

Q. Find $\frac{dy}{dx}$ if $y = \sec^2\left(\frac{x}{a}\right)$.

Solution. $\frac{dy}{dx} = \frac{d}{dx} \left(\sec^2 \frac{x}{a} \right) = 2 \sec\left(\frac{x}{a}\right) \cdot \frac{d}{dx} \left(\sec \frac{x}{a} \right)$

$$= 2 \sec\left(\frac{x}{a}\right) \cdot \sec \frac{x}{a} \cdot \tan \frac{x}{a} \cdot \frac{d}{dx} \left(\frac{x}{a} \right)$$

$$= 2 \sec^2\left(\frac{x}{a}\right) \cdot \tan\left(\frac{x}{a}\right) \cdot \left(\frac{1}{a}\right)$$

$$= \frac{2}{a} \sec^2\left(\frac{x}{a}\right) \cdot \tan\left(\frac{x}{a}\right) \leftarrow \text{Ans.}$$

Q. Find $\frac{d}{dx} \left(\frac{1}{\sin^2 x + 1} \right)$.

Solution. Suppose $y = \frac{1}{1 + \sin^2 x} = (1 + \sin^2 x)^{-1}$

$$\therefore \frac{dy}{dx} = \frac{d}{dx} (1 + \sin^2 x)^{-1} = (-1)(1 + \sin^2 x)^{-2} \frac{d}{dx} (1 + \sin^2 x)$$

$$= (-1)(1 + \sin^2 x)^{-2} \left\{ 0 + 2 \sin x \frac{d}{dx} (\sin x) \right\}$$

$$= (-1)(1 + \sin^2 x)^{-2} \{ 2 \sin x \cos x \}$$

$$= (-1)(1 + \sin^2 x)^{-2} \{ \sin 2x \}$$

($\because 2 \sin x \cdot \cos x = \sin 2x$)

$$= \frac{-\sin 2x}{(1 + \sin^2 x)^2} \leftarrow \text{Ans.}$$

Unit 5 \rightarrow

Integration:

Q. Find $\int x^4 dx$.

Solution. Suppose $I = \int x^4 dx$

$$= \frac{x^{4+1}}{4+1} = \frac{x^5}{5} \leftarrow \text{Ans.}$$

Q. Find $\int \frac{x^3 - 1}{x^2} dx$

Solution. Suppose $I = \int \frac{x^3 - 1}{x^2} dx$

or $I = \int \left[\frac{x^3}{x^2} - \frac{1}{x^2} \right] dx$

$= \int x dx - \int x^{-2} dx$

$= \frac{x^{1+1}}{1+1} - \frac{x^{-2+1}}{-2+1} + C \left[\because \int x^n dx = \frac{x^{n+1}}{n+1} \right]$

$= \frac{x^2}{2} - \frac{x^{-1}}{-1} + C$

$= \frac{x^2}{2} + \frac{1}{x} + C \quad \leftarrow \text{Ans.}$

Q. Evaluate $\int (x^{2/3} + 1) dx$

Solution. Suppose $I = \int (x^{2/3} + 1) dx$

or $I = \int x^{2/3} dx + \int 1 dx$

$= \frac{x^{\frac{2}{3}+1}}{\frac{2}{3}+1} + x + C \quad \left[\because \int dx = x \right]$

$= \frac{x^{\frac{2+3}{3}}}{\frac{2+3}{3}} + x + C$

$= \frac{x^{5/3}}{5/3} + x + C$

$= \frac{3}{5} x^{5/3} + x + C. \quad \leftarrow \text{Ans.}$

Q. Find $\int (x^{3/2} + 2e^x - \frac{1}{x}) dx$

Solution. Suppose $I = \int (x^{3/2} + 2e^x - \frac{1}{x}) dx$

$$\begin{aligned}
\text{or } I &= \int x^{3/2} dx + 2 \int e^x dx - \int \frac{1}{x} dx \\
&= \frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1} + 2e^x - \log x + C \\
&= \frac{x^{\frac{3+2}{2}}}{\frac{3+2}{2}} + 2e^x - \log x + C \\
&= \frac{x^{5/2}}{5/2} + 2e^x - \log x + C \\
&= \frac{2}{5} x^{5/2} + 2e^x - \log x + C \leftarrow \text{Ans.}
\end{aligned}$$

Q. Evaluate $\int (\sin x + \cos x) dx$.

Solution. $\int (\sin x + \cos x) dx = \int \sin x dx + \int \cos x dx$
 $= -\cos x + \sin x + C$

\leftarrow Ans.

Q. Evaluate $\int \operatorname{cosec} x (\operatorname{cosec} x + \cot x) dx$

Solution. Suppose $I = \int \operatorname{cosec} x (\operatorname{cosec} x + \cot x) dx$

$$\begin{aligned}
\text{or } I &= \int (\operatorname{cosec}^2 x + \operatorname{cosec} x \cdot \cot x) dx \\
&= -\cot x - \operatorname{cosec} x + C \leftarrow \text{Ans.}
\end{aligned}$$

Q. Find $\int \frac{1 - \sin x}{\cos^2 x} dx$.

Solution. Suppose $I = \int \frac{1 - \sin x}{\cos^2 x} dx$

$$\text{or } I = \int \left(\frac{1}{\cos^2 x} - \frac{\sin x}{\cos^2 x} \right) dx$$

$$= \int (\sec^2 x - \tan x \cdot \sec x) dx$$

$$\text{or } I = \tan x - \sec x + C \leftarrow \text{Ans.}$$

Q. Evaluate $\int \sin 2x$.

$$\text{Solution } \int \sin 2x = -\frac{\cos 2x}{2} + C \leftarrow \text{Ans.}$$

Integration by substitution:

Q. Evaluate $\int \frac{2x}{1+x^2} dx$.

$$\text{Solution. Suppose } I = \int \frac{2x}{1+x^2} dx \quad \text{--- (1)}$$

$$\text{Put } 1+x^2 = t$$

$$\Rightarrow 0 + 2x dx = dt \quad (\text{On differentiating})$$

$$\text{or } 2x dx = dt$$

Then by (1),

$$I = \int \frac{dt}{t}$$

$$= \log t + C \quad (\because \int \frac{1}{x} dx = \log x)$$

$$= \log(1+x^2) + C \quad (\because t = 1+x^2)$$

$\leftarrow \text{Ans.}$

Q. Evaluate $\int \frac{x}{9-4x^2} dx$

$$\text{Solution. Let } I = \int \frac{x}{9-4x^2} dx \quad \text{--- (1)}$$

$$\text{Put } 9-4x^2 = t$$

$$\Rightarrow 0 - 8x dx = dt \quad (\text{On differentiating})$$

$$\Rightarrow -8x dx = dt$$

$$\Rightarrow x dx = -\frac{dt}{8}$$

Then by (1),

$$\begin{aligned} I &= \int \frac{-dt/8}{t} = -\frac{1}{8} \int \frac{dt}{t} = -\frac{1}{8} \log t + C \\ &= -\frac{1}{8} \log(9 - 4x^2) + C \end{aligned}$$

Q. Evaluate $\int \frac{x}{e^{x^2}} dx$

Solution. Let $I = \int \frac{x}{e^{x^2}} dx$ — (1)

Put $x^2 = t$

$$\Rightarrow 2x dx = dt$$

$$\text{or } x dx = dt/2$$

Then by (1),

$$\begin{aligned} I &= \int \frac{dt/2}{e^t} = \frac{1}{2} \int \frac{dt}{e^t} = \frac{1}{2} \int e^{-t} dt \\ &= \frac{1}{2} \frac{e^{-t}}{(-1)} + C = -\frac{1}{2} e^{-t} + C = -\frac{1}{2} e^{-x^2} + C \leftarrow \text{Ans} \end{aligned}$$

Q. Evaluate $\int \frac{e^{\tan^{-1}x}}{1+x^2} dx$

Solution. Suppose $I = \int \frac{e^{\tan^{-1}x}}{1+x^2} dx$ — (1)

Put $\tan^{-1}x = t$

$$\Rightarrow \frac{1}{1+x^2} dx = dt$$

Then by (1),

$$I = \int e^t \cdot dt = e^t + C = e^{\tan^{-1}x} + C \leftarrow \text{Ans.}$$