Inverse Z - Transform

$$f(n) \xrightarrow{z} F(z) = \sum_{n=-\infty}^{\infty} f(n) z^{-n}$$

$$F(z) \xrightarrow{\text{inverse } z} f(n)$$

$$f(n) = \frac{1}{2\pi j} \oint_C F(z) z^{n-1} \, dz$$

Find the inverse z transform of

$$F(z) = \frac{z^{-1}}{3-4z^{-1}+z^{-2}} \quad ; \text{ ROC } |z|>1$$

$$F(z) = \frac{1/z}{3-4/z+1/z^2}$$

remove the inverse power of z

$$= \frac{1/z}{\frac{3z^2-4z+1}{z^2}}$$

$$= \frac{z}{3z^2-4z+1}$$

$$\frac{F(z)}{z} = \frac{1}{3z^2-4z+1}$$

Bring $\frac{F(z)}{z}$ form

$$\frac{F(z)}{z} = \frac{1}{3(z-1)(z-\frac{1}{3})} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{3}} \quad \rightarrow (1)$$

$$A = \left. \frac{1}{3(z-\frac{1}{3})} \right|_{z=1}$$

If ROC is not given then By default take all the signals right sided

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$$A = \frac{1}{2}$$

$$B = \left.\frac{1}{3(z-1)}\right|_{z=\frac{1}{3}}$$

$$B = -\frac{1}{2}$$

from eq ①

$$\frac{F(z)}{z} = \frac{1/2}{(z-1)} - \frac{1/2}{(z-1/3)}$$

$$F(z) = \frac{1/2\ z}{(z-1)} - \frac{1/2\ z}{(z-1/3)}$$

$$= \frac{1}{2}\left[\frac{z}{z-1}\right] - \frac{1}{2}\left[\frac{z}{z-1/3}\right] \rightarrow ②$$

$|z| = 1$ $\quad$ $|z| = 1/3$

$r = 1$ $\quad$ $r = 1/3$

$r > 1$ (given)

right sided

$$f(n) = \frac{1}{2}(1)^n u(n) - \frac{1}{2}\left(\frac{1}{3}\right)^n u(n)$$

When ROC $|z| < \frac{1}{3}$

$$= \frac{1}{2}\left[\frac{z}{z-1}\right] - \frac{1}{2}\left[\frac{z}{z-1/3}\right]$$

$r < 1/3$ (given)

left sided

$$f(n) = -\frac{1}{2}(1)^n u(-n-1) + \frac{1}{2}\left(\frac{1}{3}\right)^n u(-n-1)$$

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When ROC $\frac{1}{3} < |z| < 1$

$$= \frac{1}{2}\left[\frac{z}{z-1}\right] - \frac{1}{2}\left[\frac{z}{z-1/3}\right]$$

$$\underbrace{\frac{1}{3}}_{\text{Right sided}} < |z| < \underbrace{1}_{\text{left sided}}$$

$$f(n) = -\frac{1}{2}(+1)^n u(-n-1) - \frac{1}{2}\left(\frac{1}{3}\right)^n u(n)$$

Q. find inverse z transform of

$$F(z) = \frac{\frac{1}{6}z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{3}z^{-1}\right)} \quad |z| > \frac{1}{2}$$

$$F(z) = \frac{1/6z}{\left(1-\frac{1}{2z}\right)\left(1-\frac{1}{3z}\right)}$$

$$= \frac{1/\cancel{6}z}{\frac{(2z-1)(3z-1)}{\cancel{6}z^2}}$$

$$= \frac{z}{(2z-1)(3z-1)}$$

$$\frac{F(z)}{z} = \frac{1}{6(z-1/2)(z-1/3)}$$

$$= \frac{A}{(z-1/2)} + \frac{B}{(z-1/3)}$$

$$A = \left.\frac{1}{6\cdot(z-1/3)}\right|_{z=1/2}$$

$$= 1$$

$$B = \left.\frac{1}{6(z-1/2)}\right|_{z=1/3}$$

$$= -1$$

$$\frac{F(z)}{z} = \frac{1}{(z-1/2)} - \frac{1}{(z-1/3)}$$

$$F(z) = \frac{z}{(z-1/2)} - \frac{z}{(z-1/3)}$$

$$|z| > 1/2$$

Right sided

$$= \left(\frac{1}{2}\right)^n u(n) - \left(\frac{1}{3}\right)^n u(n)$$

$$r < \frac{1}{3}$$

$$= -\left(\frac{1}{2}\right)^n u(-n-1) + \left(\frac{1}{3}\right)^n u(-n-1)$$

$$\frac{1}{3} < r < \frac{1}{2}$$

$$= -\left(\frac{1}{2}\right)^n u(-n-1) - \left(\frac{1}{3}\right)^n u(n)$$

Q. A causal system has input $x(n)$ & o/p $y(n)$ find the system function, frequency response & impulse response of the system if.

$$x(n) = \delta(n) + \frac{1}{6}\delta(n-1) - \frac{1}{6}\delta(n-2)$$

$$y(n) = \delta(n) - \frac{2}{3}\delta(n-1)$$

$$X(z) = 1 + \frac{1}{6z} - \frac{1}{6z^2}$$

$$Y(z) = 1 - \frac{2}{3z}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - \frac{2}{3z}}{1 + \frac{1}{6z} - \frac{1}{6z^2}}$$

$$= \frac{\frac{1}{\cancel{3z}}(3z - 2)}{\frac{1}{2\,\cancel{6z^2}}(6z^2 + z - 1)}$$

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$$H(z) = \frac{2z(3z-2)}{6z^2+z-1}$$

frequency response

$$H(e^{j\omega}) = \frac{2e^{j\omega}(3e^{j\omega}-2)}{6e^{2j\omega}+e^{j\omega}-1}$$

$$= \frac{2\cos\omega + 2j\sin\omega\,(3\cos\omega + j3\sin\omega + 2)}{6\cos 2\omega + 6j\sin 2\omega + \cos\omega + j\sin\omega - 1}$$

for $h(n)$.

$$H(z) := \frac{2z(3z-2)}{6z^2+z-1}$$

$$\frac{H(z)}{z} = \frac{2(3z-2)}{(6z^2+z-1)}$$

$$\frac{2(3z-2)}{(3z-1)(2z+1)}$$

$$= \frac{1/3(3z-2)}{(z-\frac{1}{3})(z+\frac{1}{2})}$$

$$\frac{H(z)}{z} = \frac{A}{(z-1/3)} + \frac{B}{(z+1/2)}$$

$$A = \left.\frac{1/3\,(3z-2)}{(z+\frac{1}{2})}\right|_{z=1/3}$$

$$A = \frac{-2}{5}$$

$$B = \left.\frac{\frac{1}{3}(3z-2)}{(z-\frac{1}{3})}\right|_{z=-\frac{1}{2}}$$

$$B = \frac{7}{5}$$

$$\frac{H(z)}{z} = \frac{-2}{5(z-\frac{1}{3})} + \frac{7}{5(z+\frac{1}{2})}$$

$$H(z) = \frac{-2}{5}\frac{z}{(z-\frac{1}{3})} + \frac{7}{5}\frac{z}{(z+\frac{1}{2})}$$

$$h(n) = \frac{-2}{5}\left(\frac{1}{3}\right)^n u(n) + \frac{7}{5}\left(-\frac{1}{2}\right)^n u(n)$$

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