2. Evaluate the inverse z transform of $X(z)=\frac{1}{1-az^{-1}}, |z|>|a|$ using the complex inversion integral.

Long Division Method

The *z*-transform is a power series expansion,

$$X(z)=\sum_{n=-\infty}^{\infty} x(n)z^{-n} = \cdots + x(-2)\,z^{2} + x(-1)\,z + x(0) + x(1)\,z^{-1} + x(2)\,z^{-2} + \cdots$$

where the sequence values *x*(*n*)are the coefficients of z^{-n} in the expansion. Therefore, if we can find the power series expansion for *X*(*z*)*,* the sequence values *x*(*n*)may be found by simply picking off the coefficients of z^{-n}.

1. Sometimes the inverse transform of a given $X(z)$ can be obtained by long division.

$$X(z)=\frac{1}{1-az^{-1}}$$

By a long division, we get

$$1 \div (1-az^{-1}) = 1 + az^{-1} + a^2z^{-2} + \cdots$$

which converges if the ROC is $|z|>|a|$, i.e., $|az^{-1}|<1$ and we get

$$x[n]=a^n u[n]$$

. Alternatively, the long division can also be carried out as:

$$1 \div (-az^{-1}+1) = -a^{-1}z - a^{-2}z^2 - \cdots$$

which converges if the ROC is $|z|<|a|$, i.e., $|a^{-1}z|<1$ and we get

$$x[n]=-a^n u[-1-n]$$

2. To understand how an inverse Z Transform can be obtained by long division, consider the function

$$F(z) = \frac{z}{z-0.5}$$

If we perform long division

$$\begin{array}{r} 1 + 0.5z^{-1} + 0.25z^{-2} + \cdots \\ z - 0.5 \overline{\smash{\big)} z \qquad\qquad\qquad} \\ z - 0.5 \qquad\qquad \\ \hline 0.5 \qquad\quad \\ 0.5 - 0.25z^{-1} \\ \hline 0.25z^{-1} \\ 0.25z^{-1} - 0.125z^{-2} \\ \hline \vdots \end{array}$$

we can see that

$$F(z) = 1 + 0.5z^{-1} + 0.25z^{-2} + \cdots.$$

So the sequence f[k] is given by

$$f = \{1, 0.5, 0.25, \cdots\}$$

Upon inspection

$$f[k] = 0.5^k$$

3. Find the Inverse Z Transform using Long Division Method

$$F(z) = \frac{2z^2 + z}{z^2 - 1.5z + 0.5}$$

$$\begin{array}{r} 2 + 4z^{-1} + 5z^{-2} + \cdots \\ z^2 - 1.5z + 0.5 \overline{\smash{\big)} 2z^2 + z \qquad\qquad} \\ 2z^2 - 3z + 1 \qquad \\ \hline 4z - 1 \qquad \\ 4z - 6 + 2z^{-1} \\ \hline 5 - 2z^{-1} \\ 5 - 7.5z^{-1} + 2.5 \\ \hline \vdots \end{array}$$

$$F(z) = 2 + 4z^{-1} + 5z^{-2} + \cdots$$

and the sequence f[k] is given by $f = \{2, 4, 5, \cdots\}$

4. E(z) = $\frac{0.5}{(z-1)(z-0.6)}$

$$
\begin{array}{r|l}
 & 0.5z^{-2} + 0.8z^{-3} + 0.98z^{-4} + \dots \\
z^2 - 1.6z + 0.6 & 0.5 \\
 & \underline{0.5 - 0.8z^{-1} + 0.3z^{-2}} \\
 & 0.8z^{-1} - 0.3z^{-2} \\
 & \underline{0.8z^{-1} - 1.28z^{-2} + 0.48z^{-3}} \\
 & 0.98z^{-2} - 0.48z^{-3}
\end{array}
$$

e(0) = 0, e(1) = 0, e(2) = 0.5, …

Inverse Z Transform using Residue Method:

Find the solution using the formula

$$y[n] = Z^{-1}[Y(z)] = \sum_{i=1}^{k} \mathrm{Res}[Y(z)\, z^{n-1}, z_i]$$

where $z_1, z_2, \ldots, z_k$ are the poles of $f(z) = Y(z)\, z^{n-1}$.

Partial fraction method

Inverse Z Transform by Partial Fraction Expansion

This technique uses Partial Fraction Expansion to split up a complicated fraction into forms that are in the Z Transform table. As an example consider the function

$$F(z) = \frac{2z^2 + z}{z^2 - 1.5z + 0.5}$$

For reasons that will become obvious soon, we rewrite the fraction before expanding it by dividing the left side of the equation by "z."

$$\frac{F(z)}{z} = \frac{2z + 1}{z^2 - 1.5z + 0.5}$$

Now we can perform a partial fraction expansion

$$\frac{F(z)}{z} = \frac{2z+1}{z^2-1.5z+0.5}$$
$$= \frac{2z+1}{(z-1)(z-0.5)}$$
$$= \frac{A}{z-1} + \frac{B}{z-0.5}$$
$$= \frac{6}{z-1} + \frac{-4}{z-0.5}$$

These fractions are not in our table of Z Transforms. However if we bring the "z" from the denominator of the left side of the equation into the numerator of the right side, we get forms that are in the table of Z Transforms; this is why we performed the first step of dividing the equation by "z"

$$F(z) = 6\frac{z}{z-1} - 4\frac{z}{z-0.5}$$

So

$$f[k] = 6u[k] - 4 \cdot 0.5^k$$

or

$$f = \{2, 4, 5, 5.5, \cdots\}$$

PART - A

1. Distinguish between DFT and DTFT?
2. State and prove Parseval's relation for DFT.
3. Find the DFT of the sequence {0,1,0,1}
4. Define DFT and IDFT
5. Find IDFT of X(k) = {1, 0, 1, 0}.
6. Define Z transform
7. Mention the types of Z transform
8. Find the Z transform of u(n)
9. Define ROC
10. Mention the properties of ROC

PART – B

1. Explain the Properties of Z Transform
2. Explain the properties of DFT
3. Mention the Properties of DTFT
4. Find Inverse Z Transform for the following function using partial fraction method

$$X(z) = \frac{1}{\left(1-\frac{1}{4}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)} \qquad \text{ROC} : |z| > \frac{1}{2}$$

5. Find Inverse Z Transform for the following function using Long division method

$$X(z) = \frac{1 + 2z^{-1} + z^{-2}}{1 - \frac{3}{2}z^{-1} + \frac{1}{2}z^{-2}} = \frac{\left(1 + z^{-1}\right)^2}{\left(1 - \frac{1}{2}z^{-1}\right)\left(1 - z^{-1}\right)} \qquad |z| > 1$$

6. Find Inverse Z Transform for the following function using power series method

$$X(z) = z^2\left(1 - \frac{1}{2}z^{-1}\right)\left(1 + z^{-1}\right)\left(1 - z^{-1}\right)$$

$$= z^2 - \frac{1}{2}z - 1 + \frac{1}{2}z^{-1}$$