5 Resolution and Accuracy

Resolution is a measurement for how close or dense targets may be to each other, while still being able to separately detect them. Under the term resolution the ability is therefore understood for the separation of neighbouring objects. Accuracy is a measure for how large the tolerances for the determination of the information parameters, such as distance, speed, etc. are. The accuracy describes thus, with which relative or absolute errors the measured variable can be determined. Measurement uncertainty result from:

- Imprecise measurement equipment
- Quantization errors
- Noise
- Signal distortion.

For an example a sine wave oscillation with be analyzed.

5.1 Measurement Accuracy and Sine Wave Oscillations

As shown in Figure 5.1, when one observes a sine wave, with superimposed noise n(t), so differs the observed amplitude from the true amplitude around $\Delta U = n(t)$

$$u(t) = U_0 \sin(\omega t + \varphi_0) + n(t) \tag{5.1}$$

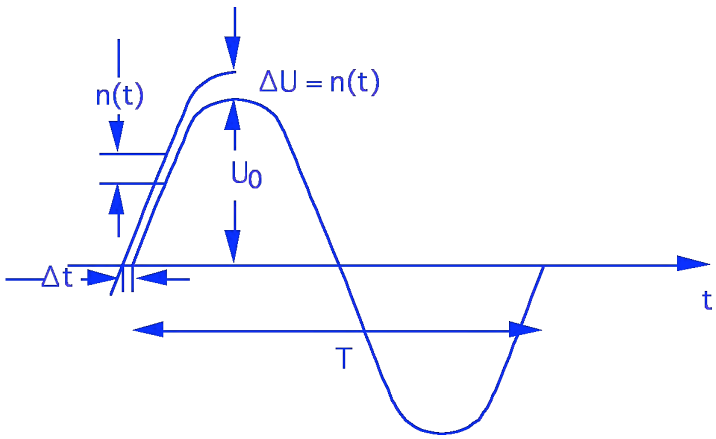

Figure 5.1 Sine wave oscillation with superimposed noise.

Amplitude measurement error: The root-mean-square error is

$$\partial U = \sqrt{\overline{n^2}} \tag{5.2}$$

The relative error then becomes

$$\frac{\partial U}{U} = \frac{\sqrt{\overline{n^2}}}{\sqrt{U_0^2}} = \frac{1}{\sqrt{\frac{(\sqrt{2}U_{eff})^2}{\overline{n^2}}}} = \frac{1}{\sqrt{2S/N}} \tag{5.3}$$

Time measurement error: The time error Δt according to Figure 5.1 is

$$\Delta t = \frac{n(t)}{slope} \tag{5.4}$$

For the sine function the is (at zero crossover crucial for the measurement) ω U_0, from which follows:

$$\partial t = \sqrt{\overline{\Delta t^2}} = \frac{\sqrt{\overline{n^2}}}{\omega U_o} = \frac{\sqrt{\overline{n^2}}}{\omega \sqrt{2} U_{eff}} = \frac{1}{\omega \sqrt{2S/N}} \tag{5.5}$$

Phase error: The phase error results directly:

$$\partial \phi = 2\pi f \cdot \partial t = \frac{1}{\sqrt{2S/N}} \tag{5.6}$$

Period error: The period error ∂T (RMS) is larger than ∂t by around a factor of $\sqrt{2}$, since two passes are to be measured.

$$\partial T = \frac{\sqrt{2}}{\omega \sqrt{2S/N}} = \frac{T}{2\pi \sqrt{S/N}} \tag{5.7}$$

The relative errors for the period and the frequency are then:

$$\frac{\partial T}{T} = \frac{1}{2\pi \sqrt{S/N}} \tag{5.8}$$

$$\frac{\partial f}{f} = \frac{1}{2\pi \sqrt{S/N}} \tag{5.9}$$

5.2 Accuracy and Range Measurement

For calculating the accuracy of the range measurement one replaces the sine wave with a video pulse as in Figure 5.2.

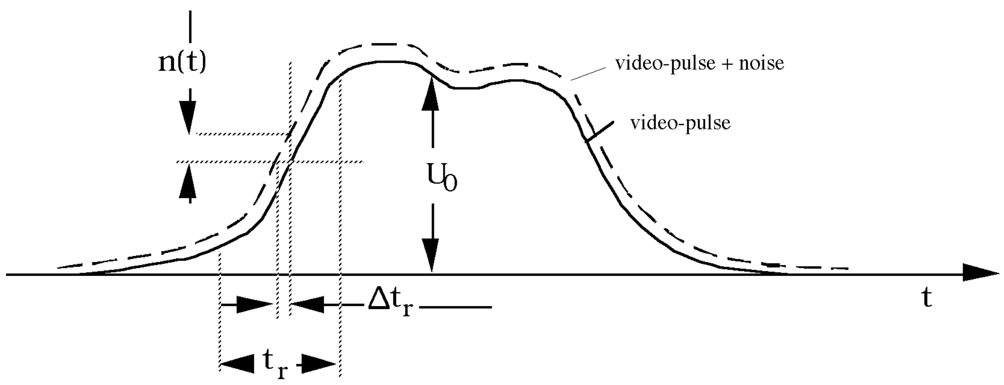

Figure 5.2 Video pulse and added noise with edge triggering.

The calculation of Δt_r is performed similar to equation (5.5)

$$\Delta t_r = \frac{t_r}{\sqrt{U_0^2/n^2}} = \frac{t_r}{\sqrt{2S/N}} \tag{5.10}$$

t_r is the linear rise time from 10% up to 90% of U_0. Δt_r is the measurement accuracy for the edge triggering. With that yields the range accuracy:

$$\Delta R = c_0 \cdot \Delta t_r / 2 \tag{5.11}$$

The other equations from above can be applied in analogous way.

5.3 Range Resolution of Two Neighbouring Targets

By the term resolution it is understood the differentiation of targets, which are in very close proximity to on another, as shown in Figure 5.3

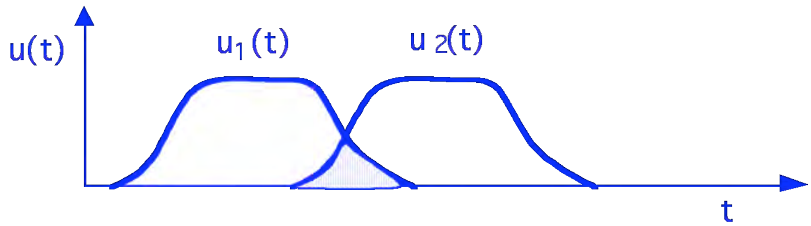

Figure 5.3 Echoes of two neighbouring objects.

$u_1(t)$ indicates the echo of target 1 and $u_2(t)$ the echo of target 2. Then for resolution optimization the integral must become a maximum.

$$\int (u_1(t) - u_2(t))^2 dt = \int \left[u_1^2(t) - 2u_1(t)u_2(t) + u_2^2(t)\right] dt \tag{5.12}$$

The integral will become a maximum when the negative part becomes minimum, thus

$$\int u_1(t)u_2(t) = \min imum \tag{5.13}$$

Assuming that both signals result from the same or similar shape (same reflection cross section) then their transit times being t_1 & t_2 yield on the left side of the equation (5.13):

$$\int u(t-t_1)u(t-t_2) \tag{5.14}$$

This integral represents the auto-correlation function of the signal with itself. For optimizing the resolution, the shape of the pulse should be optimized. The ideal result would be:

$$\int = 0 \;\; f\ddot{u}r\; t_1 \neq t_2; \int = 1 \;\; f\ddot{u}r\; t_1 = t_2 \tag{5.15}$$

If the targets are different in size, then one should bear equation (5.14) in mind. The auto-correlation will then become a cross-correlation.

With typical specifications of the threshold value for the cross-correlation, the range resolution ΔR can be approximately calculated as illustrated in Figure 5.4

$$\Delta R = c_0 \cdot \tau / 2 \tag{5.16}$$

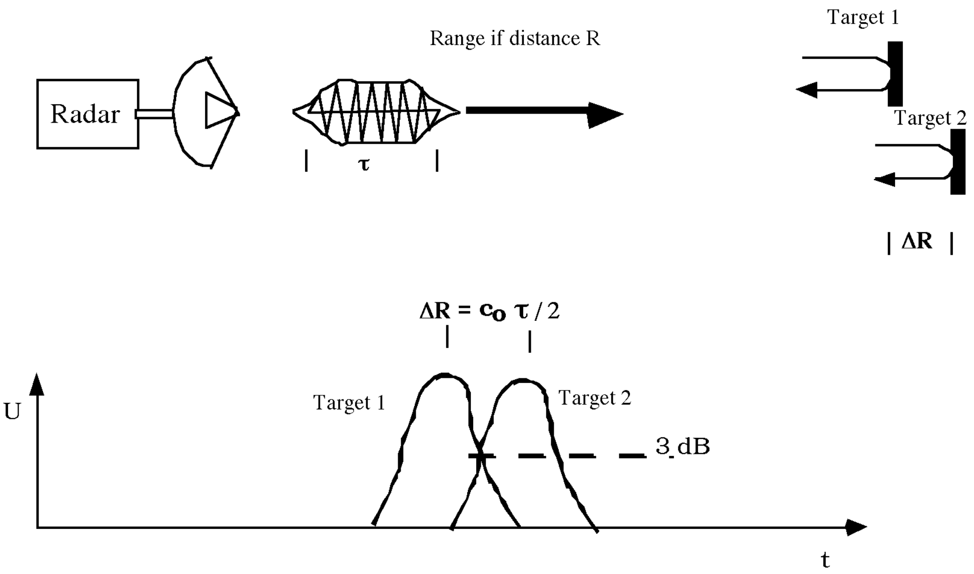

Figure 5.4 Range resolution of two targets.

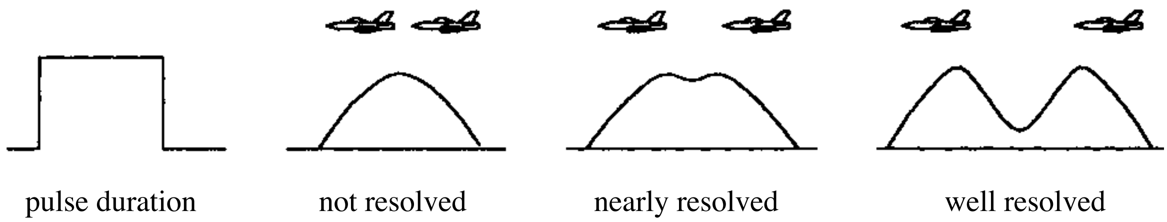

Figure 5.5 Example for the ability to resolve objects at differing ranges & separation distances.

5.4 Angular Resolution

The angular resolution can be calculated in a similar way as the range resolution, if pulse echoes are replaced with the maximum of the directivity. As a good approximation the half-power beamwidth γ_{BW} of the antenna can be utilized. This is illustrated in Figure 5.6.

$$\Delta x = R \cdot \gamma_{BW} \tag{5.17}$$

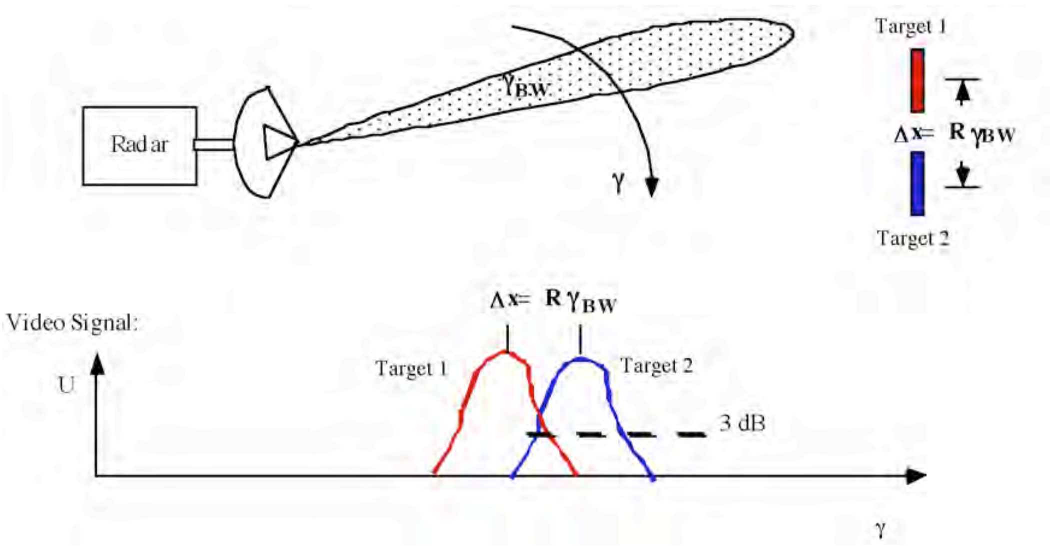

Figure 5.6 Angular resolution of two targets.

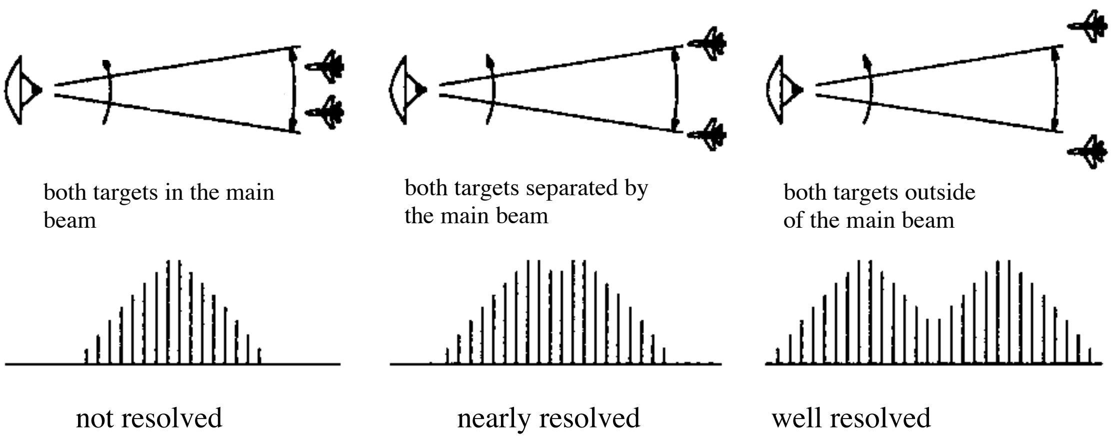

Figure 5.7 Further examples for angular resolution.