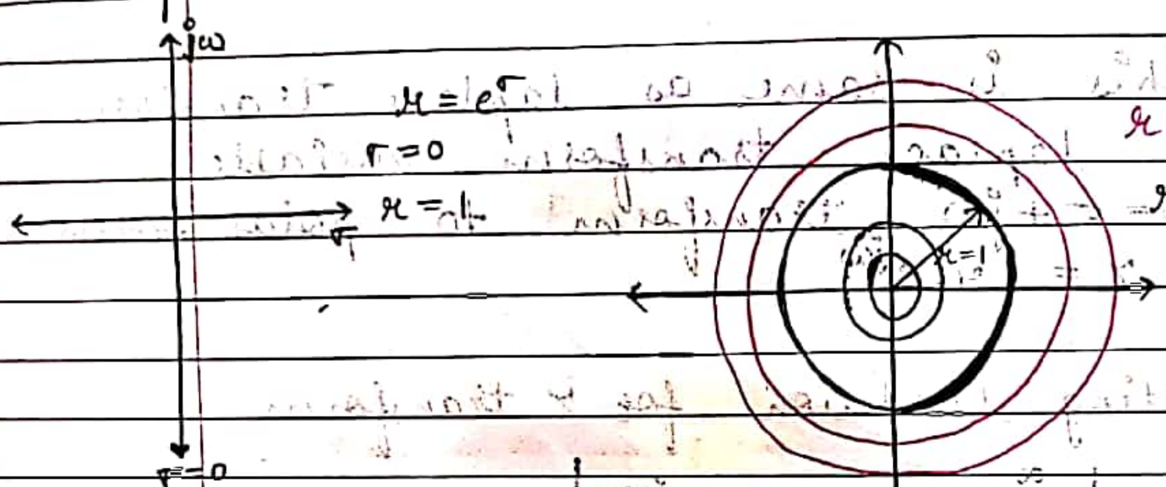
ROC can be specified in terms of r or |z|

ROC of Z-transform

Splane   Z- plane



$\sigma = 1$

$r = e^{1} = 2.7$

$r = e^{2}$

$\sigma = -1$

$r = e^{-1} = \frac{1}{2.7}$

$r < 1$ {inward directing}

$r > 1$ {outward directing}

Q Find the z-transform of unit impulse sequence $\delta(n)$

$$f(n) = \delta(n)$$

$$f(z) = \sum_{n=-\infty}^{\infty} f(n)\, z^{-n}$$

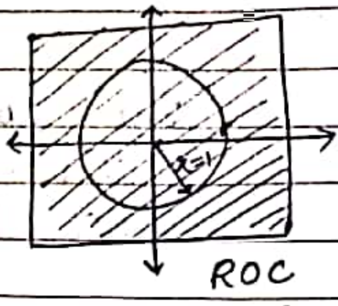
$$= \sum_{n=-\infty}^{\infty} \delta(n) \cdot z^{-n}$$

$$f(z) = \cdots 0 + 0 \cdots + \underset{\substack{\uparrow \\ \text{when } n=0}}{1} + 0 + 0 \cdots$$

$$\left.\begin{array}{l} \delta(n) = 1 \\ z^{-n} = 1 \end{array}\right\} \text{when } n=0$$

$$f(z) = 1$$

$$\sum_{n=-\infty}^{\infty} \left| f(n)\, r^{-n} \right| < \infty$$



$$\sum_{n=-\infty}^{\infty} \left| \delta(n)\, r^{-n} \right| < \infty$$

r ranges from 0 to ∞

radius is never -ve

ROC = all possible values of r

Entire z-plane

Q. find the z transform of unit step sequence.

$$f(n) = u(n)$$

$$f(z) = \sum_{n=-\infty}^{\infty} u(n)\, z^{-n}$$

$$= \sum_{n=0}^{\infty} z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + z^{-4} + \cdots \infty$$

Sum of infinite G.P $= \frac{1}{1-r}$; $|r| < 1$

$= \infty$; $|r| > 1$

sum of infinite GP is possible when

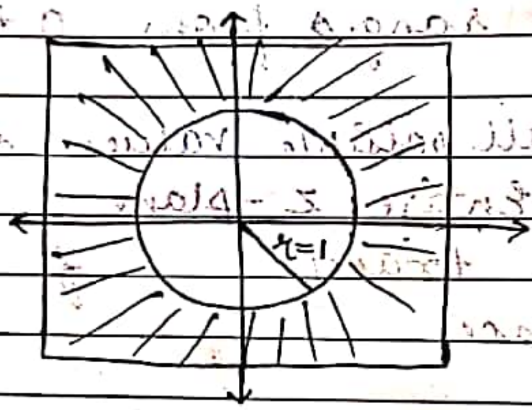
$$|z^{-1}| < 1$$

$$|z| > 1$$

ROC : $|x| > 1$ or $|z| > 1$

$$\therefore f(z) = \frac{1}{1 - z^{-1}} = \frac{z}{z-1}$$

The above series converges if $|z^{-1}| < 1$ i.e., $|z| > 1$ so the ROC is the exterior (outward) of the unit circle in the z-plane

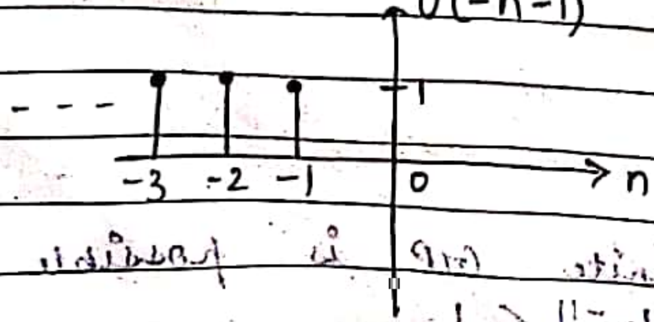


ROC of $u(n)$

$$u(n) \xrightarrow{z} \frac{z}{z-1} \quad \text{with } |z| > 1$$

Q. find the z transform of $u(-n-1)$.

$$f(n) = u(-n-1)$$



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$$u(-n-1) = \begin{cases} 1, & n<0 \\ 0, & n>-1 \end{cases}$$

$$f(z) = \sum_{n=-\infty}^{\infty} f(n)\, z^{-n}$$

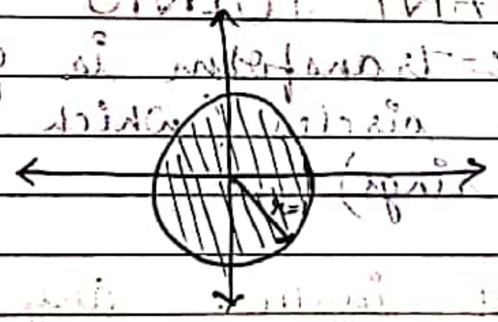
$$f(z) = \sum_{n=-\infty}^{\infty} u(-n-1)\, z^{-n}$$

$$= \sum_{n=-\infty}^{-1} z^{-n} = \sum_{n=1}^{\infty} z^{n}$$

$$= z + z^2 + z^3 + \cdots + \infty$$

ROC $= |z| < 1$; $r < 1$

$$f(z) = \frac{z}{1-z}$$



ROC (in z plane)

The above series converges if $|z|<1$ so the ROC is interior (inward) of the unit circle in the z plane

$u(-n-1) \xrightarrow{z} \frac{z}{1-z}$ with $|z|<1$.

Q. Find z transform of $-u(-n-1)$.

$$F(z) = \sum_{n=-\infty}^{\infty} -u(-n-1)\, z^{-n}$$

$$= -\sum_{n=-\infty}^{n=-1} z^{-n} = -\sum_{n=1}^{\infty} z^{n}$$

$$= -\left[\frac{z}{1-z}\right] = \frac{z}{z-1}$$

ROC $|z| < 1$

$-u(-n-1) \xrightarrow{z} \frac{z}{z-1}$ with $|z| < 1$

* ROC of $f(n)$ and $-f(n)$ is always same

~~Properties of ROC~~

IMPORTANT POINTS

- ROC of z-transform is generally a set of circles (which is in form of rings)

- ROC cannot include any pole and it is bounded by circle whose radius is equal to magnitude of pole.

- ROC of right sided signal is outward directed
  eg. $u(n)$