\frac{M \ell_{x j}}{\sum_{j=1}^{n} \ell_{x j}^{2}+\sum_{j=1}^{n} \ell_{z j}^{2}} \tag{8.17b}
\end{align*}
$$

and

$$
\begin{equation*}
F_{R j}=\sqrt{F_{R x j}^{2}+F_{R z j}^{2}} \tag{8.17c}
\end{equation*}
$$

### 8.4 DESIGN OF RIVETED AND BOLTED CONNECTIONS

Based on the considerations and analysis described above, design of a riveted or bolted joint under a given load involves

1. Determining the type of the joint and number of fasteners
2. Using predetermined allowable stresses in order to find the required area in shear, bearing, and tension for the fasteners and plates (connected parts) to be used

Example 8.2 Load Capacity for a Member in Tension A bolted connection consists of a $4 \times 4 \times \frac{1}{2}$ angle with three bolts of diameter $\frac{3}{4}$ in. and a strong plate $B$, as shown in Fig. 8-10. Determine the tension resistance capacity of the angle. Assume that the angle is made of American Iron and Steel Institute (AISI) 1015 steel.


Figure 8-10: Example 8.2.

The gross area of the $4 \times 4 \times \frac{1}{2}$ angle is

$$
\begin{equation*}
A_{g}=3.75 \mathrm{in}^{2} \tag{1}
\end{equation*}
$$

From failure mode 3 of Table 8-4,

$$
\begin{equation*}
A_{n}=t\left[b-\left(d+\frac{1}{8}\right)\right]=A_{g}-\left(d+\frac{1}{8}\right) t=3.75-\left(\frac{3}{4}+\frac{1}{8}\right)\left(\frac{1}{2}\right)=3.31 \mathrm{in}^{2} \tag{2}
\end{equation*}
$$

For 1015 steel, from Table 4-9,

$$
\begin{equation*}
\sigma_{y s}=45.5 \mathrm{ksi}, \quad \sigma_{u}=61.0 \mathrm{ksi} \tag{3}
\end{equation*}
$$

and $A_{e}=U A_{n}=0.85 \cdot 3.31=2.81 \mathrm{in}^{2}(U=0.85$ from Table 8-2 $)$.
As mentioned in Section 8.2, to prevent failure of the angle, the tensile stress should be less than $0.6 \sigma_{y s}$ on the gross area $A_{g}$ and less than $0.5 \sigma_{u}$ on the effective net area $A_{e}$, that is,

$$
\begin{equation*}
P / A_{g} \leq 0.6 \sigma_{y s} \quad \text { and } \quad P / A_{e} \leq 0.5 \sigma_{u} \tag{4}
\end{equation*}
$$

From (4)

$$
\begin{equation*}
P \leq 0.6 \sigma_{y s} A_{g}=(0.6)(45.5)(3.75)=102.38 \mathrm{kips} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
P \leq 0.5 \sigma_{u} A_{e}=(0.5)(61.0)(2.81)=85.71 \mathrm{kips} \tag{6}
\end{equation*}
$$

The tension resistance capacity $P$ is the smaller value of these two: $P=85.71 \mathrm{kips}$.

Example 8.3 Load Capacity of a Riveted Connection A riveted lap connection consists of two $\frac{3}{4} \times 12$ in. plates of A36 steel and $\frac{7}{8}$-in.-diameter A502 grade 1 rivets, as shown in Fig. 8-11. Determine the maximum tensile load $P$ that can be resisted by the connection.

The rivets may fail in shear or bearing or the plate may fail in tension or bearing. Any failure will mean the failure of the connection. Thus, each condition must be considered to determine the critical condition and the load capacity of the connection.
(a) Shear failure of the rivet: The corresponding total force $P_{T}$ is calculated as

$$
\begin{equation*}
P_{T}=\tau_{w} A N \tag{1}
\end{equation*}
$$

where $\tau_{w}$ is the allowable shear stress of the rivet, $A$ the cross-sectional area of each rivet, and $N$ the number of the rivets in the connection.


Figure 8-11: Example 8.3.

For A502 grade 1 rivets, Table 8 - 1 provides $\tau_{w}=17.5 \mathrm{ksi}$. Since

$$
A=\frac{1}{4} \pi d^{2}=\frac{1}{4}(3.14)\left(\frac{7}{8} \mathrm{in} .\right)^{2}=0.601 \mathrm{in}^{2} \quad \text { and } \quad N=9
$$

it follows from (1) that

$$
\begin{equation*}
P_{T}=(17.5)(0.601)(9)=94.66 \mathrm{kips} \tag{2}
\end{equation*}
$$

(b) Bearing and end failure:

$$
\begin{equation*}
P_{T}=\sigma_{p} A_{b} N \tag{3}
\end{equation*}
$$

where $\sigma_{p}$ is the allowable bearing stress of the plate and $A_{b}$ is the bearing area of each rivet: $A_{b}=d t$ ( $d$ is rivet diameter and $t$ is thickness of the plate).

From Eq. (8.3) the allowable bearing stress $\sigma_{p}$ is calculated as

$$
\begin{equation*}
\sigma_{p}=\frac{1}{2} \sigma_{u}(s / d-0.5) \leq 1.5 \sigma_{u} \quad \text { between two fasteners } \tag{4}
\end{equation*}
$$

For A36 steel, $\sigma_{u}=58.0 \mathrm{ksi}$,

$$
\sigma_{p}=\left(\frac{58}{2}\right)\left(\frac{3}{7 / 8}-0.5\right)=84.9 \mathrm{ksi} \leq(1.5)(58)=87.0 \mathrm{ksi}
$$

From (3)

$$
\begin{equation*}
P_{T}=84.9\left(\frac{7}{8}\right)\left(\frac{3}{4}\right) 9=501.44 \mathrm{kips} \tag{5}
\end{equation*}
$$

For the fasteners near the end, from Eq. (8.8),

$$
P=\frac{1}{2} \sigma_{u} t e=\left(\frac{58}{2}\right)\left(\frac{3}{4}\right) 2=43.5 \mathrm{kips}
$$

If it is assumed that all of the fasteners can cause tearing,

$$
\begin{equation*}
P_{T}=P N=391.5 \mathrm{kips} \tag{6}
\end{equation*}
$$

Select the smaller value from (5) and (6), $P_{T}=391.5 \mathrm{kips}$.
(c) Tensile capacity: On the gross cross-sectional area, the allowable tensile stress $\sigma_{t w}$ is [from failure mode (c) in Section 8.2 with $\sigma_{y s}=36.0 \mathrm{ksi}$ for A36 steel]

$$
\sigma_{t w}=0.6 \sigma_{y s}=(0.6)(36.0)=21.6 \mathrm{ksi}
$$

Since $A_{g}=\left(\frac{3}{4} \mathrm{in}.\right)(12 \mathrm{in})=.9 \mathrm{in}^{2}$, the tensile capacity of the gross area is

$$
\begin{equation*}
P_{T}=\sigma_{t w} A_{g}=(21.6)(9)=194.4 \mathrm{kips} \tag{7}
\end{equation*}
$$

On the effective net area

$$
\begin{aligned}
U & =1.0 \\
\sigma_{t w} & =0.5 \sigma_{u}=(0.5)(58)=29.0 \mathrm{ksi} \quad \text { [failure mode (c) in Section 8.2] } \\
A_{n} & =\left[12-3\left(\frac{7}{8}+\frac{1}{8}\right)\right]\left(\frac{3}{4}\right)=6.75 \mathrm{in}^{2} \\
A_{e} & =A_{n} U=(6.75)(1.0)=6.75 \mathrm{in}^{2} \quad[\text { Eq. (8.5)] }
\end{aligned}
$$

Therefore, the tensile capacity of the plate for the effective net area is

$$
\begin{equation*}
P_{T}=\sigma_{t w} A_{e}=(29.0)(6.75)=195.75 \mathrm{kips} \tag{8}
\end{equation*}
$$

Maximum load capacity of the connection in Fig. 8-11 is the least value of $P_{T}$ given in (2) and (5)-(8), that is,

$$
\begin{equation*}
P_{\max }=\left(P_{T}\right)_{\text {shear }}=94.66 \mathrm{kips} \tag{9}
\end{equation*}
$$

Example 8.4 Bolted Connection with Bolts in Double Shear and in Zigzag Patterns Determine the maximum value of $P$ that the bolted connection in Fig. 8-12 can carry using 1-in.-diameter A307 bolts. Use (Table 8-1) $\tau_{w}=10.0$ ksi for bolts and $\sigma_{u}=58.0 \mathrm{ksi}$ and $\sigma_{y s}=36.67 \mathrm{ksi}$ for plates. Use a procedure similar to those in Example 8.3 to investigate the critical condition in each failure mode.
(a) $P_{T}$ by shear resistance of the bolts: Since the bolts are in double shear (two cross sections are subjected to shear), the total load by shear is

$$
\begin{equation*}
P_{T}=2 \tau_{w} A N=(2)(10.0)\left[\left(\frac{1}{4} \pi\right)\left(1^{2}\right)\right](6)=94.2 \mathrm{kips} \tag{1}
\end{equation*}
$$

(b) $P_{T}$ by bearing resistance of the plate: It can be seen that bearing on the $\frac{3}{4}$-in. plate is more critical than on two $\frac{1}{2}$-in. plates:

$$
P_{T}=\sigma_{p} d t N
$$



Figure 8-12: Example 8.4.
where $\sigma_{p}$ is the allowable bearing stress, $d$ the diameter of the bolt, and $t=\frac{3}{4}$ in. the thickness of the plate. For the bolts inside the plates, from Eq. (8.3),

$$
\sigma_{p}=\frac{1}{2} \sigma_{u}\left(s / d_{b}-0.5\right)=\frac{1}{2} \sigma_{u}(4 / 1.0-0.5)=1.75 \sigma_{u}
$$

However, the condition $\sigma_{p} \leq 1.5 \sigma_{u}$ should be met, so $\sigma_{p}=1.5 \sigma_{u}=1.5(58)=$ 87.0 ksi:

$$
\begin{equation*}
P_{T}=\sigma_{p} d t N=(87.0)(1.0)\left(\frac{3}{4}\right)(6)=391.5 \mathrm{kips} \tag{2}
\end{equation*}
$$

For the bolts near the end, from Eq. (8.8),

$$
\begin{align*}
P & =\frac{1}{2} \sigma_{u} t e=\left(\frac{58}{2}\right)\left(\frac{3}{4}\right) 2=43.5 \mathrm{kips}  \tag{3}\\
P_{T} & =43.5(6)=261.0 \mathrm{kips}
\end{align*}
$$

(c) $P_{T}$ by tension of the plate:

Cross section $A B C D: \quad W_{n}=11-2\left(1.0+\frac{1}{8}\right)=8.75$ in.
Cross section EFGHI: $\quad W_{n}=11-3\left(1.0+\frac{1}{8}\right)+2\left(\frac{2^{2}}{4 \times 3.5}\right)=8.20 \mathrm{in}$.
It is seen that all other section lengths are between the values of $W_{n}$ of (4) and (5). The least value is $W_{n}=8.20 \mathrm{in}$. The maximum tensile load on the gross area is

$$
\begin{equation*}
P_{T}=0.6 \sigma_{y s} A_{g}=(22.0)\left(\frac{3}{4}\right)(11)=181.5 \mathrm{kips} \tag{6}
\end{equation*}
$$

and on the effective net area

$$
\begin{equation*}
P_{T}=0.5 \sigma_{u} A_{e}=(29.0)\left[(1.0)\left(\frac{3}{4}\right)(8.20)\right]=178.35 \mathrm{kips} \tag{7}
\end{equation*}
$$

Comparison of the values of $P_{T}$ in the cases above indicates that (1) governs the connection, that is,

$$
P_{\max }=94.2 \mathrm{kips}
$$

Example 8.5 Analysis of Eccentrically Loaded Riveted Joints The riveted joint shown in Fig. 8-13a is loaded with 10 kips at a distance of 8 in. from a vertical axis passing through the centroid $c$ of the rivet group that fastens the plate to a column flange. Find the required rivet diameter for an allowable shear stress of $11,000 \mathrm{psi}$. Assume that shear failure is the critical condition and all rivets have the same diameter.


Figure 8-13: Eccentrically loaded riveted joint: (a) riveted joint; (b) free-body diagram of a plate; (c) rivet forces.

Because of the symmetrical arrangement of the rivets, the centroid $c$ of the rivet group can be located by observation. The load $P$ generates a moment of $P L=$ 80 kip-in. about the centroid (Fig. 8-13b), so that force $P=10 \mathrm{kips}$ ( $P_{x}=0$, $\left.P_{z}=10 \mathrm{kips}\right)$ at the centroid is being resisted equally by all six rivets, and a twisting moment is resisted by the twisting forces of the rivets.

From the dimensions given,

$$
\begin{equation*}
\sum_{j=1}^{6}\left(\ell_{x j}^{2}+\ell_{z j}^{2}\right)=4\left(4^{2}+5^{2}\right)+2\left(5^{2}+0^{2}\right)=214 \mathrm{in}^{2} \tag{1}
\end{equation*}
$$

where $\ell_{x j}$ and $\ell_{z j}$ are the coordinates of rivet $j$. Therefore, from Eqs. (8.16), the components of the twisting forces on rivet 1 are

$$
\begin{align*}
& F_{x 1}=\frac{-M \ell_{z 1}}{\sum_{j=1}^{6}\left(\ell_{x j}^{2}+\ell_{z j}^{2}\right)}=\frac{-80(-4)}{214}=+1.495 \mathrm{kips}  \tag{2}\\
& F_{z 1}=\frac{M \ell_{x 1}}{\sum_{j=1}^{6}\left(\ell_{x j}^{2}+\ell_{z j}^{2}\right)}=\frac{80(-5)}{214}=-1.869 \mathrm{kips} \tag{3}
\end{align*}
$$

Similarly,

$$
\begin{align*}
& F_{x 3}=-F_{x 4}=F_{x 6}=-F_{x 1}=-1.495 \mathrm{kips}  \tag{4}\\
& F_{z 3}=-F_{z 4}=-F_{z 6}=F_{z 1}=-1.869 \mathrm{kips}  \tag{5}\\
& F_{x 2}=F_{x 5}=0 \quad \text { and } \quad F_{z 2}=-F_{z 5}=1.869 \mathrm{kips} \tag{6}
\end{align*}
$$

From Eqs. (8.17), the resultant force on rivet 1 is

$$
\begin{align*}
F_{R 1} & =\left[\left(P_{x} / n+F_{x 1}\right)^{2}+\left(P_{z} / n+F_{z 1}\right)^{2}\right]^{1 / 2} \\
& =\left[\left(\frac{0}{6}+1.495\right)^{2}+\left(\frac{10}{6}-1.869\right)^{2}\right]^{1 / 2}=1.51 \mathrm{kips} \tag{7}
\end{align*}
$$

In a similar fashion, for rivets, 4,5 , and 2 ,

$$
\begin{equation*}
F_{R 4}=3.84 \mathrm{kips}, \quad F_{R 5}=3.54 \mathrm{kips}, \quad F_{R 2}=0.2 \mathrm{kips} \tag{8}
\end{equation*}
$$

Also from Fig. 8-13c, it can be seen that

$$
\begin{equation*}
F_{R 3}=F_{R 1}, \quad F_{R 6}=F_{R 4} \tag{9}
\end{equation*}
$$

Thus, the maximum value of the shear force $F_{R}$ on the rivet is $F_{R 4}$ or $F_{R 6}$. The size of the rivets has to be determined for a shear force of 3.84 kips.

Assume that the rivet is in single shear with the given allowable shear stress of $\tau_{w}=11 \mathrm{ksi}$, and the shear failure is the critical condition. Since $\left(\frac{1}{4} \pi d^{2}\right) \tau_{w}=3.84$,
the rivet diameter $d$ is calculated as

$$
\begin{equation*}
d=\sqrt{(3.84)(4) /\left(\pi \tau_{w}\right)}=0.67 \mathrm{in} . \quad \text { or } \quad d=17 \mathrm{~mm} \tag{10}
\end{equation*}
$$

Example 8.6 Torque Necessary to Draw Up a Nut in Machine Design A set of two bolts is to be used to provide a clamping force of 6000 lb between two bolted parts (Fig. 8-14). The joint is also subjected to an additional external load of 5000 lb . Assume that (1) the forces are shared equally between the two bolts, (2) the stiffness of the bolted parts is three times that of the bolt [i.e., $k_{c}=3 k_{b}$ in Eqs. (8.10) and (8.11)], and (3) each bolt is stressed to $75 \%$ of its proof strength. Find the tensile force in the bolts and the necessary tightening torque for the nuts.

For the given conditions, the initial clamping load $F_{i}$ on each bolt is $6000 / 2=$ 3000 lb , and the external load on each is $5000 / 2=2500 \mathrm{lb}$. Then from Eq. (8.10), the final tensile force in one of the bolts is

$$
\begin{equation*}
F_{b}=F_{i}+\frac{k_{b}}{k_{b}+k_{c}} P_{e}=3000+\frac{k_{b}}{k_{b}+3 k_{b}}(2500)=3625 \mathrm{lb} \tag{1}
\end{equation*}
$$

while, from Eq. (8.11), the compressive force is

$$
\begin{equation*}
F_{c}=F_{i}-\frac{k_{c}}{k_{b}+k_{c}} P_{e}=3000-\frac{3 k_{b}}{k_{b}+3 k_{b}}(2500)=1125 \mathrm{lb} \tag{2}
\end{equation*}
$$

Thus, the compressive force in the bolted parts $F_{c}$ is greater than zero, which indicates that the joint is still tight.

If a bolt made from Society of Automotive Engineers (SAE) grade 4 steel (Table 8-5) is chosen, it will have a proof strength of 65,000 psi. Then the allowable tensile stress of the bolt is $\sigma_{w}=(0.75)(65,000)=48,750 \mathrm{psi}$ and the required tensile stress area for the bolt is

$$
\begin{equation*}
A_{t}=\frac{F_{b}}{\sigma_{t w}}=\frac{3625 \mathrm{lb}}{48,750 \mathrm{lb} / \mathrm{in}^{2}}=0.0744 \mathrm{in}^{2} \tag{3}
\end{equation*}
$$



Figure 8-14: Bolt connection for Example 8.6.

From Table $8-6$ it can be seen that the $\frac{3}{8}, 16$ UNC thread has the required tensile stress area, since $A_{t}=0.785(0.3750-0.9743 / n)^{2}=0.07745 \mathrm{in}^{2}$ is larger than $A_{t}$ of (3). Thus, the necessary torque required would be, from Eq. (8.9),

$$
\begin{equation*}
T=0.2 d F_{i}=(0.20)(0.3750)(3000)=225 \mathrm{lb} / \mathrm{in} \tag{4}
\end{equation*}
$$

References such as [8.6] can be consulted for more detailed discussions of the design of connections.

### 8.5 WELDED JOINTS AND CONNECTIONS

In contrast to bolted and riveted joints, welded joints do not necessarily require an overlap in plates, thus affording more flexibility in design. Also, welded joints are usually lighter and are particularly advantageous in that they provide continuity between connected parts.

## Types of Welded Joints and Typical Drawing Symbols

There are various types of welded joints, but the two main types are fillet and butt welds, shown in Fig. 8-15. The butt weld is usually loaded in tension, the strength of which is based on the net cross-sectional area of the thinner of the two plates being joined. If the joint is properly made with the appropriate welding metal, the joint will be stronger than the parent metal. A fillet weld subjected to shear stress would tend to fail along the shortest dimension of the weld, which is referred to as the throat of the weld, shown in Figs. 8-16 and 8-17. In most cases the legs are of equal length, since welds with legs of different lengths are less efficient than those with equal legs.

Other types of welded joints are plug welds, slot welds, and spot welds, as shown in Fig. 8-15. Plug welds are made by punching holes into one of the two plates to be welded and then filling the holes with weld metal, which fuses into both plates. Slot welds can be used when other types of welded joints are not suitable and to provide additional strength to a fillet joint. Spot welds, which are used extensively in the fabrication of sheet metal parts, are a quick and simple way to fasten light pieces together at intervals along a seam.

Welded joints are often used with various edge preparations, some of which have been qualified by the American Welding Society (AWS) [8.7]. The choice of joint type often directly affects the cost of welding. Thus, the choice is not always dominated by the design function.

The symbols representing the type of weld to be applied to a particular joint are shown in Table 8-7. These symbols, which have been standardized and adopted by the AWS, quickly indicate the exact welding details established for each joint to satisfy all necessary conditions of material strength and service. The symbols may be broken down into basic elements and combinations can be formed if desired.

(a) Fillet

(b) Fillet

(c) Butt

(d) Fillet

(e) Butt

(b) Spot

Figure 8-15: Types of welds.


Figure 8-16: Notation for fillet welds.


Figure 8-17: Stress distribution in side fillet welds. (c) Damage usually occurs along the throat of a weld.

## Analysis of Welded Joints

For butt welds, as mentioned previously, the weld is stronger than the base metals and no further analysis is required. However, the fillet welded joint needs to be analyzed to guarantee it is strong enough to sustain the applied loading. Four basic types of loading are considered here: direct tension or compression, direct vertical shear, bending, and twisting. The area of the fillet weld is calculated using leg size $w$, throat width $t$ (Fig. 8-16b), and welded seam length $L_{w}$. Usually, leg size $w$ and throat width $t$ are related, depending on the form of the welded joint. Table 8-8 gives the relationship between $w$ and the plate thickness. Table 8-9 gives the allowable shear stresses and forces on welds.

The cross section along the welded seam, the width of which equals the throat width $t$, is called the effective cross section. The stress in the effective cross section should be less than the allowable stress.

The analysis of welded joints involves the following steps:

1. Draw the effective cross section of the welded connection. It is a narrow area along the weld seam width $t$. In the case of Fig. $8-16 b, t=0.707 w$. For example, in Fig. 8-18 the area enclosed by dashed lines represents the effective cross section of a [-shaped weld.
2. Let the centroid of the effective section be the origin and set up an orthogonal reference system $x, y, z$. If the normal stress is to be considered, select $z, y$ axes as principal axes. The area and moments of inertia of the effective cross section of weld can be obtained by using Eqs. (2.1), (2.4), and (2.11); that is,


Figure 8-18: Stress components on a weld area.

$$
A_{w}=\int d A, \quad I_{y}=\int z^{2} d A, \quad I_{z}=\int y^{2} d A, \quad J_{x}=\int\left(z^{2}+y^{2}\right) d A
$$

Some geometric properties of the welded connections are provided in Table 8-10.
3. Find the forces and moments that act on the welded connection. The positive directions of forces $P_{x}, P_{y}, P_{z}$ and moments $M_{x}, M_{y}, M_{z}$ are indicated in Fig. 8-18.
4. At any point of the connection, the stress on the weld due to a single component of load can be obtained from Eqs. (3.41), (3.46), (3.47), and (3.55). These stresses are summarized in Table 8-11.
5. Determine the resultant nominal stress and the load force per unit length of weld. The nominal resultant stress $f_{r}$ is the vector sum of stress components (Fig. 8-18):

$$
\begin{equation*}
f_{r}=\sqrt{f_{x}^{2}+f_{y}^{2}+f_{z}^{2}}=\sqrt{\left(f_{x}^{\prime}+f_{x}^{\prime \prime}\right)^{2}+\left(f_{y}^{\prime}+f_{y}^{\prime \prime}\right)^{2}+\left(f_{z}^{\prime}+f_{z}^{\prime \prime}\right)^{2}} \tag{8.18}
\end{equation*}
$$

The resultant force per unit length is $q_{r}=t f_{r}$.
It is assumed that all loads acting on a fillet weld are shear forces independent of their actual direction and the critical section is always the throat of the weld. The nominal stress $f_{r}$ should be less than the allowable shear stress of the welding material (Table 8-9) to avoid failure.

The stress distribution within a fillet weld is complex, due to such factors as eccentricity of the applied load, shape of the fillet, and notch effect of the root. However, the same conditions exist in the actual fillet welds tested and have been recorded as a unit force per unit length of weld as in Table 8-9.

An alternative solution to this problem is to calculate the principal stresses and the maximum shear stresses using the formulas of Chapter 3 instead of determining $f_{r}$ and $q_{r}$ and to check if the weld is strong enough by applying a failure theory discussed in Chapter 3. However, this method is more time consuming.

Example 8.7 Weld Joint Determine the size of the required fillet weld for the bracket shown in Fig. 8-19, which carries a vertical load of 6000 lb .

Choose a [-shaped weld pattern. The weld will be subjected to direct vertical shear and twisting caused by the eccentric load $P$. Using case 5 of Table $8-10$ gives the geometric properties of the weld treated as a line:

$$
\begin{aligned}
A_{w} & =(2 b+d) t=[(2)(5)+8] t=18 t \mathrm{in}^{2} \\
J_{x} & =\frac{(2 b+d)^{3} t}{12}-\frac{b^{2}(b+d)^{2} t}{2 b+d} \\
& =\frac{(18)^{3} t}{12}-\frac{(25)(13)^{2} t}{18}=251.3 t \mathrm{in}^{4} \\
\bar{y} & =\frac{b^{2}}{2 b+d}=\frac{5^{2}}{18}=1.39 \mathrm{in} .
\end{aligned}
$$

Substitute these geometric properties into the proper formulas from Table 8-11 according to the types of loading to find the various forces on the weld.

The stress due to vertical shear is

$$
\begin{equation*}
f_{z}^{\prime}=\frac{P_{z}}{A_{w}}=\frac{P}{A_{w}}=\frac{6000}{18 t}=\frac{333.3}{t} \quad \mathrm{lb} / \mathrm{in}^{2} \tag{1}
\end{equation*}
$$

The twisting moment is

$$
\begin{align*}
T & =M_{x}=-P L=-P[6+(5-\bar{y})]=-(6000)(6+5-1.39) \\
& =-57,660 \mathrm{lb} / \mathrm{in} \tag{2}
\end{align*}
$$



Figure 8-19: Weld joint for Example 8.7. Point $c$ is the centroid of the weld pattern.

The moment $M_{x}$ causes a force to be exerted on the weld that is perpendicular to a radial line from the centroid of the weld pattern to the point of interest. The maximum combined stresses occur at point $G$ (Fig. 8-19):

$$
\begin{align*}
& f_{y}^{\prime \prime}=-\frac{M_{x}}{J_{x}} z_{G}=-\frac{(-57,660)(-4)}{251.3 t}=-\frac{918}{t} \quad \mathrm{lb} / \mathrm{in}^{2} \\
& f_{z}^{\prime \prime}=\frac{M_{x}}{J_{x}} y_{G}=\frac{(-57,660)(-5+1.39)}{251.3 t}=\frac{828}{t} \quad \mathrm{lb} / \mathrm{in}^{2} \tag{3}
\end{align*}
$$

Superimpose the stress components,

$$
\begin{align*}
& f_{x}=0  \tag{4}\\
& f_{y}=0+f_{y}^{\prime \prime}=-\frac{918}{t} \quad \mathrm{lb} / \mathrm{in}^{2}  \tag{5}\\
& f_{z}=f_{z}^{\prime}+f_{z}^{\prime \prime}=\frac{333.3+828}{t}=\frac{1161.3}{t} \quad \mathrm{lb} / \mathrm{in}^{2} \tag{6}
\end{align*}
$$

so that the nominal resultant stress becomes

$$
\begin{align*}
& f_{r}=\frac{\sqrt{(-918)^{2}+1161.3^{2}}}{t}=\frac{1480}{t} \quad \mathrm{lb} / \mathrm{in}^{2}  \tag{7}\\
& q_{r}=f_{r} t=1480 \mathrm{lb} / \mathrm{in} . \tag{8}
\end{align*}
$$

From (7) and (8) it is clear that for welded connections with uniform size, $q_{r}$ can be computed by considering $t=1$.

Suppose that the base metals of the welded joints are ASTM A36 steel. From Table 8-9, if an E60 electrode is chosen for the welding, the allowable shear stress is $13,600 \mathrm{psi}$ so that $f_{r}=1480 / t \leq 13,600 \mathrm{psi}$. Then the throat width is $t \geq$ $1480 / 13,600 \mathrm{in}$. Finally, the required leg size of the fillet weld connecting the bracket is

$$
w=\frac{t}{0.707}=\frac{1480}{13,600 \times 0.707}=0.154 \mathrm{in} .
$$

Note that if the base-metal parts are thick plates, the leg size obtained above should be specified according to Table 8-8.

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TABLE 8-1 ALLOWABLE STRESSES (SHEARING AND BEARING CAPACITIES) IN RIVETS AND BOLTS (ksi) ${ }^{a}$

| Description of Fasteners | Allowable Tension, ${ }^{b} \sigma_{\text {tw }}$ | Allowable Shear, ${ }^{b} \tau_{w}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Slip-Critical Connections ${ }^{c, d}$ |  |  |  | Bearing Connections ${ }^{d}$ |
|  |  |  | Oversized and Short-Slotted Holes | Long-Slotted Holes |  |  |
|  |  | Standard-Size Holes |  | Transverse ${ }^{e}$ Load | Parallel ${ }^{e}$ Load |  |
| A502, grade 1, hot-driven rivets | $23.0{ }^{\text {f }}$ | - | - | - | - | $17.5{ }^{\text {g }}$ |
| A502, grades 2 and 3, hot-driven rivets | $29.0{ }^{f}$ | - | - | - | - | $22.0{ }^{\text {g }}$ |
| A307 bolts | $20.0{ }^{\text {f }}$ | - | - | - | - | $10.0^{8, h}$ |
| Threaded parts meeting the requirements of Secs. A3.1 and A3.4 and A449 bolts meeting the requirements of Sec. A3.4, when threads are not excluded from shear planes | $0.33 \sigma_{u}^{f, i, j}$ | - | - | - | - | $0.17 \sigma_{u}{ }^{j}$ |
| Threaded parts meeting the | $0.33 \sigma_{u}^{f, j}$ | - | - | - | - | $0.22 \sigma_{u}{ }^{j}$ | requirements of Secs. A3.1 and A3.4 and A449 bolts meeting the requirements of Sec. A3.4 when threads are excluded from shear planes


| A325 bolts, when threads are not |  |  |  |  |  |  |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
| excluded from shear planes <br> A325 bolts, when threads are <br> excluded from shear planes | $44.0^{k}$ | 17.0 | 15.0 | 12.0 | 10.0 | $21.0^{g}$ |
| A490 bolts, when threads are not <br> excluded from shear planes | $44.0^{k}$ | $54.0^{k}$ | 17.0 | 15.0 | 12.0 | 10.0 |

${ }^{a}$ From Ref. [8.1] with permission. Citations in this table (e.g., Sec. A3.4) are from this reference.
${ }^{b}$ See Sec. A5.2.
${ }^{c}$ Class A (slip coefficient 0.33 ). Clean mill scale and blast-cleaned surfaces with class A coatings. When specified by the designer, the allowable shear stress, $\tau_{w}$, for slip-critical connections having special faying surface conditions may be increased to the applicable value given in the RCSC Specification.
${ }^{d}$ For limitations on use of oversized and slotted holes, see Sec. J3.2.
${ }^{e}$ Direction of load application relative to long axis of slot.
${ }^{f}$ Static loading only.
${ }^{g}$ When bearing-type connections used to splice tension members have a fastener pattern whose length, measured parallel to the line of force, exceeds 50 in., tabulated values shall be reduced by $20 \%$.
${ }^{h}$ Threads permitted in shear planes.
${ }^{i}$ The tensile capacity of the threaded portion of an upset rod, based on the cross-sectional area at its major thread diameter, $A_{b}$, shall be larger than the nominal body area of the rod before upsetting times $0.60 \sigma_{y s}$.
${ }^{j}$ See Table 2, Numerical Values Section, for values for specific ASTM steel specifications.
${ }^{k}$ For A325 and A490 bolts subject to tensile fatigue loading, see Appendix K4.3.

## TABLE 8-2 REDUCTION COEFFICIENT Ua

Shape, ${ }^{b}$ Number of Fasteners per
${ }^{a}$ Adapted from AISC [8.1].
${ }^{b} b_{f}$, flange width; $h$, member depth.

## TABLE 8-3 MINIMUM DISTANCE FROM CENTER OF STANDARD HOLE TO EDGE OF CONNECTED PART ${ }^{a}$

| Nominal Rivet or <br> Bolt Diameter (in.) | At Sheared <br> Edges (in.) | Shapes, Bars, Gas-Cut <br> or Saw-Cut Edges ${ }^{b}$ (in.) |
| :---: | :---: | :---: |
| $\frac{1}{2}$ | $\frac{7}{8}$ | $\frac{3}{4}$ |
| $\frac{5}{8}$ | $1 \frac{1}{8}$ | $\frac{7}{8}$ |
| $\frac{3}{4}$ | $1 \frac{1}{4}$ | 1 |
| $\frac{7}{8}$ | $1 \frac{1}{2}^{c}$ | $1 \frac{1}{8}$ |
| 1 | $1 \frac{3}{4}^{c}$ | $1 \frac{1}{4}$ |
| $1 \frac{1}{8}$ | 2 | $1 \frac{1}{2}$ |
| $1 \frac{1}{4}$ | $2 \frac{1}{4}$ | $1 \frac{5}{8}$ |
| Over $1 \frac{1}{4}$ | $1 \frac{3}{4} \times$ diameter | $1 \frac{1}{4} \times$ diameter |

[^0]
## TABLE 8-4 MODES OF FAILURE OF RIVETED AND BOLTED JOINTS

## Notation

$A=$ cross-sectional area of rivets or bolts
$A_{e}=$ effective area
$A_{n}=$ net sectional area
$A_{g}=$ gross area
$U=$ reduction factor (Table 8-2)
$\sigma_{e}=$ effective stress
$\sigma_{g}=$ stress based on gross area
$P=$ applied force
$d=$ diameter of rivets or bolts
$t=$ thickness of plate
$W_{n}=$ net width
$\sigma_{\mathrm{br}}=$ bearing stress
$\sigma_{u}=$ ultimate tensile strength
$\tau=$ shear stress

| Mode of Failure | Connection | Strength Formula |
| :---: | :---: | :---: |
| 1. <br> Fastener shearing | a. Single shear | $\tau=\frac{P}{A}=\frac{4 P}{\pi d^{2}}$ |
|  | b. Double shear | $\tau=\frac{P}{2 A}=\frac{2 P}{\pi d^{2}}$ |
| 2. <br> Bearing |  | $\sigma_{\mathrm{br}}=\frac{P}{t d}$ |

TABLE 8-4 (continued) MODES OF FAILURE OF RIVETED AND BOLTED JOINTS

| 3. |
| :--- |
| Tension |
| or tearing |
| End |
| 4. Straight with no stagger |

## TABLE 8-5 SAE GRADES OF STEELS FOR BOLTS

| Grade Number | $\begin{aligned} & \text { Bolt Size } \\ & \text { (in.) } \end{aligned}$ | Tensile <br> Strength <br> (ksi) | Yield Strength (ksi) | Proof Strength ${ }^{a}$ (ksi) | Head Marking |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | $\frac{1}{4}-1 \frac{1}{2}$ | 60 | 36 | 33 | None |
| 2 | $\frac{1}{4}-\frac{3}{4}$ | 74 | 57 | 55 | None |
|  | Over $\frac{3}{4}-1 \frac{1}{2}$ | 60 | 36 | 33 | - |
| 4 | $\frac{1}{4}-1 \frac{1}{2}$ | 115 | 100 | 65 | None |
| 5 | $\frac{1}{4}-1$ | 120 | 92 | 85 | $\sum$ |
|  | Over 1-1 $\frac{1}{2}$ | 105 | 81 | 74 |  |
|  | Over $\frac{1}{2}-3$ | 90 | 58 | 55 |  |
| 5.2 | $\frac{1}{4}-1$ | 120 | 92 | 85 | 0 |
| 7 | $\frac{1}{4}-1 \frac{1}{2}$ | 133 | 115 | 105 | $\theta$ |
| 8 | $\frac{1}{4}-1 \frac{1}{2}$ | 150 | 130 | 120 | E |

[^1]TABLE 8-6 AMERICAN STANDARD THREAD DIMENSIONS ${ }^{a}$

| Size | Basic Major Diameter (in.) | Threads per Inch ${ }^{b}$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | Coarse Threads UNC | Fine Threads UNF | Extra Fine UNEF |
| 0 | 0.0600 | - | 80 |  |
| 1 | 0.0730 | 64 | 72 |  |
| 2 | 0.0860 | 56 | 64 |  |
| 3 | 0.0990 | 48 | 56 |  |
| 4 | 0.1120 | 40 | 48 |  |
| 5 | 0.1250 | 40 | 44 |  |
| 6 | 0.1380 | 32 | 40 |  |
| 8 | 0.1640 | 32 | 36 |  |
| 10 | 0.1900 | 24 | 32 |  |
| 12 | 0.2160 | 24 | 28 | 32 |
| $\frac{1}{4}$ | 0.2500 | 20 | 28 | 32 |
| $\frac{5}{16}$ | 0.3125 | 18 | 24 | 32 |
| $\frac{3}{8}$ | 0.3750 | 16 | 24 | 32 |
| $\frac{7}{16}$ | 0.4375 | 14 | 20 | 28 |
| $\frac{1}{2}$ | 0.5000 | 13 | 20 | 28 |
| $\frac{9}{16}$ | 0.5625 | 12 | 18 | 24 |
| $\frac{5}{8}$ | 0.6250 | 11 | 18 | 24 |
| $\frac{3}{4}$ | 0.7500 | 10 | 16 | 20 |
| $\frac{7}{8}$ | 0.8750 | 9 | 14 | 20 |
| 1 | 1.000 | 8 | 12 | 20 |
| $1 \frac{1}{8}$ | 1.125 | 7 | 12 | 18 |
| $1 \frac{1}{4}$ | 1.250 | 7 | 12 | 18 |
| $1 \frac{3}{8}$ | 1.375 | 6 | 12 | 18 |
| $1 \frac{1}{2}$ | 1.500 | 6 | 12 | 18 |
| $1 \frac{3}{4}$ | 1.750 | 5 | - | 18 |
| 2 | 2.000 | $4 \frac{1}{2}$ | - | - |

${ }^{a}$ The tensile stress area $A_{t}$ is given by

$$
A_{t}=0.785\left(d-\frac{0.9743}{n}\right)^{2}
$$

where $d$ is the basic major diameter and $n$ is the number of threads per inch.
${ }^{b}$ UNC, unified coarse; UNF, unified fine; UNEF, unified extrafine. The smaller American Standard threads use a number designation from 0 to 12 . The larger sizes use fractional inch designations.


Single fillet


Closed square butt joint ( $\frac{1}{8}$-in. penetration both sides)


Double V-groove


Single V-groove (complete penetration; welded both sides)


Closed square butt joint (complete penetration both sides)


Double U-groove


Outside single bevel corner joint, fillet weld


Open square-grooved corner joint, fillet weld


Double-fillet corner joint



Single-V-corner joint, fillet weld



Single-U corner joint, fillet weld


Double J-groove (full penetration)


Open square butt joint ( $\frac{1}{8}$-in. root opening; complete penetration both sides)


Double V-groove (Full penetration)


Double fillet; 2-in. welds on 5 -in. centers opposite increments

${ }^{a}$ Dimensions of figures are given in inches.

## TABLE 8-8 MINIMUM WELD SIZES FOR THICK PLATES

|  | Plate Thickness |  |  | Minimum Leg Size $(w)$ <br> for Fillet Weld |  |
| :--- | :--- | :---: | :---: | :---: | :---: |
|  | mm |  | mm |  |  |
| in. | $\leq 12.7$ | $\frac{3}{16}$ | 4.76 |  |  |
| $\leq \frac{1}{2}$ | $>12.7-19.1$ | $\frac{1}{4}$ | 6.35 |  |  |
| $>\frac{1}{2}-\frac{3}{4}$ | $>19.1-38.1$ | $\frac{5}{16}$ | 7.94 |  |  |
| $>\frac{3}{4}-1 \frac{1}{2}$ | $>38.1-57.2$ | $\frac{3}{8}$ | 9.53 |  |  |
| $>1 \frac{1}{2}-2 \frac{1}{4}$ | $>57.2-152.4$ | $\frac{1}{2}$ | 12.70 |  |  |
| $>2 \frac{1}{4}-6$ | $>152.4$ | $\frac{5}{8}$ | 15.88 |  |  |
| $>6$ |  |  |  |  |  |

TABLE 8-9 ALLOWABLE SHEAR STRESSES AND FORCES ON WELDS

|  |  | Allowable <br> Shear Stress |  |  |
| :---: | :---: | :---: | :---: | :---: | | Allowable Force |
| :---: |
| per Inch of Leg |
| Base-Metal |

## TABLE 8-10 GEOMETRIC PROPERTIES OF WELD SEAMS

## Notation

$$
\begin{aligned}
M & =\text { applied bending moment } \\
J_{x} & =\text { polar moment of inertia } \\
T & =\text { twisting moment } \\
Z_{\text {ew }} & =\text { elastic section modulus } \\
& \text { of the weld seam } \\
t & =\text { width }(=1)
\end{aligned}
$$

Dimensions of Weld Bending
2.

$A_{w}=2 d$

$A_{w}=2 b$
b

$Z_{\text {ew }}=\frac{1}{3} d^{2}$

$J_{x}=\frac{\left(3 b^{2}+d^{2}\right) d}{6}$

$Z_{\text {ew }}=b d$

$J_{x}=\frac{1}{6}\left(b^{3}+3 b d^{2}\right)$

TABLE 8-10 (continued) GEOMETRIC PROPERTIES OF WELD SEAMS
4.

5.

$A_{w}=d+2 b$
$y_{c}=\frac{b^{2}}{2 b+d}$


$$
J_{x}=\frac{1}{12}(2 b+d)^{3}-\frac{b^{2}(b+d)^{2}}{(2 b+d)}
$$


$A_{w}=b+2 d$
$z_{c}=\frac{d^{2}}{b+2 d}$

$$
Z_{\mathrm{ew}}=b d+\frac{1}{6} d^{2}
$$



At top: $Z_{\text {ew }}=\frac{1}{3}\left(2 b d+d^{2}\right)$

$\begin{aligned} z_{c} & =\frac{d^{2}}{2(b+d)} \\ A_{w} & =b+d\end{aligned}$
$\begin{aligned} z_{c} & =\frac{d^{2}}{2(b+d)} \\ A_{w} & =b+d\end{aligned}$
At bottom: $Z_{\text {ew }}=\frac{d^{2}(2 b+d)}{3(b+d)}$

$$
\begin{aligned}
J_{x}= & \frac{1}{12}(b+2 d)^{3} \\
& -\frac{d^{2}(b+d)^{2}}{(b+2 d)}
\end{aligned}
$$



At top: $Z_{\text {ew }}=\frac{1}{6}\left(4 b d+d^{2}\right)$
At bottom: $Z_{\text {ew }}=\frac{(4 b+d) d^{2}}{6(2 b+d)}$

$J_{x}=\frac{d^{3}(4 b+d)+b^{3}(b+d)}{12(b+d)}$

TABLE 8-10 (continued) GEOMETRIC PROPERTIES OF WELD SEAMS

9.

$A_{w}=2 d+2 b$
$Z_{\text {ew }}=b d+\frac{1}{3} d^{2}$
$J_{x}=\frac{1}{6}\left(b^{3}+3 b d^{2}+d^{3}\right)$

$A_{w}=\pi d$

$Z_{\text {ew }}=\frac{1}{4} \pi d^{2}$

$J_{x}=\frac{1}{4} \pi d^{3}$

## TABLE 8-11 FORMULAS FOR DETERMINING STRESSES IN WELDED JOINTS

## Notation

$f_{x}^{\prime}, f_{y}^{\prime}, f_{z}^{\prime}=$ stress components of $x, y, z$ direction due to external forces (Fig. 8-18)
$f_{x}^{\prime \prime}, f_{y}^{\prime \prime}, f_{z}^{\prime \prime}=$ stress components of $x, y, z$ direction due to external moments
$f_{x}, f_{y}, f_{z}=$ algebraic sum of stress components in $x, y, z$ direction
$f_{r}=$ nominal resultant stress
$q_{r}=$ resultant force per unit length
$A_{w}=$ effective welded area
$I_{y}, I_{z}=$ moments of inertia of welded area
$J_{x}=$ polar moment of inertia
$P_{x}, P_{y}, P_{z}=$ applied forces in $x, y, z$ direction
$M_{x}=T, M_{y}, M_{z}=$ applied moments in $x, y, z$ direction
$\tau_{w}=$ allowable shear stress
$t=$ effective throat dimension
Stress due to forces: $f_{x}^{\prime}=P_{x} / A_{w} \quad f_{y}^{\prime}=P_{y} / A_{w} \quad f_{z}^{\prime}=P_{z} / A_{w}$
Stress due to moments: $\quad f_{x}^{\prime \prime}=\frac{M_{y}}{I_{y}} z-\frac{M_{z}}{I_{z}} y \quad f_{y}^{\prime \prime}=-\frac{M_{x}}{J_{x}} z \quad f_{z}^{\prime \prime}=\frac{M_{x}}{J_{x}} y$
Sum of stress components: $\quad f_{x}=f_{x}^{\prime}+f_{x}^{\prime \prime} \quad f_{y}=f_{y}^{\prime}+f_{y}^{\prime \prime} \quad f_{z}=f_{z}^{\prime}+f_{z}^{\prime \prime}$
Nominal resultant stress: $\quad f_{r}=\sqrt{f_{x}^{2}+f_{y}^{2}+f_{z}^{2}}$
Resultant force per unit of length: $\quad q_{r}=t f_{r}$
Design criterion: $\quad f_{r} \leq \tau_{w}$


[^0]:    ${ }^{a}$ From AISC [8.1], with permission. For oversized or slotted holes, see Table J3.6 of AISC.
    ${ }^{b}$ All edge distances in this column may be reduced $\frac{1}{8}$ in. when the hole is at a point where stress does not exceed $25 \%$ of the maximum design strength in the element.
    ${ }^{c}$ These may be $1 \frac{1}{4} \mathrm{in}$. at the ends of beam connection angles.

[^1]:    ${ }^{a}$ Defined as stress at which bolt will undergo permanent deformation: usually ranges between 0.90 and 0.95 times yield strength.

