Torque equation

- The torque produced by three phase induction motor depends upon the following three factors:

1.The part of rotating magnetic field which reacts with rotor and is responsible to produce induced e.m.f. in rotor.

2. The magnitude of rotor current in running condition.

3. The power factor of the rotor circuit in running condition.

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Torque equation

- Combining all these factors, we get the equation of torque as-

$$T \propto \phi I_2 \cos\theta_2$$

- Where,

  T is the torque produced by the induction motor,

  ϕ is flux responsible for producing induced emf,

  I_2 is rotor current,

  $\cos\theta_2$ is the power factor of rotor circuit.

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Torque equation

- The flux ϕ produced by the stator is proportional to stator emf E_1. i.e

$$\phi \propto E_1$$

We know that transformation ratio K is defined as the ratio of secondary voltage (rotor voltage) to that of primary voltage (stator voltage).

$$K = \frac{E_2}{E_1}$$

$$or,\ K = \frac{E_2}{\phi}$$

$$or,\ E_2 = \phi$$

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Torque equation

- Rotor current I_2 is defined as the ratio of rotor induced emf under running condition , sE_2 to total impedance, Z_2 of rotor side,

$$i.e\ I_2 = \frac{sE_2}{Z_2}$$

- and total impedance Z_2 on rotor side is given by ,

$$Z_2 = \sqrt{R_2^2 + (sX_2)^2}$$

- Putting this value in above equation we get,

$$I_2 = \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

s = slip of Induction motor

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Torque equation

- We know that power factor is defined as ratio of resistance to that of impedance. The power factor of the rotor circuit is

$$\cos\theta_2 = \frac{R_2}{Z_2} = \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

- Putting the value of flux ϕ, rotor current I_2, power factor $\cos\theta_2$ in the equation of torque we get,

$$T \propto E_2 \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

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Maximum Torque Condition

- In the equation of torque,

$$T = \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \times \frac{3}{2\pi n_s}$$

So, for torque to be maximum,

$$\frac{dT}{ds} = 0$$

$$T = K s E_2^2 \frac{R_2}{R_2^2 + (sX_2)^2}$$

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Maximum Torque Condition

$$\frac{dT}{ds} = \frac{(k\,s\,E_2^2\,R_2)\,\frac{d}{ds}\,(R_2^2 + s^2\,X_2^2) - (R_2^2 + s^2\,X_2^2)\,\frac{d}{ds}\,(k\,s\,E_2^2\,R_2)}{(R_2^2 + s^2\,X_2^2)^2} = 0$$

$$k\,s\,E_2^2\,R_2\,[2s\,X_2^2] - (R_2^2 + s^2\,X_2^2)\,(k\,E_2^2\,R_2) = 0$$

$$2\,s^2\,k\,X_2^2\,E_2^2\,R_2 - R_2^2\,k\,E_2^2\,R_2 - k\,s^2\,X_2^2\,E_2^2\,R_2 = 0$$

$$k\,s^2\,X_2^2\,E_2^2\,R_2 - R_2^2\,k\,E_2^2\,R_2 = 0$$

$$s^2X_2^2 - R_2^2 = 0$$ Taking $k\,E_2^2\,R_2$ common.

$$s^2 = \frac{R_2^2}{X_2^2}$$

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Maximum Torque Condition

- So, when slip $s = R_2 / X_2$, the torque will be maximum and this slip is called maximum slip Sm ,and it is defined as the ratio of rotor resistance to that of rotor reactance.
- The equation of torque is

$$T = \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

- The torque will be maximum when slip $s = R_2 / X_2$,Substituting the value of this slip in above equation we get the maximum value of torque as,

$$T_{max} = K \frac{{E_2}^2}{2X_2} \quad N-m$$

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Maximum Torque Condition

- From the above equation it is concluded that The maximum torque is directly proportional to square of rotor induced emf at the standstill.
- The maximum torque is inversely proportional to rotor reactance.
- The maximum torque is independent of rotor resistance.
- The slip at which maximum torque occur depends upon rotor resistance, R_2. So, by varying the rotor resistance, maximum torque can be obtained at any required slip.

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