Topic Name – Hydrostatic forces on Surface

Hydrostatic
- Hydro – water
- Static – rest Condition

- this chapter deals with fluid (i.e liquid and gases) at rest.
- So, we can say there will be no relative motion b/w adjacent or neighbouring layer.
- the velocity graident, which is equal to the change in velocity b/w two adjacent layer divided by distance b/w layer: will be zero.

So we can say $\boxed{\frac{du}{dy} = 0}$

due to this reason Shear Stress are applied will is also equal to zero $\boxed{\tau = \mu \frac{du}{dy} = 0}$

Then the force acting on the fluid Particle will be:-
(1) due to Pressure of fluid normal to the Surface
(2) due to gravity (Self wt of fluid Particle)

$\bar{h}^*$ = distance of centre of Pressure from free Surface of liquid.

Total Pressure :-

For determine total Pressure on the Surface dividing the entire Surface into a number of Small Parallel Strips.

The force on Small Strip is calculated and the total Pressure force on the whole area is calculated. by integrating the force on a Small Strip.

Consider a Strip of thickness dh, and width b, at a depth from free Surface of liquid

we know that

Pressure intensity on the Strip $p = \rho g h$

Area of Strip $dA = b \times dh$

total Pressure force on Strip $dF = p \times \text{Area}$

$$= \rho g h \times b \times dh$$

total Pressure force on the whole Surface

$$F = \int dF$$

$$= \int \rho g h \times b \times dh$$

$$= \rho g \int b \times h \times dh$$

So, we can Say

$\int b \times h \times dh = \int h \times dA$ = moment of Surface area about the free Surface of liquid

= Area of Surface × distance of C.G from free Surface

$= A \times \bar{h}$

$$\boxed{F = \rho g A \bar{h}}$$

- Centre of Pressure h^* :-

this is calculated by using "Principle of moments"

which states that the moment of the resultant force about an axis is equal to the sum of moments of the components about the same axis

So, the resultant force F is acting at P, at a distance h^* from free surface of the liquid (Acc^n to above fig).

Hence moment of the force F about the free surface of the liquid $= F \times h^*$ —①

Moment of force dF, acting on a strip about free surface of liquid $= dF \times h$ $(\because dF = \rho g h \times b \times dh)$

$= \rho g h \times b \times dh \times h$

Sum of moments of all such forces about free surface of liquid

$= \cancel{\int \rho g h} \int \rho g h \times b \times dh \times h$

$= \rho g \int b \times h \times h\, dh$

$= \rho g \int h^2 \cdot b \cdot dh = \rho g \int h^2 \cdot dA$

But $\int h^2 \cdot dA = \int b \cdot h^2 \cdot dA$

= moment of inertia of the Surface about free Surface of liquid

$= I_0$

∴ Sum of moments about free Surface

$= \rho g I_0$ — (2)

equating eqn (1) & (2), we get

$$F \times h^* = \rho g I_0$$

$$\rho g A \bar{h} \times h^* = \rho g I_0$$

$$h^* = \frac{\rho g I_0}{\rho g A \bar{h}} = \frac{I_0}{A\bar{h}} \quad \text{— (3)}$$

By theorem of Parallel axis,

$$I_0 = I_G + A \times \bar{h}^2$$

I_G = Moment of intertia of area about an axis passing through the C.G of the area and parallel to the free Surface of liquid

Putting the value of I_0 in eqn = (3)

$$h^* = \frac{I_G + A\bar{h}^2}{A\bar{h}^2} = \boxed{\frac{I_G}{A\bar{h}} + \bar{h}}$$

Numerical Problem

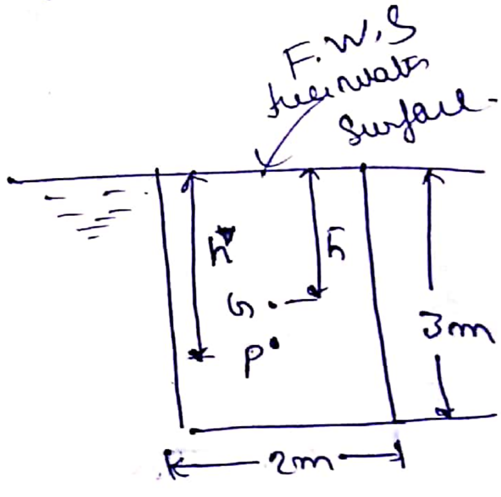
Q A rectangular plane surface is 2m wide and 3m deep. it lies in vertical plane in water. determine the total pressure and position of centre of pressure on the plane surface when its upper edge is horizontal and (a) coincide with surface (b) 2.5m below the free surface of water

Sol. given that

$b = 2m$

$d = 3m$

(a) upper edge coincide with water surface



$F = \rho g A\bar{h}$

$\rho = 1000\ kg/m^3$, $g = 9.81\ m/s^2$

$A = 3\times2 = 6m^2$

$\bar{h} = \frac{1}{2}\times3 = 1.5m$

$F = 1000\times9.81\times6\times1.5$

$\boxed{F = 88290\ N}$

Depth of centre of pressure is given by

$$h^* = \frac{I_G}{A\bar{h}} + \bar{h} \qquad \left\{ I = \frac{bd^3}{12} = \frac{2\times3^3}{12} = 4.5m^4 \right\}$$

$$\boxed{h^* = \frac{4.5}{6\times1.5} + 1.5 = 2m}$$